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SUBCLASS OF ANALYTIC FUNCTIONS DEFINED BY DIFFERENTIAL OPERATOR

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Abstract. This paper aims to explore a novel category of regular mapping characterized by negative coefficients in connection with the Differential operator within the unit disk. We will establish fundamental properties such as coefficient inequalities, extreme points, integral means inequalities and subordination results for this class.

Keywords: analytic; starlike; coefficient bound; extreme points; subordination.

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1. INTRODUCTION

Let A denote the class of all functions $u(z)$ of the form

$$u(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

in the open unit disc $E = \{z \in \mathbb{C} : |z| < 1\}$. Let S be the subclass of A consisting of univalent functions and satisfy the following usual normalization condition $u(0) = u'(0) - 1 = 0$. We denote by S the subclass of A consisting of functions $u(z)$ which are all univalent in E. A

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function $u \in A$ is a starlike function of the order $\upsilon, 0 \leq \upsilon < 1$, if it satisfy

$$\Re\left\{\frac{zu'(z)}{u(z)}\right\} > \upsilon, (z \in E). \tag{2}$$

We denote this class with $S^*(\upsilon)$.

A function $u \in A$ is a convex function of the order $\upsilon, 0 \leq \upsilon < 1$, if it satisfy

$$\Re\left\{1 + \frac{zu''(z)}{u'(z)}\right\} > \upsilon, (z \in E). \tag{3}$$

We denote this class with $K(\upsilon)$.

Let T denote the class of functions analytic in E that are of the form

$$u(z) = z - \sum_{n=2}^{\infty} a_n z^n,\ a_n \geq 0\ (z \in E) \tag{4}$$

and let $T^*(\upsilon) = T \cap S^*(\upsilon)$, $C(\upsilon) = T \cap K(\upsilon)$. The class $T^*(\upsilon)$ and allied classes possess some interesting properties and have been extensively studied by Silverman [23] and others.

Differential operators in a complex domain play a significant role in functions theory and its information. They have used to describe the geometric interpolation of analytic functions in a complex domain. Also, they have utilized to generate new formulas of holomorphic functions. Lately, Lupas [15] presented a amalgamation of two well-known differential operators prearranged by Ruscheweyh [19] and Salagean [20]. Later, these operators are investigated by researchers considering different classes and formulas of analytic functions [9, 10, 16, 18, 21, 22].

For $u \in A$ given by (1) and $g(z)$ given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \tag{5}$$

their convolution (or Hadamard product), denoted by $(u * g)$, is defined as

$$(u * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * u)(z)\ (z \in E). \tag{6}$$

Note that $u * g \in A$.

Salagean [20] introduced the following differential operator for $u(z) \in A$ which is called the

Salagean operator:

$$D^0u(z) = u(z)$$

$$D^1u(z) = Du(z) = zu'(z)$$

$$D^ku(z) = D(D^{k-1}u(z)) \qquad (k \in \mathbb{N} = 1,2,3\cdots)$$

We note that

$$D^ku(z) = z + \sum_{n=2}^{\infty} n^k a_n z^n \qquad (k \in \mathbb{N}_0 = \mathbb{N} \cup 0) \tag{7}$$

Recently, Komatu [12] introduced a certain integral operator L_a^δ defined by

$$L_a^\delta u(z) = \frac{a^\delta}{\Gamma(\delta)} \int_0^1 t^{a-2} \left(log \frac{1}{t} \right)^{\delta-1} u(zt)dt \ (a > 0,\ \delta \geq 0,\ u(z) \in A) \tag{8}$$

Thus, if $u(z) \in A$ is of the form (1), it is easily seen from (1.8) that [12]

$$L_a^\delta u(z) = z + \sum_{n=2}^{\infty} \left(\frac{a}{a+n-1} \right)^\delta a_n z^n \ (a > 0,\ \delta > 0.) \tag{9}$$

We note that:

(i). $L_a^0 u(z) = u(z)$

(ii). $L_1^1 u(z) = A[u](z)$ known as Alexander operator [3];

(iii). $L_2^1 u(z) = A[u](z)$ known as Liberal operator [13];

(iv). $L_{c+1}^1 u(z) = L_c[u](z)$ called Libera operator or Bernardi operator [7];

(v). For $a = 1$ and $\delta = k$ (k is any integer), the multiplier transformation $L_1^{-k} u(z) = I^k u(z)$ was studied by Flett [8] and Salgean [20];

(vi). For $a = 1$ and $\delta = -k$ $(k \in N_0 = N \cup 0,)$ the differential operator $L_1^{-k} u(z) = D^k u(z)$ was studied by Salagean [20];

(vii). For $a = 2$ and $\delta = k$ (k is any integer), $L_2^{-k} u(z) = L^k u(z)$ was studied by Uralegaddi and Somanatha [26];

(viii). $a = 2$, the multiplier transformation $L_2^\delta u(z) = I^\delta u(z)$ was studied by Jung et al [11].

For $D^k u(z)$ given by (1.7) and $L_a^\delta u(z)$ is given by (1.9), Arkan et al [6] defined the differential operator $D^k L_a^\delta u(z)$ as follows:

$$D^k L_a^\delta u(z) = z + \sum_{n=2}^{\infty} \aleph_k^{a,\delta}(n) a_n z^n$$

where

$$\aleph_k^{a,\delta}(n) = n^k \left(\frac{a}{a+n-1}\right)^{\delta} \tag{10}$$

Note that, by taking $\delta = 0$ and $k = 0$ in (1.10), the differential operator $D^k L_a^{\delta} u(z)$ reduces to Salagean differential operator and Komatu integral operator, respectely.

Using the operator $D^k L_a^{\delta} u$, we now introduce a new subclass of analytic functions as follows:

Inspired by the contributions of several researchers [1, 2, 7, 5, 17, 26, 25] , we propose a novel subclass of mapping within the broader class A.

Definition 1.1. For $\hbar \geq 0, 0 \leq \ell < 1$, we set $S_k^{a,\delta}(\hbar, \ell)$ be the subclass of A consisting of functions of the form (1) and satisfy

$$Re\left(\frac{D^k L_a^{\delta} u(z)}{z}\right) \geq \hbar \left| \left(D^k L_a^{\delta} u(z)\right)' - \frac{D^k L_a^{\delta} u(z)}{z} \right| + \ell \tag{11}$$

where $D^k L_a^{\delta} u(z)$ is given by (10).

We further let $TS_k^{a,\delta}(\hbar, \ell) = S_k^{a,\delta}(\hbar, \ell) \cap T$.

In this paper, we obtain coefficient inequalities, extreme points, integral means inequalities for the functions in the class $TS_k^{a,\delta}(\hbar, \ell)$ and also subordination results for the class of function $u \in S_k^{a,\delta}(\hbar, \ell)$.

2. Coefficient Estimates

Theorem 2.1. The function u defined by (1) is in the class $S_k^{a,\delta}(\hbar, \ell)$ if

$$\sum_{n=2}^{\infty} [1 + \hbar(n-1)]\, \aleph_k^{a,\delta}(n) |a_n| \leq 1 - \ell, \tag{12}$$

where $\hbar \geq 0, 0 \leq \ell < 1$ and $\aleph_k^{a,\delta}(n)$ is given by (10).

Proof. It suffices to show that

$$\hbar \left| \left(D^k L_a^{\delta} u(z)\right)' - \frac{D^k L_a^{\delta} u(z)}{z} \right| - Re\left\{ \frac{D^k L_a^{\delta} u(z)}{z} - 1 \right\} \leq 1 - \ell.$$

We have

$$\hbar \left| \left(D^k L_a^{\delta} u(z)\right)' - \frac{D^k L_a^{\delta} u(z)}{z} \right| - Re\left\{ \frac{D^k L_a^{\delta} u(z)}{z} - 1 \right\}$$

$$\leq \hbar \left| \frac{\sum_{n=2}^{\infty} (n-1) \aleph_k^{a,\delta}(n) a_n z^n}{z} \right| + \left| \frac{\sum_{n=2}^{\infty} \aleph_k^{a,\delta}(n) a_n z^n}{z} \right|$$

$$\leq \hbar \sum_{n=2}^{\infty} (n-1) \aleph_k^{a,\delta}(n)|a_n| + \sum_{n=2}^{\infty} \aleph_k^{a,\delta}(n)|a_n|$$

$$= \sum_{n=2}^{\infty} [1+\hbar(n-1)] \aleph_k^{a,\delta}(n)|a_n|.$$

The last expression is bounded above by $(1-\ell)$ if

$$\sum_{n=2}^{\infty} [1+\hbar(n-1)] \aleph_k^{a,\delta}(n)|a_n| \leq 1-\ell$$

and the proof of theorem is completed. □

In the following theorem, we obtain necessary and sufficient conditions for functions in $TS_k^{a,\delta}(\hbar,\ell)$.

Theorem 2.2. For $\hbar \geq 0, 0 \leq \ell < 1$, a function u of the form (4) to be in the class $TS_k^{a,\delta}(\hbar,\ell)$ if and only if

$$\sum_{n=2}^{\infty} [1+\hbar(n-1)] \aleph_k^{a,\delta}(n)|a_n| \leq 1-\ell.$$

Proof. Suppose $u(z)$ of the form (4) is in the class $TS_k^{a,\delta}(\hbar,\ell)$. Then

$$Re\left\{ \frac{D^k L_a^\delta u(z)}{z} \right\} - \hbar \left| \left(D^k L_a^\delta u(z) \right)' - \frac{D^k L_a^\delta u(z)}{z} \right| \geq \ell.$$

Equivalently

$$Re\left[1 - \sum_{n=2}^{\infty} \aleph_k^{a,\delta}(n)|a_n| z^{n-1} \right] - \hbar \left[\sum_{n=2}^{\infty} (n-1) \aleph_k^{a,\delta}(n) a_n z^{n-1} \right] \geq \ell.$$

Letting z to be real values and as $|z| \to 1$, we have

$$1 - \sum_{n=2}^{\infty} \aleph_k^{a,\delta}(n)|a_n| - \hbar \sum_{n=2}^{\infty} (n-1) \aleph_k^{a,\delta}(n)|a_n| \geq \ell$$

which implies

$$\sum_{n=2}^{\infty} [1+\hbar(n-1)] \aleph_k^{a,\delta}(n)|a_n| \leq 1-\ell,$$

where $\hbar \geq 0, 0 \leq \ell < 1$, $\aleph_k^{a,\delta}(n)$ is given by (10) and the sufficiency follows from Theorem 2.1. □

Corollary 2.1. If $u \in TS_k^{a,\delta}(\hbar,\ell)$ then

$$a_n \leq \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}.$$

Equality holds for the function

$$u(z) = z - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} z^n,$$

$\hbar \geq 0, 0 \leq \ell < 1$, $\aleph_k^{a,\delta}(n)$ is given by (10).

3. Extreme Points

Theorem 3.1. Let $u_1(z) = z$ and $u_n(z) = z - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} z^n, n \geq 2$ for $\hbar \geq 0, 0 \leq \ell < 1$, $\aleph_k^{a,\delta}(n)$ is given by (10). Then $u(z)$ is in the class $\aleph_k^{a,\delta}(n)$ if and only if it can be expressed in the form $u(z) = \sum_{n=1}^{\infty} \lambda_n u_n(z)$, where λ_n and $\sum_{n=1}^{\infty} \lambda_n = 1$.

Proof. If $u(z) = \sum_{n=1}^{\infty} \lambda_n u_n(z)$ with $\lambda_n \geq 0$ and $\sum_{n=1}^{\infty} \lambda_n = 1$. Then

$$\begin{aligned}
u(z) &= \sum_{n=1}^{\infty} \lambda_n u_n(z) \\
&= \lambda_1 u_1(z) + \sum_{n=2}^{\infty} \lambda_n u_n(z) \\
&= \left(1 - \sum_{n=2}^{\infty} \lambda_n\right) z + \sum_{n=2}^{\infty} \left[\lambda_n \left(z - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} z^n\right)\right] \\
&= z - \sum_{n=2}^{\infty} \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} z^n.
\end{aligned}$$

$$\begin{aligned}
\text{Now } \sum_{n=2}^{\infty} & \frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{1-\ell} \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} \lambda^n \\
&= \sum_{n=2}^{\infty} \lambda_n = 1 - \lambda_1 \leq 1.
\end{aligned}$$

Then $u \in TS_k^{a,\delta}(\hbar,\ell)$. Conversely suppose that $u \in TS_k^{a,\delta}(\hbar,\ell)$. Then Corollary 2.1, gives

$$a_n \leq \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}, \ n \geq 2$$

$$\text{set } \lambda_n = \frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{1-\ell} a_n, \ n \geq 2$$

$$\text{where } \lambda_n = 1 - \sum_{n=2}^{\infty} \lambda_n.$$

Then

$$\begin{aligned} u(z) &= z - \sum_{n=2}^{\infty} a_n z^n \\ &= z - \sum_{n=2}^{\infty} \lambda_n \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} \\ &= z - \left[1 - \sum_{n=2}^{\infty} \lambda_n\right] + \sum_{n=2}^{\infty} \lambda_n u_n(z) \\ &= \lambda_1 u_1(z) + \sum_{n=2}^{\infty} \lambda_n u_n(z) \\ &= \sum_{n=1}^{\infty} \lambda_n u_n(z). \end{aligned}$$

The poof of theorem is completed □

4. Integral Means Inequalities

Definition 4.1. (Subordination principle) for analytic function g and h with $g(0) = h(0),\ g$ is said to be subordinate to h, denoted by $g \prec h$ if there exists an analytic function ω such that $\omega(0) = 0, |\omega(z)| < 1$ and $g(z) = h(\omega(z))$, for all $z \in U$.

Lemma 4.1.[14] If the function $u(z)$ and $g(z)$ are analytic in U with $g(z) \prec h(z)$ then

$$\int_0^{2\pi} \left|g(re^{i\theta})\right|^p d\theta \leq \int_0^{2\pi} \left|u(re^{i\theta})\right|^p d\theta \ (0 \leq r < 1, p > 0).$$

Theorem 4.1. Suppose $u \in TS_k^{a,\delta}(\hbar,\ell), p > 0, \hbar \geq 0, 0 \leq \ell < 1$ and $u(z)$ is defined by

$$u_2(z) = z - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}.$$

Then for $z = re^{i\theta},\ 0 \leq r < 1$,

$$\int_0^{2\pi} |u(z)|^p d\theta \leq \int_0^{2\pi} |u_2(z)|^p d\theta \tag{13}$$

Proof. For $u(z) = z - \sum_{n=2}^{\infty} |a_n| z^n$, (13) is equivalent to proving that

$$\int_0^{2\pi} \left| z - \sum_{n=2}^{\infty} |a_n| z^n \right|^p d\theta \leq \int_0^{2\pi} \left| z - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} \right|^p d\theta, \ (p > 0).$$

By applying Little wood's subordination theorem (Lemma 4.1),

it would be sufficient to show that

$$1 - \sum_{n=2}^{\infty} |a_n| z^{n-1} \prec 1 - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} z. \tag{14}$$

Setting

$$1 - \sum_{n=2}^{\infty} |a_n| z^{n-1} \prec 1 - \frac{1-\ell}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} \omega(z).$$

We have $\omega(z) = \frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{1-\ell} \sum_{n=2}^{\infty} a_n z^{n-1}$ and and $\omega(z)$ is analytic in U with $\omega(0) = 0$. Moreover it suffices to prove that $\omega(z)$ satisfies $|\omega(z)| < 1, z \in U$. Now

$$\begin{aligned} |\omega(z)| &= \left| \sum_{n=2}^{\infty} \frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{1-\ell} a_n z^{n-1} \right| \\ &\leq |z| \sum_{n=2}^{\infty} \frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{1-\ell} |a_n| \qquad (15) \\ &\leq |z| < 1. \end{aligned}$$

Thus is view of the inequality (15) the subordination (14) follows, which proves the Theorem. □

5. Subordination Results

Definition 5.1. (Subordination factor sequence) A sequence $\left\{ b_n \right\}_{n=2}^{\infty}$ of complex numbers is said to be a subordinating sequence if, whenever $u(z) = \sum_{n=2}^{\infty} a_n z^n, a_1 = 1$ is regular, univalent and convex in U, we have $\sum_{n=1}^{\infty} b_n a_n z^n \prec u(z),\ z \in U$.

Theorem 5.1.[24] The sequence $\left\{ b_n \right\}_{n=2}^{\infty}$ is a subordinating factor sequence if and only if

$$Re\left\{ 1 + 2\sum_{n=1}^{\infty} b_n z^n \right\} > 0, z \in U.$$

Theorem 5.2. Let $u \in D^k L_a^\delta u(z)(\hbar, \ell)$ and $g(z)$ any function in the usual class of convex function $\mathbb{C}$. Then

$$\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}(u*g)(z) \prec g(z) \tag{16}$$

where $\hbar \geq 0, 0 \leq \ell < 1$ with $\aleph_k^{a,\delta}(n)$ is given by (6)

$$Re\{u(z)\} > -\frac{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)},\ z \in E. \tag{17}$$

The constant $\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}$ is the best estimate.

Proof. Let $u \in D^k L_a^\delta u(z)(\hbar, \ell)$ and $g(z) = z + \sum\limits_{n=2}^{\infty} c_n z^n \in \mathbb{C}$.
Then

$$\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}(u*g)(z) = \frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}\left(z+\sum_{n=2}^{\infty} c_n a_n z^n\right).$$

Then by Definition 5.1, the subordination result holds true if $\left\{\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}\right\}_{n=1}^{\infty}$ is a subordinating factor sequence with $a_1 = 1$.
In view of Theorem 5.2, this is equivalent to the following inequality.

$$Re\left\{1+\sum_{n=1}^{\infty}\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}a_n z^n\right\} > 0,\ z \in U. \tag{18}$$

Now for $|z| = r < 1$, we have

$$\begin{aligned}
& Re\left\{1+\sum_{n=1}^{\infty}\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}a_n z^n\right\} \\
&= Re\left\{1+\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}z+\frac{\sum\limits_{n=2}^{\infty}[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)a_n z^n}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}\right\} \\
&\geq 1-\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}r-\frac{\sum\limits_{n=2}^{\infty}[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)a_n r^n}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)} \\
&\geq 1-\frac{[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}r-\frac{1-\ell}{(1-\ell)+[1+\hbar(n-1)]\,\aleph_k^{a,\delta}(n)}r \\
&> 0.
\end{aligned}$$

Using (12) and the fact that $[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)$ is increasing function for $n \geq 2$.

This proves the inequality (18) and hence also the subordination result (16) asserted by Theorem 5.

The inequality (17) follows from (16) by taking

$$g(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n \in \mathbb{C}.$$

Now we consider the function $u(z) = z - \frac{1-\ell}{[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)} z^2$, where $\hbar \geq 0, 0 \leq \ell < 1$.

Clearly $F \in D^k L_a^\delta u(z)(\hbar, \ell)$. For the function (16) becomes

$$\frac{[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)} u(z) \prec \frac{z}{1-z}.$$

It is easily verified that

$$\min Re\left\{\frac{[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)} u(z)\right\} = \frac{-1}{2}, z \in U.$$

This shows that the constant $\frac{[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)}{2(1-\ell)+[1+\hbar(n-1)]\aleph_k^{a,\delta}(n)} u(z) \prec \frac{z}{1-z}$ is best possible. □

Conflict of Interests

The authors declare that there is no conflict of interests.

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