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STABILITY OF SEMILINEAR DISCRETE INCLUSIONS IN A HILBERT SPACE

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Abstract. In this paper we consider a class of semilinear discrete (difference) inclusions in a Hilbert space. As is well-known, the discrete inclusions, in particular, are mathematical models of discrete-time switched systems. For the considered inclusions we suggest explicit exponential stability conditions. Our results are new even in the finite-dimensional case. An illustrative example is given.

Keywords: discrete inclusions; switched discrete-time systems; nonlinear systems; exponential stability; distributed parameter systems.

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1. Introduction and statement of the main result

This paper deals with the stability of discrete (difference) semilinear inclusions in a Hilbert space. Such inclusions, in particular, are mathematical models of discrete-time switched systems with distributed parameters, cf. [7].

The literature on the stability of discrete inclusions is rather rich, but mainly inclusions in the finite dimensional case have been investigated, cf. [1, 2], [8]-[14] and references given therein.

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At the same time, to the best of our knowledge, the stability theory of nonlinear infinite dimensional discrete inclusions is at early stage of the development. In the present paper we derive explicit exponential stability conditions for a class of discrete inclusions in a Hilbert space. We also obtain a norm estimate for the solutions of the considered inclusions. Our results are new even in the finite-dimensional case.

Let $\mathscr{H}$ be a complex separable Hilbert space, with a scalar product $(.,.)$, the norm $\|.\| = \sqrt{(.,.)}$ and unit operator I. By $\mathscr{B}(\mathscr{H})$ we denote the set of all bounded linear operators in $\mathscr{H}$. For an $A \in \mathscr{B}(\mathscr{H})$, $\sigma(A)$ is the spectrum, $\rho_s(A)$ is the spectral radius; A^* is the operator adjoint to A, and $\|A\| = \sup_{h\in\mathscr{H}} \|Ah\|/\|h\|$ is the operator norm. In addition, $A_I = (A - A^*)/2i$ and $N_2(A) := (\text{trace}\, A^*A)^{1/2}$ is the Hilbert-Schmidt norm of a Hilbert-Schmidt operator A. The ideal of the Hilbert-Schmidt operators is denoted by SN_2. For a positive number $r \le \infty$ put $\omega(r) = \{x \in \mathscr{H} : \|x\| \le r\}$.

Let $\mathscr{F} = \{f_1, ..., f_{m_1}\}$ be a finite set of mappings $f_j : \omega(r) \to \mathscr{H}$ $(j = 1, ..., m_1)$ and $\mathscr{A} = \{A_1, ..., A_m\}$ be a finite set of operators from $\mathscr{B}(\mathscr{H})$. As is well-known [7], a wide class of switched nonlinear systems can be described by the discrete inclusion

$$u_{k+1} \in \{Bu_k + f(u_k) : \ B \in \mathscr{A}, f \in \mathscr{F}\} \ \ (k = 0, 1, 2, ...) \tag{1.1}$$

with a given $u_0 \in \mathscr{H}$. Note that the existence of the solutions for differential and differential-difference inclusions requires that the nonlinearity satisfies some rigorous conditions, but the solutions of the considered discrete inclusion are defined recurrently directly from (1.1) and solution estimates derived below.

We assume that

$$\|f_j(x)\| \le \nu\|x\| \ \ (f_j \in \mathscr{F}, j = 1, ..., m_1; x \in \omega(r)) \tag{1.2}$$

with a non-negative constant ν independent of j.

The zero solution to inclusion (1.1) under condition (1.2) is said to be exponentially stable, if there are constants $d_0 \ge 1$, $d_1 \in (0, 1)$ and $d_2 > 0$, such that any solution u_k to (1.1) satisfies the inequality, $\|u_k\| \le d_1 d_2^k \|u_0\|$ $(k = 1, 2, ...)$, provided the condition $\|u_0\| < r/d_2$ holds. If $r = \infty$, then the last inequality can be omitted.

Let $A \in \mathscr{B}(\mathscr{H})$ be a Schur-Kohn stable operator, i.e. $\rho_s(A) < 1$. As is well-known, cf. [4], there exists a linear operator $X = X(A)$, such that

$$X - A^*XA = I. \tag{1.3}$$

Note that $\|X\| > 1$. Now we are in a position to formulate the main result of the paper.

Theorem 1.1. *Let condition (1.2) hold and there be an* $A \in \mathscr{B}(\mathscr{H})$ *with* $\rho_s(A) < 1$*, such that*

$$2\|A\|\|A - A_j\| + \|A - A_j\|^2 + 2\nu\|A_j\| + \nu^2 < 1 - \frac{1}{\|X\|} \ \ (A_j \in \mathscr{A}; j = 1, ..., m). \tag{1.4}$$

Then the zero solution to inclusion (1.1) is exponentially stable.

The proof of this theorem is presented in the next section. In particular, in (1.4) one can take $A = A_j \in \mathscr{A}$ for some index j.

Definition 1.2. *We will say that inclusion (1.1) is quasi-linear, if*

$$\overline{\lim}_{w \to 0} \|f_j(w)\|/\|w\| = 0 \ \ (f_j \in \mathscr{F}, j = 1, ..., m). \tag{1.5}$$

Corollary 1.3. *Let (1.1) be quasi-linear and there be an* $A \in \mathscr{B}(\mathscr{H})$ *with* $\rho_s(A) < 1$*, such that*

$$2\|A\|\|A - A_j\| + \|A - A_j\|^2 < 1 - \frac{1}{\|X\|} \ \ (A_j \in \mathscr{A}; j = 1, ..., m). \tag{1.6}$$

Then the zero solution to inclusion (1.1) is exponentially stable.

Indeed, according to (1.5) we have

$$\|f_j(w)\| \le \hat{\nu}(r)\|w\| \ \ (w \in \omega(r))$$

with a $\hat{\nu}(r) \to 0$ as $r \to 0$. So for a sufficiently small r we have condition (1.2) with a sufficiently small ν. Now Theorem 1.1 yields the required result.

2. Proof of Theorem 1.1

Rewrite (1.1) as the equation

$$u_{k+1} = B_k u_k + h_k(u_k) \ \ (k = 0, 1, ...), \tag{2.1}$$

where for each k, $B_k \in \mathscr{A}$ and $h_k(.) \in \mathscr{F}$. So for any k we have $B_k = A_j \in \mathscr{A}$ and $h_k(.) = f_{j_1}(.) \in \mathscr{F}$ for some $j \le m$, $j_1 \le m_1$.

Lemma 2.1. *Let condition (1.2) be fulfilled with $r = \infty$ and condition (1.4) hold. Then for any solution u_k of (2.1) one has*

$$(Xu_k, u_k) \le c_0^k (Xu_0, u_0) \ \ (k = 1, 2, ...), \tag{2.2}$$

where

$$c_0 = 1 - \frac{1}{\|X\|} + \max_{A_j \in \mathscr{A}} (2\|A\| \|A - A_j\| + \|A - A_j\|^2 + 2\nu \|A_j\|) + \nu^2 < 1.$$

Proof. Multiplying (2.1) by X and doing the scalar product, we have.

$$(Xu_{k+1}, u_{k+1}) = (X(B_k u_k + h_k(u_k)), B_k u_k + h_k(u_k)) = (XB_k u_k, B_k u_k) + \Phi_k(u_k),$$

where

$$\Phi_k(x) = (Xh_k(x), B_k x) + (XB_k(x), h_k(x)) + (Xh_k(x), h_k(x)) \quad (x \in \mathscr{H}).$$

But according to (1.3)

$$B_k^* X B_k = (A + Z_k)^* X (A + Z_k) = A^* X A + W_k = X - I + W_k,$$

where $Z_k = B_k - A$ and $W_k = Z_k^* X A + A^* X Z_k + Z_k^* X Z_k$. Thus,

$$(Xu_{k+1}, u_{k+1}) = (XB_k u_k, B_k u_k) + \Phi_k(u_k) = ((X - I + W_k) u_k, u_k) + \Phi_k(u_k)$$

$$\le (Xu_k, u_k) - (u_k, u_k) + \|W_k\| (u_k, u_k) + |\Phi_k(u_k)|. \tag{2.3}$$

Take into account that due to (1.2),

$$|\Phi_k(x)| \le \|X\| (2\|B_k\| \|h_k(x)\| \|x\| + \|h_k(x)\|^2) \le \|X\| (2\|B_k\| \nu + \nu^2) \|x\|^2$$

and, in addition,

$$\|W_k\| \le \|X\| (2\|A\| \|A - B_k\| + \|A - B_k\|^2) \ \ (B_k \in \mathscr{A}).$$

Thus $|\Phi_k(x)| + \|W_k\| \|x\|^2 \le b_k \|X\| \|x\|^2$, where

$$b_k = 2\|A\| \|A - B_k\| + \|A - B_k\|^2 + 2\nu \|B_k\| + \nu^2.$$

Since $X \ge I$, X is invertible. From (2.3), according to (1.4) it follows

$$(Xu_{k+1}, u_{k+1}) \le (Xu_k, u_k) - (1 - \|X\| b_k)(u_k, u_k) \le (Xu_k, u_k) - (1 - b_k \|X\|)(\frac{1}{\|X\|} Xu_k, u_k)$$

$$\leq (Xu_k,u_k)(1-\frac{1}{\|X\|}+2\|A\|\|A-B_k\|+\|A-B_k\|^2+\nu\|B_k\|+\nu^2)\leq c_0(Xu_k,u_k).$$

Hence (2.2) follows, as claimed. □

Lemma 2.2. *Under the hypothesis of Theorem 1.1, let*

$$\|u_0\| < \frac{r}{(\|X^{-1}\|\|X\|)^{1/2}}. \tag{2.4}$$

Then for any solution u_k of equation (2.1) one has

$$\|u_k\| \leq (\|X^{-1}\|\|X\|)^{1/2} c_0^{k/2}\|u_0\| \ \ (k=1,2,...). \tag{2.5}$$

Proof. First let $r=\infty$. We have

$$\frac{1}{\|X^{-1}\|}(u_k,u_k)=\frac{1}{\|X^{-1}\|}(X^{-1}Xu_k,u_k)\leq (Xu_k,u_k).$$

Hence and from the latter lemma we get

$$(u_k,u_k)\leq \|X^{-1}\|(Xu_k,u_k)\leq \|X^{-1}\|c_0^k(Xu_0,u_0).$$

This implies (2.5). So for $r=\infty$ the lemma is proved.

Now let $r<\infty$. By the Urysohn theorem [3, p. 15], there is a scalar-valued function ψ_r defined on $\mathscr{H}$, such that

$$\psi_r(w)=1 \ \ (w\in\mathscr{H}, \|w\|<r) \text{ and } \psi_r(w)=0 \ \ (\|w\|\geq r).$$

Put $h_{kr}(t,w)=\psi_r(h)h_k(w)$. Consider the equation

$$v_{k+1}=B_k v_{k+1}+h_{kr}(v_k), v(0)=u_0. \tag{2.6}$$

Besides, (1.2) yields the condition

$$\|h_{kr}(w))\|\leq \nu\|w\| \ \ (w\in\mathscr{H}; k\geq 0)$$

for some $j\leq m$. A solution v_k of equation (2.6) satisfies the estimate (2.5). Since $c_0<1$, according to (2.4) and (2.5), $\|v_k\|<(\|X^{-1}\|\|X\|)^{1/2}\|u_0\|<r$. So solutions of (2.6) and (2.1) under (2.4) coincide. This proves the required result. □

The assertion of Theorem 1.1 follows from Lemma 2.2.

3. Auxiliary results

In this section we suggest norm estimates for the powers and resolvents of some operators. In Section 4 these estimates give us bounds for solutions of the discrete Lyapunov equation (1.3) with concrete operators.

3.1. The finite dimensional case. . Let $\mathscr{H} = \mathbb{C}^n$-the n-dimensional Euclidean space. So $\mathscr{B}(\mathscr{H}) = \mathbb{C}^{n\times n}$-the set of $n\times n$-matrices. For an $A \in \mathbb{C}^{n\times n}$ put

$$g(A) = [N_2^2(A) - \sum_{k=1}^{n} |\lambda_k(A)|^2\,]^{1/2},$$

where $\lambda_k(A)\ \ (k = 1,...,n)$ are the eigenvalues of A, counted with their multiplicities and enumerated in an arbitrary order. The following relations are valid [6, Section 3.1]: $g^2(A) \le N_2^2(A) - |\text{trace } A^2|$. If A is a normal matrix: $AA^* = A^*A$, then $g(A) = 0$. Moreover, $g^2(A) \le 2N_2^2(A_I)$. Due to Example 3.3 from [6] for any $A \in \mathrm{SN}_2$ we have

$$\|A^j\| \le \sum_{k=0}^{j} \frac{j!\rho_s^{j-k}(A)g^k(A)}{(j-k)!(k!)^{3/2}} \ \ (j = 1,2,...).$$

Furthermore, by Theorem 3.2 from [6] for any $A \in \mathbb{C}^{n\times n}$ we have

$$\|R_\lambda(A)\| \le \sum_{k=0}^{n-1} \frac{g^k(A)}{(\text{dist}(A,\lambda))^{k+1}\sqrt{k!}} \ \ (\lambda \notin \sigma(A)).$$

3.2. Hilbert-Schmidt operators. In the infinite dimensional case for an $A \in \mathrm{SN}_2$ put

$$g(A) = [N_2^2(A) - \sum_{k=1}^{\infty} |\lambda_k(A)|^2\,]^{1/2},$$

where $\lambda_k(A)\ \ (k = 1,2,...)$ are the eigenvalues of A, counted with their multiplicities and enumerated in the non-increasing order of their absolute values. Moreover, in the infinite dimensional case also $g^2(A) \le 2N_2^2(A_I)$ and $g^2(A) \le N_2^2(A) - |\text{trace } A^2|$, cf [6, Section 7.1]: If A is a normal Hilbert-Schmidt operator: $AA^* = A^*A$, then $g(A) = 0$. Due to Corollary 7.4 from [6] for any $A \in \mathrm{SN}_2$ we have

$$\|A^j\| \le \sum_{k=0}^{j} \frac{j!\rho_s^{j-k}(A)g^k(A)}{(j-k)!(k!)^{3/2}} \ \ (j = 1,2,...). \tag{3.1}$$

Furthermore, by Theorem 7.1 from [6] for any $A \in \mathrm{SN}_2$ we have

$$\|R_\lambda(A)\| \le \sum_{k=0}^{\infty} \frac{g^k(A)}{(\text{dist}(A,\lambda))^{k+1}\sqrt{k!}} \ \ (\lambda \notin \sigma(A)). \tag{3.2}$$

By the Schwarz inequality

$$(\sum_{j=0}^{\infty} \frac{b^j g^j(A)}{b^j \sqrt{j!} x^j})^2 \le \sum_{k=0}^{\infty} b^{2k} \sum_{j=0}^{\infty} \frac{g^{2j}(A)}{b^{2j} j! x^{2j}} =$$

$$\frac{1}{1-b^2} \exp\,[\frac{g^2(A)}{b^2 x^2}] \;\; (x > 0, b \in (0,1)).$$

Taking $b^2 = 1/2$ and making use of (3.2), we arrive at the inequality

$$\|R_\lambda(A)\| \le \frac{\sqrt{2}}{\operatorname{dist}(A,\lambda)} \exp\,[\frac{g^2(A)}{(\operatorname{dist}(A,\lambda))^2}] \;\; (\lambda \notin \sigma(A)). \tag{3.3}$$

Similarly, making use of Theorems 7.2 and7.3 from [6] one can consider the Shatten-von Neumann operators.

3.3. Operators with Hilbert-Schmidt components. In this subsection we suppose that $A_I = (A - A^*)/(2i) \in \mathrm{SN}_2$. Introduce the quantity

$$g_I(A) := \sqrt{2}\left[N_2^2(A_I) - \sum_{k=1}^{\infty} (\Im\, \lambda_k(A))^2\right]^{1/2} \;\; (\Im\, \lambda_k(A) = \frac{1}{2i}(\lambda_k(A) - \overline{\lambda_k}(A)).$$

Obviously, $g_I(A) \le \sqrt{2} N_2(A_I)$. Due to Example 10.2 from [6],

$$\|A^j\| \le \sum_{k=0}^{j} \frac{j!\rho_s^{j-k}(A) g_I^k(A)}{(j-k)!(k!)^{3/2}} \;\; (A_I \in \mathrm{SN}_2; \mathrm{j} = 1,2,...). \tag{3.4}$$

Furthermore, by Theorem 9.1 from [6] under condition (3.9) we have,

$$\|R_\lambda(A)\| \le \sum_{k=0}^{\infty} \frac{g_I^k(A)}{(\operatorname{dist}(A,\lambda))^{k+1}\sqrt{k!}} \tag{3.5}$$

and

$$\|R_\lambda(A)\| \le \frac{\sqrt{e}}{\operatorname{dist}(A,\lambda)} \exp\,[\frac{g_I^2(A)}{2(\operatorname{dist}(A,\lambda))^2}] \;\; (\lambda \notin \sigma(A)). \tag{3.6}$$

Some other classes of operators can be considered, in particular, via norm estimates for operator functions from [6].

4. Solution estimates for the discrete Lyapunov equation

As it is well-known (see for instance [4], [5, Sec. 7.1]), if $\rho_s(A)<1$, then for any $C\in\mathscr{B}(\mathscr{H})$, there exists a linear operator $X=X(A,C)$, such that

$$X-A^*XA=C. \tag{4.1}$$

Moreover,

$$X(A,C)=\sum_{k=0}^{\infty}(A^*)^kCA^k \tag{4.2}$$

and

$$X(A,C)=\frac{1}{2\pi}\int_0^{2\pi}(Ie^{-i\omega}-A^*)^{-1}C(Ie^{i\omega}-A)^{-1}d\omega. \tag{4.3}$$

Due to representations (4.2) and (4.3) we have

$$\|X(A,C)\|\le\|C\|\sum_{k=0}^{\infty}\|A^k\|^2. \tag{4.4}$$

and

$$\|X(A,C)\|\le\frac{\|C\|}{2\pi}\int_0^{2\pi}\|(e^{it}I-A)^{-1}\|^2dt,$$

respectively. From the latter inequality it follows

$$\|X(A,C)\|\le\|C\|\sup_{|z|=1}\|(zI-A)^{-1}\|^2. \tag{4.5}$$

Let there be monotonically increasing non-negative continuous function $\phi(x)$ $(x\ge 0)$, such that $\phi(0)=0$, $\phi(\infty)=\infty$ and

$$\|(\lambda I-A)^{-1}\|\le\phi(1/\text{dist}(A,\lambda))\ \ (\lambda\not\in\sigma(A)),$$

where $\text{dist}(A,\lambda)=\inf_{s\in\sigma(A)}|s-\lambda|$. If $\rho_s(A)<1$ and $|z|=1$, then obviously, $\text{dist}(A,z)\ge 1-\rho_s(A)$ and therefore, $\|(Iz-A)^{-1}\|\le\phi(1/(1-\rho_s(A)))$. Now (4.5) implies

$$\|X(A,C)\|\le\|C\|\phi^2\left(\frac{1}{1-\rho_s(A)}\right). \tag{4.6}$$

In particular, if A is normal, then $\|A\|=\rho_s(A)$ and from (4.4.) it follows

$$\|X(A,C)\|\le\|C\|\frac{1}{1-\rho_s^2(A)}. \tag{4.7}$$

Relations (3.1) and (4.4) yield.

Corollary 4.1. *Let $A \in \mathrm{SN}_2$. Then*

$$\|X(A,C)\| \le \|C\| \sum_{j=1}^{\infty} \left(\sum_{k=0}^{j} \frac{j!\rho_s^{j-k}(A) g^k(A)}{(j-k)!(k!)^{3/2}} \right)^2 . \tag{4.8}$$

If A is normal, then (4.8) gives us inequality (4.7). Furthermore, (4.6) and (3.6) imply

$$\|X(A,C)\| \le \|C\| \frac{2}{(1-\rho_s(A))^2} \exp\, [\frac{g^2(A)}{(1-\rho_s(A))^2}] \quad (A \in \mathrm{SN}_2). \tag{4.9}$$

Relations (3.4) and (4.4) yield

Corollary 4.2. *Let $A_I \in \mathrm{SN}_2$. Then*

$$\|X(A,C)\| \le \|C\| \sum_{j=0}^{\infty} \left(\sum_{k=0}^{j} \frac{j!\rho_s^{j-k}(A) g^k(A)}{(j-k)!(k!)^{3/2}} \right)^2 . \tag{4.10}$$

If A is normal, then from (4.10) we get (4.7).

Furthermore, (4.5) along with (3.5) and (3.6) give us the inequalities

$$\|X(A,C)\| \le \|C\| \sum_{j=0}^{\infty} \left(\frac{g_I^j(A)}{\sqrt{j!}(1-\rho_s(A))^{j+1}} \right)^2 \quad (A_I \in \mathrm{SN}_2),$$

and

$$\|X(A,C)\| \le \|C\| \frac{e}{(1-\rho_s(A))^2} \exp\, [\frac{g_I^2(A)}{(1-\rho_s(A))^2}],$$

respectively. Similarly one can consider the finite dimensional case.

Let us point the lower bound for $X(A,C)$: if $C = C^* > 0$, then (4.1) gives us the inequality

$$(X(A,C)x,x) \ge \sum_{k=0}^{\infty} (CA^k x, A^k x) \ge \lambda_{min}(C) \sum_{k=0}^{\infty} \lambda_{min}((A^*)^k A^k)(x,x)$$

$$\ge \lambda_{min}(C)(1+\lambda_{min}(A^*A))(x,x) \quad (x \in \mathscr{H}).$$

Here $\lambda_{min}(C)$ means the smallest eigenvalue of C.

5. An example

Let $L^2 = L^2(0,1)$ be the space of square integrable complex functions defined on $[0,1]$. Consider the inclusion (1.1) with $\mathscr{H} = L^2(0,1)$, $(A_j u)(y) = a_j(y)u(y)$ and

$$[f_j(u)](y) = (\int_0^1 K_j(y,s)u(s)ds)^{2p} \quad (u \in L^2, j=1,2;\ p>1;\ y \in [0,1]),$$

where $a_j(y)$ are bounded measurable functions, $K_j(.,.) : [0,1]^2 \to \mathbb{C}$ are kernels satisfying the conditions

$$\eta_j := (\int_0^1 (\int_0^1 |K_j(y,s)|^2 ds)^{2p} dy)^{1/2} < \infty.$$

By the Schwarz inequality

$$|\int_0^1 K_j(y,s)u(s)ds|^2 \le \int_0^1 |K_j(y,s)|^2 ds \|u\|^2.$$

So

$$\|f_j(u)\| = (\int_0^1 |f_j(u)](y)|^2 dy)^{1/2} \le \|u\|^p \eta_j \ \ (u \in L^2).$$

Thus for any $r < \infty$ condition (1.2) holds with $\nu = r^{p-1} \max_{j=1,2} \eta_j$ and therefore the considered system is quasi-linear. Assume that

$$\rho_s(A_j) = \sup_y |a_j(y)| < 1 \ (j = 1,2).$$

We have $\|A_1 - A_2\| = q := \sup_y |a_1(y) - a_2(y)|$. Operators A_1, A_2 in the present example are normal and consequently, $\|A_1\| = \rho_s(A_1)$. With $A = A_1$, due to (4.7), we can write $\|X\| \le \frac{1}{1-\rho_s^2(A_1)}$. So if

$$2q\rho_s(A_1) + q^2 < \rho_s^2(A_1),$$

then making use of Corollary 1.3, we can assert that the considered inclusion is exponentially stable.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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