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MODELING AND EVALUATION OF MTSF OF A REPAIRABLE 2-OUT-OF-4 WARM STANDBY SYSTEM ATTENDED BY REPAIR MACHINES AND REPAIRMEN

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Abstract: Studies on reliability characteristics of a redundant repairable warm standby system are numerous. Little literature is found in the use of repair machines and a repairman to repair the failed unit and failed repair machine. This paper presents analysis and evaluation of mean time to system failure (MTSF) of a repairable 2-out-of-4 warm standby system. The system comprises of two subsystems A and B arranged in series. Subsystems A and B are two units warm standby. The system work provided one unit each of subsystem A and B are working while the remaining two units are in warm standby. The system is attended by two repair machines and two repairmen assigned to each subsystem to repair any failed unit and failed repair machine respectively. An explicit expression for mean time to system failure (MTSF) is derived and analyzed using Kolmogorov's forward equation method. Numerical simulations using assumed numerical values giving to the system parameters have been obtained.

Keywords: mean time to system failure, repair machines, warm standby

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1. Introduction

There are systems of three/four units in which two/three units are sufficient to perform the entire function of the system. Examples of such systems are 2-out-of-3, 2-out-of-4, or 3-out-of-4 redundant systems which have wide application in the real world especially in industries. Furthermore, a communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system. Many research results have been reported on reliability of 2-out-of-3,2-out-of-4, 3-out-of-4 redundant systems. For example, Chander and Bhardwaj [2] present reliability

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and economic analysis of 2-out-of-3 redundant system with priority to repair. Bhardwaj and Malik [1] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Yusuf and Hussaini [9] have analyzed reliability characteristics of 2-out-of-3 system under perfect repair option. Yusuf [10] analyzed the availability and profit of 3-out-of-4 system with preventive maintenance.

There are situations where repair machines are employed to repair the failed unit. Example of such situations can be seen in nuclear reactor, marine equipments, etc, (see Gupta and Chaudhary [7]). However, on the course repairing the failed unit, the repair machines are also subjected to failure due to another reason. When the repair machines failed on the course of repairing the failed units, their repair has priority over the repair of the individual unit. Little attention is paid on the failure of repair machines.

In this paper, we construct a redundant 2-out-of-4 warm standby system and derived its corresponding mathematical model. In the paper, two repair machines and two repairmen are employed to repair failed unit in both subsystem A and B and failed repair machine respectively. Thus, the repair of the failed repair machine has priority over the repair of the individual unit. We derived the explicit expression for mean time to system failure (MTSF). Numerical simulations using assumed numerical values giving to the system parameters have been obtained.The organization of the paper is as follows. In section 2, we give the notations, assumptions and the states of the system. Expression for mean time to system failure (MTSF) is derived in section3. The results of our numerical simulations and discussions of the results are presented in sections 4. Finally, we give a concluding remark in Section 5.

2. Notations, assumptions and states of the system

2.1 Notations

β_1, α_1 : Failure and repair rate of unit A_1

β_2, α_2 : Failure and repair rate of unit A_2

β_3, α_3 : Failure and repair rate of unit B_1

β_4, α_4 : Failure and repair rate of unit B_2

λ_1, μ_1 : Failure and repair rate of Repair Machine I

λ_2, μ_2 : Failure and repair rate of Repair Machine II

A_{iO}, A_{iS}, A_{iR}, A_{iW} : Units in subsystem A are: in operation, Standby, under repair, waiting for
repair, $i=1,2$

B_{iG}, B_{iO}, B_{iS}, B_{iR}, B_{iW} : Units in subsystem B are :good, in operation, Standby, under repair,
waiting for repair, $i=1,2$

RM_i : Repair machine, $i=1,2$

RM_{iG}, RM_{iO}, RM_{iR} : Repair Machine is : good (idle),in operation, under repair , $i=1,2$

2.2 Assumptions

1. The system consists of two subsystems each comprises of two units in warm standby.
2. One unit from each subsystem combines together for the system to operate
3. The system is attended by two repair machines
4. Repair machines are repaired by two repairmen assigned to each repair machine
5. The system failed when more than two units failed
6. Failure and repair time assumed exponential
7. Failure rates and repair rates are constant
8. Repair of the repair machine has priority over the repair of the units in the subsystems

2.3 States of the system

Table 1: Transition rate table

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
S_0		β_1								
S_1	α_1			β_4		β_2				
S_2	α_2				β_4	β_1				
S_3		α_4					β_2	β_3		

S4			α_4				β_1			β_3
S5		α_2	α_1				λ_1			
S6				α_2	α_1	μ_1			λ_2	
S7				α_3						
S8							μ_2			
S9					α_3					

Up State:

$$S_0 = \begin{pmatrix} A_{1O}, A_{2S}, B_{1O}, B_{2S} \\ RM_{1G}, RM_{2G} \end{pmatrix} S_1 = \begin{pmatrix} A_{1R}, A_{2O}, B_{1O}, B_{2S} \\ RM_{1O}, .RM_{2G} \end{pmatrix} S_2 = \begin{pmatrix} A_{1O}, A_{2R}, B_{1O}, B_{2S} \\ RM_{1O}, RM_{2G} \end{pmatrix}$$

$$S_3 = \begin{pmatrix} A_{1R}, A_{2O}, B_{1O}, B_{2R} \\ RM_{1O}, RM_{2O} \end{pmatrix} S_4 = \begin{pmatrix} A_{1O}, A_{2R}, B_{1O}, B_{2R} \\ RM_{1O}, RM_{2O} \end{pmatrix}$$

Failed State:

$$S_5 = \begin{pmatrix} A_{1R}, A_{2W}, B_{1G}, B_{2S} \\ RM_{1O}, RM_{2G} \end{pmatrix} S_6 = \begin{pmatrix} A_{1R}, A_{2W}, B_{1G}, B_{2S} \\ RM_{1R}, RM_{2G} \end{pmatrix} S_7 = \begin{pmatrix} A_{1R}, A_{2O}, B_{1W}, B_{2R} \\ RM_{1O}, RM_{2O} \end{pmatrix}$$

$$S_8 = \begin{pmatrix} A_{1W}, A_{2W}, B_{1G}, B_{2S} \\ RM_{1R}, RM_{2R} \end{pmatrix} S_9 = \begin{pmatrix} A_{1G}, A_{2R}, B_{1W}, B_{2R} \\ RM_{1O}, RM_{2O} \end{pmatrix}$$

3. Mean time to system failure analysis

From table 1, let $P_i(t)$ to be the probability that the System at time $t \geq 0$ is in state S_i. Also let $P(t)$ be the probability row vector at time t, we have the following initial condition.

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0), P_9(0)]$$

$$= \left[1, 0, 0, 0, 0, 0, 0, 0, 0, 0\right]$$

We obtain the following differential equations using Table 1:

$$\frac{dp_0(t)}{dt} = -\beta_1 p_0(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t)$$

$$\frac{dp_1(t)}{dt} = -(\alpha_1 + \beta_2 + \beta_4)\, p_1(t) + \beta_1 p_0(t) + \alpha_4 p_3(t) + \alpha_2 p_5(t)$$

$$\frac{dp_2(t)}{dt} = -(\alpha_2 + \beta_1 + \beta_4)\, p_2(t) + \alpha_4 p_4(t) + \alpha_1 p_5(t)$$

$$\frac{dp_3(t)}{dt} = -(\alpha_4 + \beta_2 + \beta_3)\, p_3(t) + \beta_4 p_1(t) + \alpha_2 p_6(t) + \alpha_3 p_7(t)$$

$$\frac{dp_4(t)}{dt} = -(\alpha_4 + \beta_1 + \beta_3)\, p_4(t) + \beta_4 p_2(t) + \alpha_1 p_6(t) + \alpha_3 p_9(t)$$

$$\frac{dp_5(t)}{dt} = -(\lambda_1 + \alpha_1 + \alpha_2)\, p_5(t) + \beta_2 p_1(t) + \beta_1 p_2(t) + \mu_1 p_6(t)$$

$$\frac{dp_6(t)}{dt} = -(\alpha_1 + \alpha_2 + \lambda_2 + \mu_1)\, p_6(t) + \beta_2 p_3(t) + \beta_1 p_4(t) + \lambda_1 p_5(t) + \mu_2 p_8(t)$$

$$\frac{dp_7(t)}{dt} = -\alpha_3 p_7(t) + \beta_3 p_3(t)$$

$$\frac{dp_8(t)}{dt} = -\mu_2 p_8(t) + \lambda_2 p_6(t)$$

$$\frac{dp_9(t)}{dt} = -\alpha_3 p_9(t) + \beta_3 p_4(t) \qquad (1)$$

Equation (1) can be written in matrix form as

$$P' = AP \qquad (2)$$

where

$$A = \begin{bmatrix} -\beta_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -Y_1 & 0 & \alpha_4 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_2 & 0 & \alpha_4 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_4 & 0 & -Y_3 & 0 & 0 & \alpha_2 & \alpha_3 & 0 & 0 \\ 0 & 0 & \beta_4 & 0 & -Y_4 & 0 & \alpha_1 & 0 & 0 & \alpha_3 \\ 0 & \beta_2 & \beta_1 & 0 & 0 & -Y_5 & \mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & \beta_1 & \lambda_1 & -Y_6 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & -\alpha_3 & \mu_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & \lambda_1 & -\mu_2 & \lambda_2 \\ 0 & 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & \mu_2 & -\alpha_3 \end{bmatrix}$$

Where $Y_1=(\alpha_1+\beta_2+\beta_4)$, $Y_2=(\alpha_2+\beta_1+\beta_4)$, $Y_3=(\alpha_4+\beta_2+\beta_3)$, $Y_4=(\alpha_4+\beta_1+\beta_3)$, $Y_5=(\lambda_1+\alpha_1+\alpha_2)$, $Y_6=(\alpha_1+\alpha_2+\lambda_2+\mu_1)$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q (see El said [3, 4], Haggag [5,6], Wang et al [8]. The expected time to reach an absorbing state is obtained from:

$$E\left[T_{P(0)\to P(absorbing)}\right]=P(0)\int_0^{\infty}e^{Qt}dt \tag{3}$$

and

$$\int_0^{\infty}e^{Qt}dt=Q^{-1}\text{, since } Q^{-1}<0 \tag{4}$$

Explicit expression for the $MTSF$ is given by

$$E\left[T_{P(0)\to P(absorbing)}\right]=MTSF=P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} \tag{5}$$

$$MTSF=\frac{\beta_2\beta_3+\beta_2^2+\alpha_1\alpha_2+\alpha_4\beta_2+\beta_3\beta_4+\alpha_1\beta_2+\alpha_1\beta_3+\beta_2\beta_4+\beta_1(\alpha_4+\beta_2+\beta_3)+\beta_1\beta_4}{\beta_1(\beta_2^2+\beta_2\beta_3+\beta_2\beta_4+\alpha_4\beta_2+\beta_3\beta_4)}$$

$$Q=\begin{bmatrix}-\beta_1 & \beta_1 & 0 & 0 & 0\\ \alpha_1 & -(\alpha_1+\beta_1+\beta_4) & 0 & \beta_4 & 0\\ \alpha_2 & 0 & -(\alpha_2+\beta_1+\beta_4) & 0 & \beta_4\\ 0 & \alpha_4 & 0 & -(\alpha_4+\beta_2+\beta_3) & 0\\ 0 & 0 & \alpha_4 & 0 & -(\alpha_4+\beta_1+\beta_3)\end{bmatrix}$$

4. Numerical Simulations

In this section, we numerically obtained the results for mean time to system failure (MTSF) for the developed model. For the model analysis the following set of parameters values are fixed throughout the simulations for consistency:

$\alpha_1 = 0.99$, $\alpha_2 = 0.94$, $\alpha_4 = 0.83$, $\beta_1 = 0.99$, $\beta_2 = 0.5$, $\beta_3 = 0.99$, $\beta_4 = 0.8$ and vary the parameter in question.

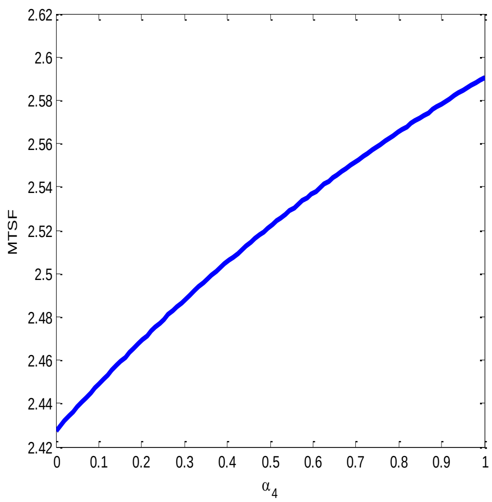

Fig. 1 effect of α_4 on MTSF

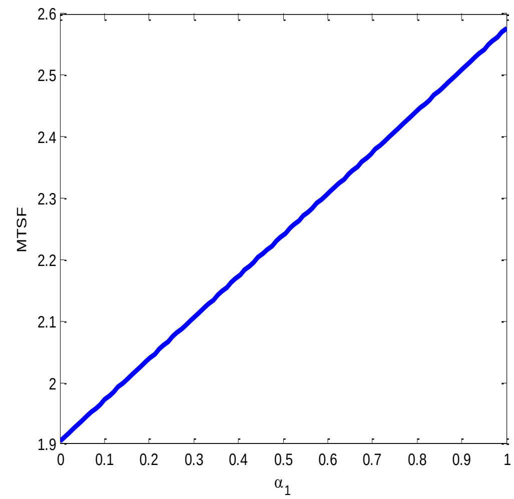

Fig. 2 effect of α_1 on MTSF

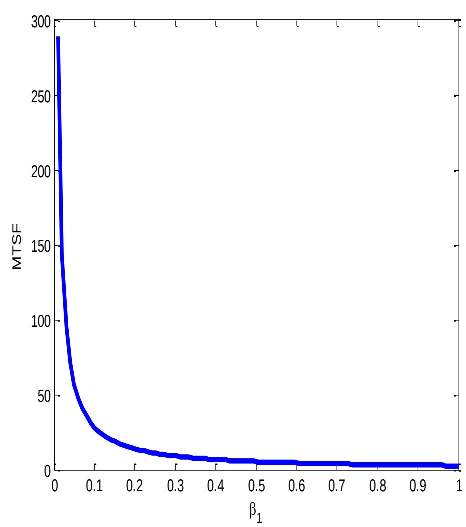

Fig. 3 effect of β_1 on MTSF

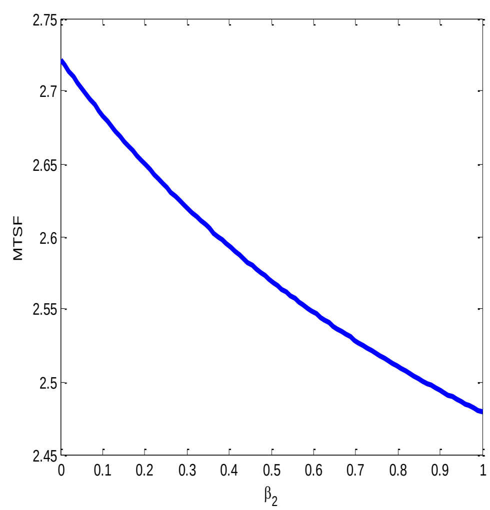

Fig. 4 effect of β_2 on MTSF

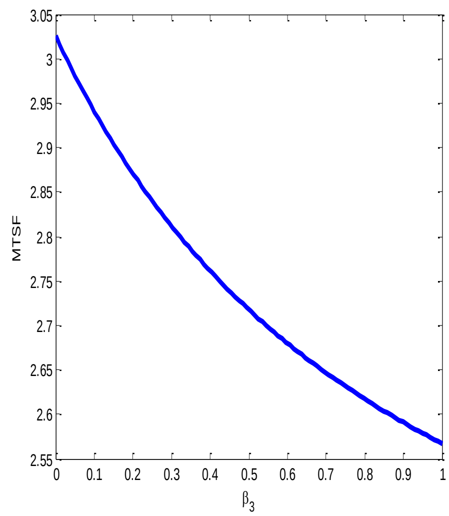

Fig. 5 effect of β_3 on MTSF

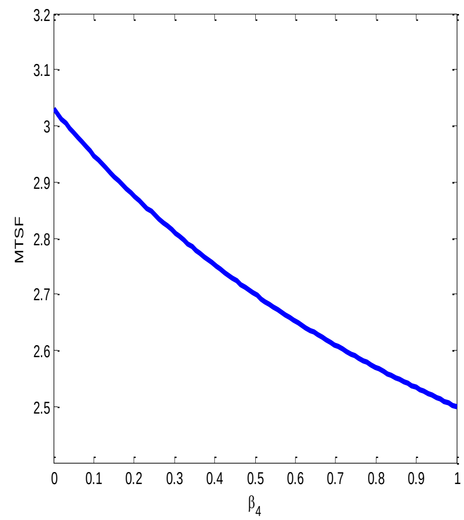

Fig. 6 effect of β_4 on MTSF

It is evident from figures 1 – 2 that the increase in repair rates α_1 and α_4 induces the increase in MTSF, while from figures 4 – 7, the increase in failure rates β_1, β_2, β_3 and β_4 induces decrease in MTSF.

5. Conclusion

In this paper, a redundant 2-out-of-4 warm standby repairable system has been constructed. Explicit expression for mean time to system failure (MTSF) is developed. Effect of both failure and repair rates on MTSF re captured.

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