Available online at http://scik.org
Eng. Math. Lett. 2025, 2025:4
https://doi.org/10.28919/eml/9173
ISSN: 2049-9337

SOME FIXED POINT RESULTS ON $\tau - \varphi$-BERINDE-CONTRACTION MAPPINGS IN PARTIAL METRIC SPACES

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Abstract. In this work, we introduce $\tau - \varphi$-Berinde-contraction mappings and establish some fixed point results on such mappings in the setting of partial metric spaces. These results substantially generalize the results in the existing literature. We provide an example in support of our result.

Keywords: $\tau - \psi$-contraction mapping; $\tau - \varphi$-Berinde-contraction mappings; partial metric spaces.

2010 AMS Subject Classification: 47H10, 54H25.

1. INTRODUCTION AND PRELIMINARIES

One of the most well-known and important theorems in nonlinear analysis is the Banach contraction principle [1].Many extensions and generalizations of the Banach contraction principle have been made. Samet et al. [5] introduced a new class of contractive type mappings called $\alpha - \psi$ contractive type mappings, which expanded and generalized the Banach contraction principle. Additionally, Kumam et al. [6] established fixed point results for this class of mappings while establishing the idea of weak $\alpha - \psi$-contractive mappings. However, Baiz et al.[7, 8] established results in rectangular quasi b-metric spaces and rectangular M-metric spaces and

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Received February 04, 2025

developed a novel generalization of contraction mappings as $\tau-\psi$-contraction and generalized $\tau-\psi$-contraction mappings.Berinde [15, 16] established the idea of almost contractions on metric spaces. Generalized Berinde-type contractions have been studied by Aydi et al. [17] in regard to partial metric spaces. Recently, Mebawondu et al. [9] introduced (α,β)-Berinde-φ-contraction mappings by employing a new class of mappings called (α,β)-cyclic admissible mappings, and provided some results for such mappings.

Matthews [2, 3] established the partial metric version of the Banach fixed point theorem and presented a very interesting generalization of the metric space called partial metric space, in which the self distance need not be zero.

Boyd and Wong [4] introduced a class of mappings called the φ-contraction mapping and obtained following result:

Definition 1.1. Let (U,d) be a metric space and $\varphi:[0,\infty)\to[0,\infty)$ be a function such that $\varphi(x)<x$ for $x>0$. A self map $T:U\to U$ is called φ-contraction if

$$d(Ts,Tt))\leq\varphi(\rho(s,t))$$

for all $s,t\in U$.

Theorem 1.2. *Let (U,d) be a complete metric space and $T:U\to U$ a φ-contraction such that φ is upper semicontinuous from the right on $[0,\infty)$ and satisfies $\varphi(x)<x$ for all $x>0$, Then T has a unique fixed point.*

More so, Berinde introduced and studied the following class of contraction mappings:

Definition 1.3. Let (U,d) be a metric space. A mapping $T:U\to U$ is said to be generalized contraction if there exist $\delta\in[0,1)$ and $L\geq 0$ such that

$$d(Ts,Tt)\leq\delta d(s,t)+L\min\{d(s,Ts),d(t,Tt),d(s,Tt),d(t,Ts)\} \tag{1.1}$$

for all $s,t\in U$.

Recently, In 2020, Mebawondu et. al. [9] introduced a new class of mappings called (α,β)-cyclic admissible mappings and presented following results:

Definition 1.4. Let T be a self-mapping on U and let $\alpha,\beta : U\times U\to[0,\infty)$. We say T is an (α,β)-cyclic admissible mapping if

(1) $\alpha(s,t)\geq 1\Rightarrow\beta(Ts,Tt)\geq 1$;

(2) $\beta(s,t)\geq 1\Rightarrow\alpha(Ts,Tt)\geq 1$.

Definition 1.5. Let Let (U,d) be a metric space. $\alpha,\beta : U\times U:\to[0,\infty)$ be two functions and T be a self map on U. The mapping T is said to be an (α,β)-Berinde-φ-contraction mapping, if there exists $L\geq 0$ such that for $s,t\in U$ with $Ts\neq Tt$, we have

(1.2)
$$\alpha(s,Ts)\beta(t,Tt)\geq 1\Rightarrow d(Ts,Tt))\leq\varphi(d(s,t))+L\min\{d(s,Ts),d(t,Tt),d(s,Tt),d(t,Ts)\}$$

where $\varphi : [0,\infty)\to[0,\infty)$ is a continuous function which satisfies $\varphi(x)<x$ for all $x>0$ and $\varphi(0)=0$.

Theorem 1.6. *Let (U,d) be a complete metric space and $T:U\to U$ be an (α,β)-Berinde-φ-contraction mapping. Suppose the following conditions hold:*

(1) T is an (α,β)-cyclic admissible mapping;

(2) there exists $s_0\in U$ such that $\alpha(s_0,Ts_0)\geq 1$ and $\beta(s_0,Ts_0)\geq 1$;

(3) T is continuous.

Then T has a fixed point.

Theorem 1.7. *Let (U,d) be a complete metric space and $T:U\to U$ be an (α,β)-Berinde-φ-contraction mapping. Suppose the following conditions hold:*

(1) T is an (α,β)-cyclic admissible mapping;

(2) there exists $s_0\in U$ such that $\alpha(s_0,Ts_0)\geq 1$ and $\beta(s_0,Ts_0)\geq 1$;

(3) if for any sequence $\{s_n\}$ in U such that $s_n\to s$ as $n\to\infty$, then $\beta(s,Ts)\geq 1$ and $\alpha(s,Ts)\geq 1$

Then T has a fixed point.

Further, Baiz et al. [7] introduced the concepts of $\tau-\psi$-contraction mappings as follows:

Definition 1.8. Let (U,σ) be a rectangular quasi b-metric space and $T: U \to U$ be a self mapping. T is said to be a generalized $\tau - \psi$ contractive mapping if there exists $\psi \in \Psi$ and $\tau > 1$ such that

$$\tau\sigma(Ts,Tt) \leq \psi(\Sigma(s,t))$$

for all $s,t \in U$. Where

$$\Sigma(s,t) = \max\left\{\sigma(s,t), \frac{\sigma(s,Ts)\sigma(s,Tt)}{1+\sigma(s,Tt)+\sigma(t,Ts)}, \sigma(s,Ts), \sigma(t,Tt)\right\}$$

and Ψ=$\{\psi : \mathbb{R}^+ \to \mathbb{R}^+$, ψ is non-decreasing, continuous, $\sum_{k=1}^{\infty} s^k \psi^k(t) < \infty$, $s\psi(x) < x$ and $\psi(0) = 0$ if and only if $x = 0$, where ψ^k is the k^{th} iterate of ψ, $s \geq 1\}$.

Now, we give some basic properties and results on the concept of partial metric space.

Definition 1.9. [2] Let U be a non-empty set. A function $\rho : U \times U \to [0,\infty)$ is said to be a partial metric on U if the following conditions hold:

(1) $s = t \Leftrightarrow \rho(s,s) = \rho(t,t) = \rho(s,t)$;
(2) $\rho(s,s) \leq \rho(s,t)$;
(3) $\rho(s,t) = \rho(t,s)$;
(4) $\rho(s,t) \leq \rho(s,r) + \rho(r,t) - \rho(r,r)$. for all $s,t,r \in U$.

The set U equipped with the metric ρ defined above is called a partial metric space and it is denoted by (U,ρ) (in short PMS).

Example 1.10. [13] Let $U = \{[a,b] : a,b \in \mathbb{R}, a \leq b\}$ and define $\rho([a,b],[c,d]) = \max\{b,d\} - \min\{a,c\}$. Then (U,ρ) is a partial metric space.

Example 1.11. [13] Let $U = [0,\infty)$ and define $\rho(s,t) = \max\{s,t\}$. Then (U,ρ) is a partial metric space.

Lemma 1.12. [2, 6] *Let (U,ρ) be a partial metric space.*

(1) A sequence $\{s_n\}$ in (U,ρ) converges to a point $s \in U$ if

$$\rho(s,s) = \lim_{n\to\infty} \rho(s_n,s),$$

(2) A sequence $\{s_n\}$ in (U,ρ) is a Cauchy sequence if $\lim_{m,n\to\infty}\rho(s_n,s_m)$ *exists and finite ,*

(3) (U,ρ) is complete if every Cauchy $\{s_n\}$ in U converges to a point $s\in U$, such that

$$\rho(s,s)=\lim_{m,n\to\infty}\rho(s_m,s_n)=\lim_{n\to\infty}\rho(s_n,s)=\rho(s,s).$$

Lemma 1.13. [2, 3, 12] *Let ρ be a partial metric on U, then the functions $d_{\rho_k},d_{\rho_m}:U\times U\to\mathbb{R}^+$ such that*

$$d_{\rho_k}(s,t)=2\rho(s,t)-\rho(s,s)-\rho(t,t)$$

and

$$\begin{aligned}d_{\rho_m}(s,t)=&\max\{\rho(s,t)-\rho(s,s),\rho(s,t)-\rho(t,t)\}\\=&2\rho(s,t)-\min\{\rho(s,s),\rho(t,t)\}\end{aligned}$$

are metric on U. Furthermore (U,d_{ρ_k}) and (U,d_{ρ_m}) are metric spaces. It is clear that d_{ρ_k} and d_{ρ_m} are equivalent.

Let (U,ρ) be a partial metric space. Then

(1) A sequence $\{s_n\}$ in (U,ρ) is a Cauchy sequence $\Leftrightarrow$ $\{s_n\}$ is a Cauchy sequence in (U,d_{ρ_k}),

(2) (U,ρ) is complete $\Leftrightarrow$ (U,d_{ρ_k}) is complete. Moreover $\lim_{n\to\infty}d_{\rho_k}(s_n,s)=0\Leftrightarrow\rho(s,s)=\lim_{n\to\infty}\rho(s_n,s)=\lim_{n,m\to\infty}\rho(s_n,s_m)$.

Lemma 1.14. [11] *Assume that $s_n\to s$ as $n\to\infty$ in a partial metric space (U,ρ) such that $\rho(s,s)=0$. Then $\lim_{n\to\infty}\rho(s_n,r)=\rho(s,r)$ for every $r\in U$.*

Lemma 1.15. [14] *Let (U,ρ) be a partial metric space.*

(1) if $\rho(s,t)=0$ then $s=t$,

(2) If $s\neq t$ then $\rho(s,t)>0$.

Lemma 1.16. [10] *If $\{s_n\}$ with $\lim_{n\to\infty}\rho(s_n,s_{n+1})=0$ is not a Cauchy sequence in (U,ρ) and two sequences $\{n(k)\}$ and $\{m(k)\}$ of positive integers such that $n(k)>m(k)>k$, then following four sequences*

$$\rho(s_{m(k)},s_{n(k)+1}),\rho(s_{m(k)},s_{n(k)}),$$

$$\rho(s_{m(k)-1}, s_{n(k)+1}), \rho(s_{m(k)-1}, s_{n(k)})$$

tend to $\mu > 0$ *when* $k \to \infty$.

2. MAIN RESULTS

[9] Let Φ be the family of functions $\varphi : [0,\infty) \to [0,\infty)$ such that

(1) $\varphi(x) < x$ for any $x > 0$;

(2) φ is continuous;

(3) $\varphi(0) = 0$.

Definition 2.1. Let (U, ρ) be a partial metric space and $T : U \to U$ be a given self map. We say that T is an $\tau - \varphi$-Berinde-contraction mapping on U, if there exists $\varphi \in \Phi$ and $\tau > 1$ such that for all $s,t \in U$ we have

$$\tau\rho(Ts,Tt) \leq \varphi(\max\{\rho(s,t), \rho(s,Ts), \rho(t,Tt)\}) + L(\min\{d_{\rho_m}(s,Ts), d_{\rho_m}(t,Ts)\}) \tag{2.1}$$

Theorem 2.2. *Let* (U,ρ) *be a complete partial metric space and* $T : U \to U$ *be self map. Suppose that*

(1) T *is an* $\tau - \varphi$*-Berinde-contraction mapping;*

(2) There exists $s_n \in U$ *such that* $s_n = Ts_{n-1} = T^{n+1}s_0$ *for all* $n \in \mathbb{N}$*;*

(3) T *is continuous;*

Then T *has a unique fixed point in* U.

Proof. Based on *(ii)*, we have a sequence $\{s_n\}$ in U such that $s_{n+1} = Ts_n$ for all $n \in \mathbb{N}$. If $s_n = s_{n+1}$ for some $n \in \mathbb{N}$, then s_n is a fixed point of T, and the existence proof is complete. Assume $s_n \neq s_{n+1}$ for all $n \in \mathbb{N}$.

Now, from *(i)* and *(ii)* we get

$$\begin{aligned}
\rho(s_n, s_{n+1}) = \rho(Ts_{n-1}, Ts_n) \leq \tau\rho(Ts_{n-1}, Ts_n) \\
&\leq \varphi(\max\{\rho(s_{n-1}, s_n), \rho(s_{n-1}, Ts_{n-1}), \rho(s_n, Ts_n)\}) \\
&\quad + L(\min\{d_{\rho_m}(s_{n-1}, Ts_{n-1}), d_{\rho_m}(s_n, Ts_{n-1})\}) \\
&= \varphi(\max\{\rho(s_{n-1}, s_n), \rho(s_{n-1}s_n), \rho(s_n, s_{n+1})\})
\end{aligned}$$

$$+L(\min\{d_{\rho_m}(s_{n-1},s_n),d_{\rho_m}(s_n,s_n)\})$$

$$=\varphi(\max\{\rho(s_n,s_{n+1}),\rho(s_{n-1},s_n)\}) \tag{2.2}$$

Now, if $\rho(s_n,s_{n+1})>\rho(s_{n-1},s_n)$ then

$$\rho(s_n,s_{n+1})\leq\varphi(\rho(s_n,s_{n+1}))<\rho(s_n,s_{n+1})$$

which is a contradiction, therefore taking (2.2) into consideration

$$\rho(s_n,s_{n+1})\leq\varphi(\rho(s_{n-1},s_n))<\rho(s_{n-1},s_n) \tag{2.3}$$

Since the sequence $\rho(s_n,s_{n+1})$ decreasing and bounded from below, we conclude that it converges to some nonnegative number μ i.e.

$$\lim_{n\to\infty}\rho(s_n,s_{n+1})=\lambda \tag{2.4}$$

We claim that $\lambda=0$

Suppose on contrary, that $\lambda>0$. Taking $n\to\infty$ in (2.3) and using definition of φ we get that

$$\lambda\leq\varphi(\lambda)<\lambda$$

a contradiction. Hence we conclude that

$$\lim_{n\to\infty}\rho(s_n,s_{n+1})=0 \tag{2.5}$$

Now, we show that $\{s_n\}$ is a Cauchy sequence in U i.e. We prove that $\lim_{n,m\to\infty}\rho(s_n,s_m)=0$. We prove it by contradiction.

Let

$$\lim_{n,m\to\infty}\rho(s_n,s_m)\neq 0.$$

Then sequences in Lemma 1.16 tends to $\mu>0$, when $k\to\infty$.

So we can see that

$$\lim_{k\to\infty}\rho(s_{n(k)},s_{m(k)})=\mu \tag{2.6}$$

Further corresponding to $n(k)$, we can choose $m(k)$ in such a way that it is smallest integer with $m(k)>n(k)>k$. Then

$$\lim_{k\to\infty}\rho(s_{m(k)-1},s_{n(k)})=\mu \tag{2.7}$$

Again,

$$\rho(s_{n(k)-1},s_{m(k)-1})\leq\rho(s_{n(k)-1},s_{m(k)})+\rho(s_{m(k)},s_{m(k)-1})$$

Letting $k\to\infty$ and using Lemma 1.16 we get

$$\lim_{k\to\infty}\rho(s_{n(k)-1},s_{m(k)-1})=\mu \tag{2.8}$$

In (2.1) replacing s by $s_{m(k)-1}$ and t by $s_{n(k)-1}$ respectively we get

$$\begin{aligned}\rho(s_{m(k)},s_{n(k)})&=\rho(Ts_{m(k)-1},Ts_{n(k)-1})\\&\leq\tau\rho(Ts_{m(k)-1},Ts_{n(k)-1})\\&\leq\varphi(\max\{\rho(s_{m(k)-1},s_{n(k)-1}),\rho(s_{m(k)-1},Ts_{m(k)-1}),\rho(s_{n(k)-1},Ts_{n(k)-1})\})\\&\quad+L(\min\{d_{\rho_m}(s_{m(k)-1},Ts_{m(k)-1}),d_{\rho_m}(s_{n(k)-1},Ts_{m(k)-1})\})\\&=\varphi(\max\{\rho(s_{m(k)-1},s_{n(k)-1}),\rho(s_{m(k)-1},s_{m(k)}),\rho(s_{n(k)-1},s_{n(k)})\})\\&\quad+L(\min\{d_{\rho_m}(s_{m(k)-1},s_{m(k)}),d_{\rho_m}(s_{n(k)-1},s_{m(k)})\})\end{aligned} \tag{2.9}$$

Letting $k\to\infty$ in (2.9) and applying Lemma 1.16 with (2.5), (2.7), and (2.8) yields

$$\mu\leq\varphi(\mu)<\mu$$

This implies that $\{s_n\}$ is a Cauchy sequence in the metric space (U,ρ) and hence in (U,d_{ρ_k}) which is complete.Therefore the sequence $\{s_n\}$ is convergent in the space (U,d_{ρ_k}). This implies that there exists $s^*\in U$ such that $\lim_{n\to\infty}d_{\rho_k}(s_n,s^*)=0$. Again from Lemma 1.13 we get

$$\rho(s^*,s^*)=\lim_{n\to\infty}\rho(s_n,s^*)=\lim_{n,m\to\infty}\rho(s_n,s_m)=0$$

As T is continuous, we have

$$s^*=\lim_{n\to\infty}s_{n+1}=\lim_{n\to\infty}Ts_n=Ts^*$$

Now, we show that the uniqueness of a fixed point of T. Assume that T has two distinct fixed points s^* and t^* such that $Ts^*=s^*$ and $Tt^*=t^*$ then replacing s by s^* and t by t^* in (2.1) we get

$$\begin{aligned}\rho(s^*,t^*)=\rho(Ts^*,Tt^*)&\leq\tau\rho(Ts^*,Tt^*)\\&\leq\varphi(\max\{\rho(s^*,t^*),\rho(s^*,Ts^*),\rho(t^*,Tt^*)\})\\&\quad+L(\min\{d_{\rho_m}(s^*,Ts^*),d_{\rho_m}(t^*,Ts^*)\})\\&=\varphi(\max\{\rho(s^*,t^*),\rho(s^*,s^*),\rho(t^*,t^*)\})\\&\quad+L(\min\{d_{\rho_m}(s^*,s^*),d_{\rho_m}(t^*,s^*)\})\\&=\varphi(\rho(s^*,t^*))<\rho(s^*,t^*)\end{aligned}\tag{2.10}$$

which is a contradiction. Hence T has a unique fixed point. This completes the proof. □

Now, we state the following fixed point theorem by removing the continuity assumption of T from Theorem 2.2.

Theorem 2.3. *Let (U,ρ) be a complete partial metric space and $T:U\to U$ be self map. Suppose that*

(1) T is an $\tau-\varphi$-Berinde-contraction mapping;
(2) There exists $s_n\in U$ such that $s_n=Ts_{n-1}=T^{n+1}s_0$ for all $n\in\mathbb{N}$;
(3) $\{s_n\}$ is a sequence in U such that $s_n\to s$ as $n\to\infty$.

Then T has a unique fixed point in U.

Proof. Following the proof of Theorem 2.2 we know that the sequence $\{s_n\}$ given by $s_n=s_{n+1}$ is a Cauchy sequence in the complete partial metric space (U,ρ). Consequently, there exists $s\in U$ such that

$$\rho(s,s)=\lim_{n\to\infty}\rho(s_n,s)=\lim_{n,m\to\infty}\rho(s_n,s_m)=0$$

Therefore, It is sufficient to show that T admits a fixed point.

Now, using (2.1) we get

$$\begin{aligned}\rho(s_{n+1},Ts)&=\rho(Ts_n,Ts)\\&\leq\tau\rho(Ts_n,Ts)\end{aligned}$$

$$\leq \varphi(\max\{\rho(s_n,s),\rho(s_n,Ts_n),\rho(s,Ts)\})$$
$$+L(\min\{d_{\rho_m}(s_n,Ts_n),d_{\rho_m}(s,Ts_n)\})$$
$$=\varphi(\max\{\rho(s_n,s),\rho(s_n,s_{n+1}),\rho(s,Ts)\})$$
$$+L(\min\{d_{\rho_m}(s_n,s_{n+1}),d_{\rho_m}(s,s_{n+1})\}) \tag{2.11}$$

Taking $n \to \infty$ in (2.11) we get

$$\rho(s,Ts) \leq \varphi(\rho(s,Ts)) < (\rho(s,Ts)$$

This is a contradiction, and so we obtain $Ts = s$. □

The uniqueness can be shown as Theorem 2.2.

Example 2.4. Let $U = [0,1]$ and $\rho(s,t) = \max\{s,t\}$. Then (U,ρ) is a complete partial metric space. Consider the mapping $T : U \to U$ defined by $T(s) = \frac{s}{10}$ for all s and $\varphi : [0,\infty) \to [0,\infty)$ be such that $\varphi(x) = \frac{x}{2}$ and $\tau = \frac{3}{2}$.Without loss of generality we assume that $s \geq t$.

Now,

$$\tau\rho(Ts,Tt) = \frac{3}{2}\rho(\frac{s}{10},\frac{t}{10}) = \frac{3}{20}s \tag{2.12}$$

On the other side

$$\varphi(\max\{\rho(s,t),\rho(s,Ts),\rho(t,Tt)\}) = \varphi(\max\{\rho(s,t),\rho(s,\frac{s}{10}),\rho(t,\frac{t}{10})\})$$
$$= \varphi(s) = \frac{1}{2}s \tag{2.13}$$

and

$$\min\{d_{\rho_m}(s,Ts),d_{\rho_m}(t,Ts)\} = \min\{d_{\rho_m}(s,\frac{s}{10}),d_{\rho_m}(t,\frac{s}{10})\} \tag{2.14}$$

Therefore from (2.12), (2.13) and (2.14) it is clear that it satisfies all the conditions of Theorem 2.2. Hence T has a fixed point, which in this case is 0.

3. Conclusion

Motivated by the work of Mebawondu et al. [9], Berinde [15, 16], and Baiz et al. [7, 8], this paper provides some results for $\tau-\varphi$-Berinde contraction mappings in partial metric spaces.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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