Available online at http://scik.org
Eng. Math. Lett. 2026, 2026:2
https://doi.org/10.28919/eml/9689
ISSN: 2049-9337

COMMON FIXED POINT THEOREMS IN FUZZY CONE METRIC SPACES VIA ψ-CONTRACTIONS

HEENA SONI*, PRACHI SINGH, SHOBHA RANI

Department of Mathematics, Govt. V.Y.T. PG Autonomous College, Durg (C.G.), 491001, India

Copyright © 2026 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. In this paper, we establish some common fixed point theorems for occasionally weakly compatible mappings in fuzzy cone metric spaces by employing the technique of ψ-contraction.

Keywords: t-norm; fuzzy cone metric space; occasionally weakly compatible mappings; ψ-contractions; fixed point.

2020 AMS Subject Classification: 46S40, 54H25.

1. INTRODUCTION

The concept of fuzzy sets was introduced by L. A. Zadeh [13] in 1965. Subsequently, many researchers developed the theory of fuzzy sets and explored its applications in various branches of mathematics and applied sciences. In 1975, Kramosil and Michalek [8] introduced the notion of fuzzy metric spaces. In 1999, R. Vasuki [12] established fixed point theorems for R-weakly commuting mappings. In 2007, Huang and Zhang [5] introduced the concept of cone metric spaces and proved several fixed point theorems for contractive mappings. In 2015, Tarkan Öner

*Corresponding author
E-mail address: heenasoni349@gmail.com
Received November 10, 2025

et al. [9] introduced the concept of fuzzy cone metric spaces, which generalizes the corresponding notion of fuzzy metric spaces due to George and Veeramani [4] and established the fuzzy cone Banach contraction theorem.

In recent years, several authors have obtained fixed point results in fuzzy cone metric spaces by employing various generalized contraction conditions. Among them, ψ-contraction type conditions have attracted considerable attention due to their ability to unify and extend many existing contractive frameworks. Motivated by these developments, the aim of this paper is to establish some common fixed point theorems for occasionally weakly compatible mappings in fuzzy cone metric spaces via ψ-contractions.

2. PRELIMINARIES

In this section, we recall some basic definitions and results that will be used throughout the paper.

Definition 2.1 ([13])**.** Let X be any set. A *fuzzy set* A in X is a function with domain X and codomain $[0,1]$.

Definition 2.2 ([9])**.** Let E be a real Banach space, θ the zero element of E and $P \subset E$ a subset. Then P is called a *cone* if and only if:

(1) P is closed, nonempty and $P \neq \{\theta\}$,

(2) if $a,b \in \mathbb{R}$ with $a,b \geq 0$ and $x,y \in P$, then $ax+by \in P$,

(3) if $x \in P$ and $-x \in P$, then $x = \theta$.

For a given cone P, a partial ordering "$\preceq$" on E with respect to P is defined by

$$x \preceq y \quad \text{if and only if} \quad y - x \in P.$$

We write $x \prec y$ if $x \preceq y$ and $x \neq y$ and $x \ll y$ if $y - x \in \text{int}(P)$. Throughout this paper, we assume that all cones have nonempty interior.

A cone P is called *normal* if there exists a constant $K > 0$ such that for all $t,s \in E$,

$$\theta \preceq t \preceq s \quad \Rightarrow \quad \|t\| \leq K\|s\|,$$

and the least positive number K satisfying this property is called the *normal constant* of P [9].

Definition 2.3 ([11])**.** A binary operation $*: [0,1]\times[0,1]\to[0,1]$ is called a *continuous t-norm* if it satisfies:

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a*1=a$ for all $a\in[0,1]$,

(4) if $a\le c$ and $b\le d$, then $a*b\le c*d$ for all $a,b,c,d\in[0,1]$.

Example 2.4. An example of a continuous t-norm is

$$a*b=\min\{a,b\}.$$

Definition 2.5 ([4])**.** A 3-tuple $(X,M,*)$ is said to be a *fuzzy metric space* if:

- X is a nonempty set,
- $*$ is a continuous t-norm,
- M is a fuzzy set on $X^2\times(0,\infty)$,

and the following conditions hold for all $x,y,z\in X$ and $t,s>0$:

(1) $M(x,y,t)>0$,

(2) $M(x,y,t)=1$ if and only if $x=y$,

(3) $M(x,y,t)=M(y,x,t)$,

(4) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$,

(5) $M(x,y,\cdot):(0,\infty)\to[0,1]$ is continuous.

Definition 2.6 ([9])**.** A 3-tuple $(X,M,*)$ is said to be a *fuzzy cone metric space* if:

- E is a real Banach space,
- P is a cone in E,
- X is a nonempty set,
- $*$ is a continuous t-norm,
- M is a fuzzy set on $X^2\times\operatorname{int}(P)$,

and the following conditions hold for all $x,y,z\in X$ and $t,s\in\operatorname{int}(P)$:

(1) $M(x,y,t)>0$,

(2) $M(x,y,t)=1$ if and only if $x=y$,

(3) $M(x,y,t) = M(y,x,t)$,

(4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,

(5) $M(x,y,\cdot) : \text{int}(P) \to [0,1]$ is continuous.

Example 2.7 ([9])**.** Let $E = \mathbb{R}^2$ and $P = \{(k_1,k_2) : k_1 \geq 0, k_2 \geq 0\}$, which is a normal cone with normal constant $K = 1$. Let $X = \mathbb{R}$, define $a * b = ab$ and let

$$M(x,y,t) = \frac{1}{e^{\frac{|x-y|}{\|t\|}}}$$

for all $x,y \in X$ and $t \gg \theta$. Then $(X,M,*)$ is a fuzzy cone metric space.

Definition 2.8 ([3])**.** Let X be a nonempty set. An element $x \in X$ is called a *common fixed point* of mappings $F : X \times X \to X$ and $T : X \to X$ if

$$x = T(x) = F(x,x).$$

Definition 2.9 ([3])**.** Let X be a nonempty set. The mappings $F : X \times X \to X$ and $T : X \to X$ are called *commutative* if

$$T(F(x,y)) = F(T(x),T(y)) \quad \text{for all } x,y \in X.$$

Definition 2.10. Let X be a set and let F,T be self-maps of X. A point $x \in X$ is called a *coincidence point* of F and T if and only if $F(x) = T(x)$. The value $w = F(x) = T(x)$ is called a *point of coincidence* of F and T.

Definition 2.11. A pair of maps U and V is called a *weakly compatible pair* if they commute at their coincidence points.

Definition 2.12 ([7])**.** Two self-maps F and T of a set X are *occasionally weakly compatible* if and only if there exists a point $x \in X$ which is a coincidence point of F and T at which F and T commute.

Definition 2.13 ([2])**.** A mapping $\psi : [0,1) \to [0,1)$ is called a ψ*-function* if:

- ψ is continuous and strictly increasing,
- $\psi(t) < t$ for all $t \in (0,1)$,

- $\lim_{t\to 0^+} \psi(t) = 0$.

A. Al-Thagafi and N. Shahzad [1] showed that occasionally weakly compatible mappings are weakly compatible, but the converse is not true.

Lemma 2.14 ([1])**.** *Let X be a set and let F,T be occasionally weakly compatible self-maps of X. If F and T have a unique point of coincidence $w = F(x) = T(x)$, then w is the unique common fixed point of F and T.*

3. Main Result

Definition 3.1. (ψ-Contraction in Fuzzy Cone Metric Spaces).
Let $(X,M,*)$ be a fuzzy cone metric space. A pair of self-mappings $A,B : X \to X$ is said to satisfy the ψ-contractive condition if there exists $\psi \in \Psi$ such that for all $x,y \in X$ and $t \gg \theta$:

$$M(Ax,By,k(t)) \geq \psi\big(M(Ux,Vy,t)\big).$$

Theorem 3.2. *Let $(X,M,*)$ be a complete fuzzy cone metric space and let A,B,U,V be self-mappings of X. Suppose the pairs $\{A,U\}$ and $\{B,V\}$ are occasionally weakly compatible. Let $\psi : [0,1) \to [0,1)$ be a ψ-function in the sense of* [2] *and suppose that for all $x,y \in X$ and all $t \gg \theta$,*

$$M\big(A(x),B(y),k(t)\big) \geq \psi\left(\min\left\{\begin{array}{l} M(U(x),V(y),t), \\ M(U(x),A(x),t), \\ M(B(y),V(y),t), \\ M(A(x),V(y),t), \\ M(B(y),U(x),t) \end{array}\right\}\right) \tag{3.1}$$

for some $k \in (0,1)$. Then there exists a unique point $w \in X$ such that

$$A(w) = U(w) = w \quad \text{and} \quad B(w) = V(w) = w.$$

In particular, A,B,U,V have a unique common fixed point in X.

Proof. Let the pairs $\{A,U\}$ and $\{B,V\}$ be occasionally weakly compatible, so there exist points $x,y \in X$ such that

$$A(x) = U(x) \quad \text{and} \quad B(y) = V(y).$$

We claim that $A(x) = B(y)$. If not, by the ψ-contraction condition (3.1), we have

$$M\big(A(x),B(y),k(t)\big) \geq \psi\left(\min \left\{ \begin{matrix} M(U(x),V(y),t), \\ M(U(x),A(x),t), \\ M(B(y),V(y),t), \\ M(A(x),V(y),t), \\ M(B(y),U(x),t) \end{matrix} \right\} \right). \tag{3.2}$$

Using $A(x) = U(x)$ and $B(y) = V(y)$, the right-hand side becomes

$$\psi\left(\min \left\{ \begin{matrix} M(A(x),B(y),t), \\ M(A(x),A(x),t), \\ M(B(y),B(y),t), \\ M(A(x),B(y),t), \\ M(B(y),A(x),t) \end{matrix} \right\} \right) = \psi\big(M(A(x),B(y),t)\big). \tag{3.3}$$

Since $\psi(s) < s$ for all $s \in (0,1)$, this implies

$$M(A(x),B(y),k(t)) \geq \psi(M(A(x),B(y),t)) < M(A(x),B(y),t),$$

a contradiction unless $M(A(x),B(y),t) = 1$, i.e., $A(x) = B(y)$.

Thus

$$A(x) = U(x) = B(y) = V(y).$$

Now suppose there exists another point z such that $A(z) = U(z)$. Applying (3.1) with $(x,y) = (z,y)$, we similarly deduce

$$A(z) = U(z) = B(y) = V(y),$$

so $A(x) = A(z)$ and hence $w := A(x) = U(x)$ is the unique point of coincidence of A and U. By Lemma 2.14, w is the only common fixed point of A and U.

Similarly, there exists a unique $z \in X$ such that $B(z) = V(z) = z$.

Assume $w \neq z$. Then, using the ψ-contraction condition again,

$$M(w,z,k(t)) = M(A(w),B(z),k(t))$$

$$\geq \psi\left(\min\begin{Bmatrix} M(U(w),V(z),t), \\ M(U(w),A(z),t), \\ M(B(z),V(z),t), \\ M(A(w),V(z),t), \\ M(B(z),U(w),t) \end{Bmatrix}\right) \tag{3.4}$$

Since $A(w) = U(w) = w$ and $B(z) = V(z) = z$, the minimum inside ψ is $M(w,z,t)$, so we get

$$M(w,z,k(t)) \geq \psi(M(w,z,t)).$$

By the defining property $\psi(s) < s$ for $s \in (0,1)$, this is impossible unless $M(w,z,t) = 1$, which means $w = z$.

Therefore $z = w$ is the unique common fixed point of A,B,U,V. Uniqueness follows from the same ψ-contraction property. □

Theorem 3.3. *Let $(X,M,*)$ be a complete fuzzy cone metric space and let A,B,U,V be self-mappings of X. Suppose the pairs $\{A,U\}$ and $\{B,V\}$ are occasionally weakly compatible. If there exists $k \in (0,1)$ and a ψ-function $\psi : [0,1) \to [0,1)$ such that*

$$M(A(x),B(y),k(t)) \geq \psi\left(\min\begin{Bmatrix} M(U(x),V(y),t), \\ M(U(x),A(x),t), \\ M(B(y),V(y),t), \\ M(A(x),V(y),t), \\ M(B(y),U(x),t) \end{Bmatrix}\right) \tag{3.5}$$

for all $x,y \in X$ and all $t \gg \theta$, then there exists a unique point $w \in X$ such that

$$A(w) = U(w) = B(w) = V(w) = w,$$

i.e., A,B,U,V have a unique common fixed point.

Proof. Let the pairs $\{A,U\}$ and $\{B,V\}$ be occasionally weakly compatible. Then there exist points $x,y\in X$ such that $A(x)=U(x)$ and $B(y)=V(y)$. We claim that $A(x)=B(y)$. If not, by the inequality in condition (3.5) we have

$$\begin{aligned} M\big(A(x),B(y),k(t)\big) &\geq \psi\left(\min\left\{\begin{matrix} M(U(x),V(y),t), \\ M(U(x),A(x),t), \\ M(B(y),V(y),t), \\ M(A(x),V(y),t), \\ M(B(y),U(x),t) \end{matrix}\right\}\right) \\ &= \psi\left(\min\left\{\begin{matrix} M(A(x),B(y),t), \\ M(A(x),A(x),t), \\ M(B(y),B(y),t), \\ M(A(x),B(y),t), \\ M(B(y),A(x),t) \end{matrix}\right\}\right) \\ &= \psi\big(M(A(x),B(y),t)\big). \end{aligned}$$

Since $\psi(t)<t$ for all $t\in(0,1)$, we obtain

$$M(A(x),B(y),k(t))\geq\psi(M(A(x),B(y),t))<M(A(x),B(y),t),$$

a contradiction. Therefore $A(x)=B(y)$ and hence $A(x)=U(x)=B(y)=V(y)$.

Suppose there is another point $z\in X$ such that $A(z)=U(z)$. Then by the contractive condition, we have $A(z)=U(z)=B(y)=V(y)$, so $A(x)=A(z)$ and

$$w=A(x)=U(x)$$

is the unique point of coincidence of A and U. By Lemma 2.14, w is the unique common fixed point of A and U.

Similarly, there exists a unique point $z \in X$ such that $B(z) = V(z) = z$. Assume $w \neq z$. Then

$$\begin{aligned} M(w,z,k(t)) &= M\big(A(w),B(z),k(t)\big) \\ &\geq \psi\left(\min \begin{Bmatrix} M(U(w),V(z),t), \\ M(U(w),A(z),t), \\ M(B(z),V(z),t), \\ M(A(w),V(z),t), \\ M(B(z),U(w),t) \end{Bmatrix} \right) \\ &= \psi\big(M(w,z,t)\big) \\ &< M(w,z,t). \end{aligned}$$

a contradiction. Therefore $w = z$ and this point is the unique common fixed point of A, B, U and V. □

Theorem 3.4. *Let $(X,M,*)$ be a complete fuzzy cone metric space and let A,B,U,V be self-mappings of X. Suppose the pairs $\{A,U\}$ and $\{B,V\}$ are occasionally weakly compatible. If there exists $k \in (0,1)$ such that for all $x,y \in X$ and $t \gg \theta$,*

$$M(A(x),B(y),k(t)) \geq \psi\left(\min \begin{Bmatrix} M(U(x),V(y),t), \\ M(A(x),U(x),t), \\ M(B(y),V(y),t), \\ M(A(x),V(y),t), \\ M(B(y),U(x),t) \end{Bmatrix} \right) \tag{3.6}$$

where $\psi : [0,1] \to [0,1]$ satisfies $\psi(t) > t$ for all $\theta \ll t < 1$, then A,B,U,V have a unique common fixed point in X.

Proof. Let the pairs $\{A,U\}$ and $\{B,V\}$ be occasionally weakly compatible. Then there exist points $x,y \in X$ such that $A(x) = U(x)$ and $B(y) = V(y)$. We claim that $A(x) = B(y)$.

By inequality (3.6), we have

$$M\big(A(x),B(y),k(t)\big) \geq \psi\left(\min\begin{Bmatrix} M(U(x),V(y),t), \\ M(U(x),A(x),t), \\ M(B(y),V(y),t), \\ M(A(x),V(y),t), \\ M(B(y),U(x),t) \end{Bmatrix}\right)$$

$$= \psi\left(\min\begin{Bmatrix} M(A(x),B(y),t), \\ M(A(x),A(x),t), \\ M(B(y),B(y),t), \\ M(A(x),B(y),t), \\ M(B(y),A(x),t) \end{Bmatrix}\right)$$

$$= \psi\big(M(A(x),B(y),t)\big).$$

Since $\psi(t) > t$ for all $\theta \ll t < 1$, it follows that

$$M(A(x),B(y),k(t)) > M(A(x),B(y),t),$$

which is a contradiction unless $A(x) = B(y)$. Thus $A(x) = U(x) = B(y) = V(y)$.

Suppose there exists another point $z \in X$ such that $A(z) = U(z)$. Then by (3.6) we similarly obtain $A(z) = U(z) = B(y) = V(y)$, so $A(x) = A(z)$. Let $w = A(x) = U(x)$. Then w is the unique point of coincidence of A and U and by Lemma 2.14, w is the unique common fixed point of A and U.

Similarly, there exists a unique $z \in X$ such that $z = B(z) = V(z)$. Arguing as before, we conclude $w = z$ and hence w is the unique common fixed point of A, B, U and V. □

Corollary 3.5. *Let $(X, M, *)$ be a complete fuzzy cone metric space and let A, B, U, V be self-mappings of X. Let the pairs $\{A, U\}$ and $\{B, V\}$ be occasionally weakly compatible. If there* *exists* $k \in (0,1)$ *such that, for all* $x, y \in X$ *and* $t \gg \theta$*,*

$$M\big(A(x),B(y),k(t)\big) \geq \psi\left(\min\left\{\begin{array}{l} M(U(x),V(y),t), \\ M(A(x),U(x),t), \\ M(B(y),V(y),t), \\ M(B(y),U(x),2t), \\ M(A(x),V(y),t) \end{array}\right\}\right). \tag{3.7}$$

where $\psi : [0,1]^5 \to [0,1]$ *satisfies* $\psi(t,1,1,1,t) > t$ *for all* $\theta \ll t < 1$*, then there exists a unique common fixed point of A, B, U and V.*

Proof. Let the pairs $\{A,U\}$ and $\{B,V\}$ be occasionally weakly compatible. Then there exist points $x, y \in X$ such that $A(x) = U(x)$ and $B(y) = V(y)$.

From inequality (3.7) we have

$$M\big(A(x),B(y),k(t)\big) \geq \psi\left(\min\left\{\begin{array}{l} M(U(x),V(y),t), \\ M(A(x),U(x),t), \\ M(B(y),V(y),t), \\ M(B(y),U(x),2t), \\ M(A(x),V(y),t) \end{array}\right\}\right). \tag{3.8}$$

Since $A(x) = U(x)$ and $B(y) = V(y)$, the above becomes

$$M\big(A(x),B(y),k(t)\big) \geq \psi\left(\min\left\{\begin{array}{l} M(A(x),B(y),t), \\ 1, \\ 1, \\ M(B(y),A(x),2t), \\ M(A(x),B(y),t) \end{array}\right\}\right) \tag{3.9}$$

By the assumption on ψ, we have

$$\psi\left(\min\begin{Bmatrix} M(A(x),B(y),t), \\ 1, \\ 1, \\ M(B(y),A(x),2t), \\ M(A(x),B(y),t) \end{Bmatrix}\right) > M\big(A(x),B(y),t\big). \tag{3.10}$$

whenever $\theta \ll M(A(x),B(y),t) < 1$. This is a contradiction unless $M(A(x),B(y),t) = 1$, which implies $A(x) = B(y)$.

Thus $A(x) = U(x) = B(y) = V(y)$. By the same reasoning as in Theorem 3.4, there exists a unique point w which is a common fixed point of A, B, U and V. □

Theorem 3.6. *Let $(X,M,*)$ be a complete fuzzy cone metric space. Then continuous self-mappings U and V of X have a common fixed point in X if and only if there exists a self-mapping A of X such that:*

(1) $A(X) \subset V(X) \cap U(X)$,

(2) the pairs $\{A,U\}$ and $\{A,V\}$ are weakly compatible,

(3) there exists $k \in (0,1)$ such that for every $x,y \in X$ and $t \gg \theta$,

$$M\big(A(x),A(y),k(t)\big) \geq \psi\left(\min\begin{Bmatrix} M(U(x),V(y),t), \\ M(A(x),U(x),t), \\ M(A(y),V(y),t), \\ M(A(x),V(y),t) \end{Bmatrix}\right), \tag{3.11}$$

where $\psi : [0,1]^4 \to [0,1]$ satisfies $\psi(t,1,1,t) > t$ for all $\theta \ll t < 1$.

Then A, U and V have a unique common fixed point.

Proof. Since weak compatibility implies occasional weak compatibility, the result follows directly from Theorem 3.4 by noting that the inequality (3.11) is of the same type as in Theorem 3.4, with $B = A$ and the minimum over the terms

$$M(U(x),V(y),t), \quad M(A(x),U(x),t), \quad M(A(y),V(y),t), \quad M(A(x),V(y),t)$$

in place of the multiplicative condition. Therefore, A, U and V have a unique common fixed point. □

Theorem 3.7. *Let $(X, M, *)$ be a complete fuzzy cone metric space and let A and U be self-mappings of X. Suppose that A and U are occasionally weakly compatible. If there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t \gg \theta$,*

$$M\big(U(x), U(y), k(t)\big) \geq \alpha M\big(A(x), A(y), t\big) + \beta \min \begin{Bmatrix} M(A(x), A(y), t), \\ M(U(x), A(x), t), \\ M(U(y), A(y), t) \end{Bmatrix} \tag{3.12}$$

where $\alpha, \beta > 0$ and $\alpha + \beta > 1$, then A and U have a unique common fixed point.

Proof. Since the pair $\{A, U\}$ is occasionally weakly compatible, there exists $x \in X$ such that $A(x) = U(x)$. Suppose there exists another point $y \in X$ with $A(y) = U(y)$. We claim that $U(x) = U(y)$.

From (3.12), we have

$$M\big(U(x), U(y), k(t)\big) = \alpha M\big(A(x), A(y), t\big) + \beta \min \begin{Bmatrix} M(A(x), A(y), t), \\ M(U(x), A(x), t), \\ M(U(y), A(y), t) \end{Bmatrix}.$$

Since $A(x) = U(x)$ and $A(y) = U(y)$, this becomes

$$M\big(U(x), U(y), k(t)\big) = \alpha M\big(U(x), U(y), t\big) + \beta \min \begin{Bmatrix} M(U(x), U(y), t), \\ M(U(x), U(x), t), \\ M(U(y), U(y), t) \end{Bmatrix}.$$

Noting that $M(U(x), U(x), t) = M(U(y), U(y), t) = 1$, we obtain

$$M(U(x), U(y), k(t)) = (\alpha + \beta) M(U(x), U(y), t).$$

Since $\alpha + \beta > 1$, the above inequality leads to a contradiction unless $M(U(x), U(y), t) = 1$, i.e., $U(x) = U(y)$.

Therefore $A(x) = A(y)$ and $A(x)$ is unique. By Lemma 2.14, A and U have a unique common fixed point. □

4. Numerical Example

Example 4.1. Let

$$X = [0,\infty), \quad E = \mathbb{R}, \quad P = [0,\infty)$$

with the usual cone P in $\mathbb{R}$ and the standard norm $\|\cdot\|$. Define $M : X \times X \times (0,\infty) \to [0,1]$ by

$$M(x,y,t) = \frac{t}{t+|x-y|}, \quad t > 0.$$

It is well-known that $(X,M,*)$, with the product t-norm

$$a * b = a \cdot b,$$

is a complete fuzzy cone metric space.

Let the ψ-function be defined by

$$\psi(s) = \frac{s}{2}, \quad s \in [0,1),$$

which clearly satisfies $\psi(s) < s$ for all $s \in (0,1)$.

Define four self-mappings $A,B,U,V : X \to X$ by

$$A(x) = \frac{x}{4}, \quad U(x) = \frac{x}{4}, \quad B(x) = \frac{x}{4}, \quad V(x) = \frac{x}{4}.$$

Here:

- We have $A = U$ and $B = V$, so the pairs $\{A,U\}$ and $\{B,V\}$ are occasionally weakly compatible (in fact, they commute everywhere).
- The mappings have a common fixed point $w = 0$ since $A(0) = 0$, $B(0) = 0$, $U(0) = 0$ and $V(0) = 0$.

Now, for any $x,y \in X$ and $t > 0$, we have

$$M(Ax,By,k(t)) = \frac{k(t)}{k(t)+\frac{|x-y|}{4}}.$$

The five terms inside the "min" in (3.1) are:

$$\begin{aligned}
M(U(x),V(y),t) &= M\left(\frac{x}{4},\frac{y}{4},t\right) = \frac{t}{t+\frac{|x-y|}{4}},\\
M(U(x),A(x),t) &= M\left(\frac{x}{4},\frac{x}{4},t\right) = 1,\\
M(B(y),V(y),t) &= M\left(\frac{y}{4},\frac{y}{4},t\right) = 1,\\
M(A(x),V(y),t) &= \frac{t}{t+\frac{|x-y|}{4}},\\
M(B(y),U(x),t) &= \frac{t}{t+\frac{|x-y|}{4}}.
\end{aligned}$$

Hence

$$\min\left\{\begin{matrix} M(U(x),V(y),t),\\ M(U(x),A(x),t),\\ M(B(y),V(y),t),\\ M(A(x),V(y),t),\\ M(B(y),U(x),t) \end{matrix}\right\} = \frac{t}{t+\frac{|x-y|}{4}}.$$

Therefore

$$\psi\left(\min\left\{\begin{matrix} M(U(x),V(y),t),\\ M(U(x),A(x),t),\\ M(B(y),V(y),t),\\ M(A(x),V(y),t),\\ M(B(y),U(x),t) \end{matrix}\right\}\right) = \frac{1}{2}\cdot\frac{t}{t+\frac{|x-y|}{4}}.$$

Choosing $k = \frac{1}{2} \in (0,1)$, we have

$$M(Ax,By,k(t)) = \frac{\frac{t}{2}}{\frac{t}{2}+\frac{|x-y|}{4}} \geq \frac{1}{2}\cdot\frac{t}{t+\frac{|x-y|}{4}} = \psi\left(\frac{t}{t+\frac{|x-y|}{4}}\right).$$

Thus the ψ-contraction condition (3.1) is satisfied.

By Theorem 3.2, A,B,U,V have a unique common fixed point in X, which is $w = 0$.

5. Conclusion

We have proved common fixed point theorems for occasionally weakly compatible mappings in fuzzy cone metric spaces using ψ-contraction. These results generalize existing fixed point theorems and highlight the utility of ψ-contractions in this setting.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] M. A. Al-Thagafi and N. Shahzad, Generalized I-nonexpansive selfmaps and invariant approximations, Acta Math. Sin. (Engl. Ser.) 24 (2008), 867–876.
[2] P. N. Dutta and B. S. Choudhury, A generalisation of contraction principle in metric spaces, Fixed Point Theory Appl. (2008), art. 406368.
[3] J. X. Fang, Common fixed point theorems of compatible and weakly compatible maps in Menger spaces, Nonlinear Anal. 71 (2009), 1833–1843.
[4] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets Syst. 64 (1994), 395–399.
[5] L.-G. Huang and X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332 (2007), 1468–1476.
[6] G. Jungck and B. E. Rhoades, Fixed point theorems for occasionally weakly compatible mappings, Fixed Point Theory 7 (2006), 287–296.
[7] G. Jungck and B. E. Rhoades, Fixed point for set valued functions without continuity, Indian J. Pure Appl. Math. 29 (1998), 771–779.
[8] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika 11 (1975), 336–344.
[9] T. Oner, M. B. Kandemir and B. Tanay, Fuzzy cone metric spaces, J. Nonlinear Sci. Appl. 8 (2015), 610–616.
[10] S. Rezapour and R. Hamlbarani, Some notes on paper Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 345 (2008), 719–724.
[11] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific J. Math. 10 (1960), 313–334.
[12] R. Vasuki, Common fixed points for R-weakly commuting maps in fuzzy metric spaces, Indian J. Pure Appl. Math. 30 (1999), 419–423.
[13] L. A. Zadeh, Fuzzy sets, Inform. Control 8 (1965), 338–353.