Available online at http://scik.org
J. Math. Comput. Sci. 3 (2013), No. 3, 788-798
ISSN: 1927-5307

CONVERGENCE THEOREM ON TOTAL ASYMPTOTICALLY PSEUDOCONTRACTIVE MAPPINGS

ZI-MING WANG

Department of Foundation, Shandong Yingcai University, Jinan 250104, P.R. China

Abstract. In this paper, a modified Ishikawa iterative algorithm is introduced for finding a fixed point of a total asymptotically pseudocontractive mapping. Furthermore, strong convergence result is obtained in a real Hilbert space. Our result improves the corresponding result of Qin, Cho, and Kang [4].

Keywords: Total asymptotically pseudocontractive mapping; Strong convergence; Fixed point; Hilbert space.

2000 AMS Subject Classification: 47H05, 47H09.

1. Introduction

Throughout this paper, we always assume that H is a real Hilbert space, whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$. The symbols $\rightarrow$ and $\rightharpoonup$ are denoted by strong convergence and weak convergence, respectively. $w_w(x_n) = \{x : \exists\ x_{n_i} \rightharpoonup x\}$ denotes the weak $w-$limit set of $\{x_n\}$. Let C be a nonempty closed and convex subset of H and $T : C \rightarrow C$ a mapping. In this paper, we denote the fixed point set of T by $F(T)$.

Recall the following definitions. In 1991, Schu [1] (see also [2]) introduced the class of asymptotically pseudocontractive mappings.

Received April 14, 2013

Definition 1.1 (see [1, 2]) T *is said to be asymptotically pseudocontractive if there exists a sequence* $\{k_n\} \subset [1, \infty)$ *with* $k_n \to 1$ *as* $n \to \infty$ *such that*

$$\langle T^n x - T^n y, x - y\rangle \leq k_n \|x - y\|^2, \quad \forall\ x,\ y \in C. \tag{1.1}$$

In 2010, Qin et al. [3] introduced the class of asymptotically pseudocontractive mappings in the intermediate sense.

Definition 1.2 (see [3]) T *is said to be an asymptotically pseudocontractive mapping in the intermediate sense if there exists a sequence* $\{k_n\} \subset [1, \infty)$ *with* $k_n \to 1$ *as* $n \to \infty$ *such that*

$$\limsup_{n\to\infty} \sup_{x,y\in C} (\langle T^n x - T^n y, x - y\rangle - k_n \|x - y\|^2) \leq 0, \quad \forall\ x,\ y \in C. \tag{1.2}$$

Put

$$\xi_n = \max\{0, \sup_{x,y\in C} (\langle T^n x - T^n y, x - y\rangle - k_n \|x - y\|^2)\}.$$

It follows that $\xi_n \to 0$, as $n \to \infty$. Then,(1.2) is reduced to the following:

$$\langle T^n x - T^n y, x - y\rangle \leq k_n \|x - y\|^2 + \xi_n. \quad \forall\ n \geq 1,\ x,\ y \in C. \tag{1.3}$$

It is easy to see that (1.3) is equivalent to

$$\|T^n x - T^n y\|^2 \leq (2k_n - 1)\|x - y\|^2 + \|x - y - (T^n x - T^n y)\|^2 + 2\xi_n, \quad \forall\ n \geq 1,\ x,\ y \in C.$$

In 2011, Qin, Cho, and Kang [4] introduced the class of total asymptotically pseudocontractive mappings.

Definition 1.3 (see [4]) T *is said to be total asymptotically pseudocontractive if there exist two sequences* $\{\mu_n\} \subset [0, \infty)$ *and* $\{\xi_n\} \subset [0, \infty)$ *with* $\mu_n \to 0$ *and* $\xi_n \to 0$ *as* $n \to \infty$ *such that*

$$\langle T^n x - T^n y, x - y\rangle \leq \|x - y\|^2 + \mu_n \phi(\|x - y\|) + \xi_n, \quad \forall\ n \geq 1,\ x,\ y \in C, \tag{1.4}$$

where $\phi : [0, \infty) \to [0, \infty)$ *is a continuous and strictly increasing function with* $\phi(0) = 0$.

It is easy to see that (1.4) is equivalent to the following: for all $n \geq 1$, $x,\ y \in C$,

$$\|T^n x - T^n y\|^2 \leq \|x - y\|^2 + 2\mu_n \phi(\|x - y\|) + \|x - y - (T^n x - T^n y)\|^2 + 2\xi_n, \tag{1.5}$$

If $\phi(\lambda) = \lambda^2$, then (1.4) is reduced to

$$\langle T^n x - T^n y, x - y\rangle \leq (1 + \mu_n)\|x - y\|^2 + \xi_n, \quad \forall\ n \geq 1,\ x,\ y \in C. \tag{1.6}$$

Put

$$\xi_n = \max\{0, \sup_{x,\ y \in C} (\langle T^n x - T^n y, x - y\rangle - (1 + \mu_n)\|x - y\|^2)\}. \tag{1.7}$$

If $\phi(\lambda) = \lambda^2$, then the class of total asymptotically pseudocontractive mappings is reduced to the class of asymptotically pseudocontractive mappings in the intermediate sense.

In recent years, iterative methods for approximating fixed points of total asymptotically pseudocontractive mapping have been studied by some authors. In 2011, Qin, Cho, and Kang [4] proved a weak convergence theorem for a total asymptotically pseudocontractive mapping by the modified Ishikawa iterative process which was introduced by Schu [1]. Very recently, Ding and Quan [5] introduced a modified Mann iterative algorithm for a total asymptotically pseudocontractive mapping. Moreover, they proved a strong convergence theorem for finding the fixed point of a total asymptotically pseudocontractive mapping.

Motivated and inspired by the above facts, the purpose of this paper will introduce a new modified Ishikawa iterative algorithm for a total asymptotically pseudocontractive mapping. And by the new iterative algorithm, the strong convergence theorem for find the fixed point of a total asymptotically pseudocontractive mapping can be obtained. The result of this paper improves the corresponding result in Qin, Cho, and Kang [4].

2. Preliminaries

A mapping $T : C \to C$ is said to be uniformly $L-$Lipschitzian if there exists some $L > 0$ such that

$$\|T^n x - T^n y\| \leq L\|x - y\|, \quad \forall\ x,\ y \in C,\ n \geq 1. \tag{2.1}$$

Let C be a nonempty closed convex subset of a real Hilbert space H. For every point $x \in H$, there exists a unique nearest point in C, denoted by $P_C x$, such that $\|x - P_C x\| \leq \|x - y\|$ holds for all $y \in C$, where P_C is said to be the metric projection of H onto C.

In order to prove our main results, we also need the following lemmas.

Lemma 2.1. *In a real Hilbert space, the following inequality holds:*

$$\|ax + (1-a)y\|^2 = a\|x\|^2 + (1-a)\|y\|^2 - a(1-a)\|x-y\|^2, \quad \forall\, a \in [0,1],\ x,\ y \in C.$$

Lemma 2.2. (see [6]) *Let C be a nonempty closed convex subset of a real Hilbert space H. Given $x \in H$ and $z \in C$, then $z = P_C x$ if and only if*

$$\langle x - z, y - z \rangle \leq 0$$

holds for all $y \in C$.

Lemma 2.3. (see [7]) *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $\{x_n\}$ be a sequence in H and $u \in H$. Let $q = P_C u$. If $\{x_n\}$ is such that $\omega_w(x_n) \subset C$ and satisfies the condition*

$$\|x_n - u\| \leq \|u - q\| \quad \text{for all } n.$$

Then $x_n \to q$.

Lemma 2.4. (Demiclosedness principle) (see [5]) *Let C be a nonempty bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian and total asymptotically pseudocontractive mapping. Suppose there exists $M^* > 0$ such that $\phi(\lambda_n) \leq M^* \lambda_n$ for an arbitrary positive real sequence λ_n, then $I - T$ is demiclosed at zero, where I is the identical mapping.*

Lemma 2.5. (see [5]) *Let C be a nonempty bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian and total asymptotically pseudocontractive mapping. Suppose there exists $M^* > 0$ such that $\phi(\lambda_n) \leq M^* \lambda_n$ for an arbitrary positive real sequence λ_n, then $F(T)$ is a closed convex subset of C.*

3. Main results

Theorem 3.1. *Let C be a nonempty bounded and closed convex subset of a real Hilbert space H, and let $T : C \to C$ be a uniformly $L-$Lipschitzian and total asymptotically pseudo-contraction. Suppose there exists $M^* > 0$ such that $\phi(\lambda_n) \le M^*\lambda_n$ for an arbitrary positive real sequence λ_n, and $F(T) \neq \emptyset$. For an arbitrary $x_1 \in C$, let $\{x_n\}$ be a sequence generated by the following manner:*

$$\begin{cases} z_n = (1-\beta_n)x_n + \beta_n T^n x_n, \\ y_n = (1-\alpha_n)x_n + \alpha_n T^n z_n, \\ C_n = \{u \in C : \|y_n - u\|^2 \le \|x_n - u\|^2 - \alpha_n\beta_n(1-2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n\}, \\ Q_n = \{u \in C : \langle x_1 - x_n, x_n - u\rangle \ge 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1. \end{cases} \tag{3.1}$$

where $\theta_n = 2\alpha_n(1+\beta_n)[\xi_n + \mu_n M^*(diam\ C)]$ for each $n \ge 1$. Assume that $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in (0,1) satisfying $a \le \alpha_n \le \beta_n \le b$ for some $a > 0$ and some $b \in (0, L^{-2}[\sqrt{1+L^2} - 1])$. Then$\{x_n\}$ converges strongly to $\{P_{F(T)}x_1\}$, where $P_{F(T)}$ is the projection of H onto $F(T)$.

Proof. We split the proof into eight steps.

Step 1. Show that $P_{F(T)}x_1$ is well defined for every $x_1 \in C$.

In view of Lemma 2.5, we know that $F(T)$ is a closed and convex subset of C. Therefore, $P_{F(T)}x_1$ is well defined for every $x_1 \in C$.

Step 2. Show that $C_n \cap Q_n$ is closed and convex for each $n \ge 1$.

It is obvious that Q_n is closed and convex and C_n is closed for each $n \ge 1$. Therefore, we only need to prove that C_n is convex for each $n \ge 1$. Let $w_1 \in C_n$ and $w_2 \in C_n$. Put $w = tw_1 + (1-t)w_2$, where $t \in (0,1)$. Next, we show that $w \in C_n$. From the constructions of C_n, we have

$$\|y_n - w_1\|^2 \leq \|x_n - w_1\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n \tag{3.2}$$

and

$$\|y_n - w_2\|^2 \leq \|x_n - w_2\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n. \tag{3.3}$$

In view of (3.2) and (3.3), we have

$$\begin{aligned}
\|y_n - w\|^2 &= \|y_n - (tw_1 + (1-t)w_2)\|^2 \\
&= \|t(y_n - w_1) + (1-t)(y_n - w_2)\|^2 \\
&= t\|y_n - w_1\|^2 + (1-t)\|y_n - w_2\|^2 - t(1-t)\|w_2 - w_1\|^2 \\
&\leq t[\|x_n - w_1\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n] \\
&\quad + (1-t)[\|x_n - w_2\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n] \\
&\quad - t(1-t)\|w_2 - w_1\|^2 \\
&= t\|x_n - w_1\|^2 + (1-t)\|x_n - w_2\|^2 - t(1-t)\|w_2 - w_1\|^2 \\
&\quad - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n \\
&= \|x_n - w\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n.
\end{aligned}$$

This implies $w \in C_n$, that is, C_n is convex for each $n \geq 1$. Hence, we obtain that $C_n \cap Q_n$ is closed and convex for each $n \geq 1$. Therefore, $P_{C_n \cap Q_n} x_1$ is well defined for every $n \geq 1$ and $x_1 \in C$.

Step 3. Show that $F(T) \subset C_n \cap Q_n$ for each $n \geq 1$.

Let $p \in F(T)$. From Lemma 2.1, and the algorithm (3.1), we see that

$$\begin{aligned}
\|y_n - p\|^2 &= \|(1-\alpha_n)(x_n - p) + \alpha_n(T^n z_n - p)\|^2 \\
&= (1-\alpha_n)\|x_n - p\|^2 + \alpha_n\|T^n z_n - p\|^2 - \alpha_n(1-\alpha_n)\|T^n z_n - x_n\|^2 \\
&\le \alpha_n(\|z_n - p\|^2 + 2\mu_n\phi(\|z_n - p\|) + \|z_n - T^n z_n\|^2 + 2\xi_n) \\
&\quad + (1-\alpha_n)\|x_n - p\|^2 - \alpha_n(1-\alpha_n)\|T^n z_n - x_n\|^2 \\
&\le \alpha_n(\|z_n - p\|^2 + 2\mu_n M^*\|z_n - p\| + \|z_n - T^n z_n\|^2 + 2\xi_n) \\
&\quad + (1-\alpha_n)\|x_n - p\|^2 - \alpha_n(1-\alpha_n)\|T^n z_n - x_n\|^2 \\
&\le \alpha_n(\|z_n - p\|^2 + 2\mu_n M^*(diam\ C) + \|z_n - T^n z_n\|^2 + 2\xi_n) \\
&\quad + (1-\alpha_n)\|x_n - p\|^2 - \alpha_n(1-\alpha_n)\|T^n z_n - x_n\|^2
\end{aligned}$$

$$\begin{aligned}
&= \alpha_n\|z_n - p\|^2 + \alpha_n\|z_n - T^n z_n\|^2 + (1-\alpha_n)\|x_n - p\|^2 \\
&\quad - \alpha_n(1-\alpha_n)\|T^n z_n - x_n\|^2 + 2\alpha_n\mu_n M^*(diam\ C) + 2\alpha_n\xi_n,
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
\|z_n - p\|^2 &= \|(1-\beta_n)(x_n - p) + \beta_n(T^n x_n - p)\|^2 \\
&= (1-\beta_n)\|x_n - p\|^2 + \beta_n\|T^n x_n - p\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\
&\le \beta_n(\|x_n - p\|^2 + 2\mu_n\phi(\|x_n - p\|) + \|x_n - T^n x_n\|^2 + 2\xi_n) \\
&\quad + (1-\beta_n)\|x_n - p\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\
&\le \beta_n\|x_n - p\|^2 + 2\beta_n\mu_n M^*\|x_n - p\| + \beta_n\|z_n - T^n z_n\|^2 + 2\beta_n\xi_n \\
&\quad + (1-\beta_n)\|x_n - p\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\
&\le \beta_n\|x_n - p\|^2 + 2\beta_n\mu_n M^*(diam\ C) + \beta_n\|x_n - T^n x_n\|^2 + 2\beta_n\xi_n \\
&\quad + (1-\beta_n)\|x_n - p\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\
&= \|x_n - p\|^2 + \beta_n^2\|x_n - T^n x_n\|^2 + 2\beta_n\mu_n M^*(diam\ C) + 2\beta_n\xi_n,
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}\|z_n - T^n z_n\|^2 &= \|(1-\beta_n)(x_n - T^n z_n) + \beta_n(T^n x_n - T^n z_n)\|^2 \\ &= (1-\beta_n)\|x_n - T^n z_n\|^2 + \beta_n\|T^n x_n - T^n z_n\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\ &\leq (1-\beta_n)\|x_n - T^n z_n\|^2 + \beta_n L^2\|x_n - z_n\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\ &= (1-\beta_n)\|x_n - T^n z_n\|^2 + \beta_n L^2\|x_n - T^n x_n\|^2 - \beta_n(1-\beta_n)\|T^n x_n - x_n\|^2 \\ &= (1-\beta_n)\|x_n - T^n z_n\|^2 + \beta_n(\beta_n^2 L^2 + \beta_n - 1)\|x_n - T^n x_n\|^2. \end{aligned} \tag{3.6}$$

Substituting (3.5) and (3.6) into (3.4), we obtain

$$\begin{aligned}\|y_n - p\|^2 \leq\ & \alpha_n[\|x_n - p\|^2 + \beta_n^2\|x_n - T^n x_n\|^2 + 2\beta_n\mu_n M^*(diam\ C) + 2\beta_n\xi_n] \\ & + \alpha_n[(1-\beta_n)\|x_n - T^n z_n\|^2 + \beta_n(\beta_n^2 L^2 + \beta_n - 1)\|x_n - T^n x_n\|^2] \\ & + (1-\alpha_n)\|x_n - p\|^2 - \alpha_n(1-\alpha_n)\|T^n z_n - x_n\|^2 \\ & + 2\alpha_n\mu_n M^*(diam\ C) + 2\alpha_n\xi_n \\ \leq\ & \|x_n - p\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n, \end{aligned}$$

where $\theta_n = 2\alpha_n(1+\beta_n)[\xi_n + \mu_n M^*(diam\ C)]$ for each $n \geq 1$. This follows $p \in C_n$ for each $n \geq 1$; that is $F(T) \subset C_n$ for each $n \geq 1$.

Next, we show that $F(T) \subset Q_n$ for each $n \geq 1$. By inductions, it is obvious that $F(T) \subset Q_1 = C$. Suppose that $F(T) \subset Q_k$ for some $k > 1$. Since x_{k+1} is the projection of x_1 onto $C_k \cap Q_k$, we see from Lemma 2.2 that

$$\langle x_1 - x_{k+1}, x_{k+1} - x\rangle \geq 0, \qquad \forall\ x \in C_k \cap Q_k.$$

By the induction assumption, we know that $F(T) \subset C_k \cap Q_k$. In particular, for any $q \in F(T) \subset C$, we also have

$$\langle x_1 - x_{k+1}, x_{k+1} - q\rangle \geq 0,$$

which implies that $y \in Q_{k+1}$; that is, $F(T) \subset Q_{k+1}$. This follows that $F(T) \subset Q_n$ for each $n \geq 1$. Hence, $F(T) \subset C_n \cap Q_n$ for each $n \geq 1$.

Step 4. Show that $\lim_{n\to\infty} \|x_n - x_1\|$ exists.

In view of the algorithm (3.1), we have $x_n = P_{Q_n}x_1$ and $x_{n+1} \in Q_n$ which imply that

$$\|x_1 - x_n\| \le \|x_1 - x_{n+1}\|.$$

This shows that the sequence $\{\|x_n - x_1\|\}$ is nondecreasing. Since the set C is bounded, one has $\lim_{n\to\infty} \|x_n - x_1\|$ exists.

Step 5. Show that $x_{n+1} - x_n \to 0$ as $n \to \infty$.

Since $x_n = P_{Q_n}x_1$ and $x_{n+1} = P_{C_n \cap Q_n}x_1 \in Q_n$. This implies that

$$\langle x_{n+1} - x_n, x_1 - x_n\rangle \le 0, \tag{3.7}$$

From (3.7), we have

$$\begin{aligned}\|x_{n+1} - x_n\|^2 &= \|(x_{n+1} - x_1) + (x_1 - x_n)\|^2\\ &= \|x_{n+1} - x_1\|^2 + \|x_1 - x_n\|^2 + 2\langle x_{n+1} - x_1, x_1 - x_n\rangle\\ &= \|x_{n+1} - x_1\|^2 + \|x_1 - x_n\|^2 + 2\langle x_{n+1} - x_n + x_n - x_1, x_1 - x_n\rangle\\ &= \|x_{n+1} - x_1\|^2 - \|x_1 - x_n\|^2 + 2\langle x_{n+1} - x_n, x_1 - x_n\rangle\\ &\le \|x_{n+1} - x_1\|^2 - \|x_1 - x_n\|^2.\end{aligned}$$

From Step 3, we have $x_{n+1} - x_n \to \infty$ as $n \to \infty$.

Step 6. Show that $T^n x_n - x_n \to 0$ as $n \to \infty$.

In view of $x_{n+1} \in C_n$, we see that

$$\|y_n - x_{n+1}\|^2 \le \|x_n - x_{n+1}\|^2 - \alpha_n\beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2 + \theta_n. \tag{3.8}$$

On the other hand, we have

$$\begin{aligned}\|y_n - x_{n+1}\|^2 &= \|y_n - x_n + x_n - x_{n+1}\|^2\\ &= \|y_n - x_n\|^2 + \|x_n - x_{n+1}\|^2 + 2\langle y_n - x_n, x_n - x_{n+1}\rangle.\end{aligned} \tag{3.9}$$

Since $y_n = (1 - \alpha_n)x_n + \alpha_n T^n z_n$, from (3.8) and (3.9), we have

$$\alpha_n\|T^n z_n - x_n\|^2 + 2\langle T^n z_n - x_n, x_n - x_{n+1}\rangle \le \frac{\theta_n}{\alpha_n} - \beta_n(1 - 2\beta_n - \beta_n^2 L^2)\|x_n - T^n x_n\|^2. \tag{3.10}$$

From the assumption, we have

$$1-2\beta_n-\beta_n^2L^2 \geq 1-2b-b^2L^2 > 1-2\frac{\sqrt{1+L^2}}{L^2}-(\frac{\sqrt{1+L^2}}{L^2})^2L^2=0.$$

It follows from (3.10) that

$$a(1-2b-b^2L^2)\|T^nx_n-x_n\|^2 \leq \frac{\theta_n}{\alpha_n}+2\|T^nz_n-x_n\|\|x_n-x_{n+1}\|.$$

Since $\mu_n,\ \xi_n \to 0$, as $n\to\infty$, It follows

$$\lim_{n\to\infty}\frac{\theta_n}{\alpha_n}=\lim_{n\to\infty}2(1+\beta_n)[\xi_n+\mu_nM^*(diam\ C)]=0.$$

And using Step 4, we obtain

$$\lim_{n\to\infty}\|T^nx_n-x_n\|=0.$$

Step 7. Show that $Tx_n-x_n\to 0$ as $n\to\infty$.

Noticing that

$$\begin{aligned}\|x_n-Tx_n\| &= \|x_n-x_{n+1}\|+\|x_{n+1}-T^{n+1}x_{n+1}\|\\ &\quad+\|T^{n+1}x_{n+1}-T^{n+1}x_n\|+\|T^{n+1}x_n-Tx_n\|\\ &\leq (1+L)\|x_n-x_{n+1}\|+\|x_{n+1}-T^{n+1}x_{n+1}\|+L\|T^nx_n-x_n\|.\end{aligned}$$

From Step 4 and Step 5, $\lim_{n\to\infty}\|x_n-Tx_n\|=0$. That is, $Tx_n-x_n\to 0$ as $n\to\infty$.

Step 8. Show that $x_n\to\widehat{p}$, where $\widehat{p}=P_{F(T)}x_1$ as $n\to\infty$.

Let x_{n_k} be a subsequence of x_n such that $x_{n_k}\rightharpoonup\widehat{x}\in C$ as $k\to\infty$, then by Lemma 2.4, we have $\widehat{x}\in F(T)$; that is $\omega_w(x_n)\subset F(T)$. Let $\widehat{p}=P_{F(T)}x_1$, from $x_n=P_{Q_n}x_1$ and $F(T)\subset Q_n$, we see that $\|x_n-x_1\|\leq\|\widehat{p}-x_1\|$. From Lemma 2.3, we can obtain $x_n\to\widehat{p}$, where $\widehat{p}=P_{F(T)}x_1$ as $n\to\infty$. This completes the proof.

4. Acknowledgements

The author is supported by the Project of Shandong Province Higher Educational Science and Technology Program (grant No. J13LI51) and the Foundation of Shandong Yingcai University (grant No. 12YCZDZR03).

References

[1] J. Schu, Iterative construction of fixed points of asymptotically nonexpansive mappings, Journal of Mathematical Analysis and Applications, 158 (1991) 407-413.

[2] J. Schu, Weak and strong convergence to fixed points of asymptotically nonexpansive mappings, Bulletin of the Australian Mathematical Society, 43 (1991) 153-159.

[3] X. Qin et al., Convergence theorems on asymptotically pseudocontractive mappings in the intermediate sense, Fixed Point Theory and Applications, vol. 2010, Article ID 186874, 14 pages, 2010.

[4] Xiaolong Qin, Sun Young Cho, and Shin Min Kang, Weak Convergence Theorem for Total Asymptotically Pseudocontractive Mappings in Hilbert Spaces, Fixed Point Theory and Applications, Vol. 2011, Article ID 859795, 11 pages, 2011.

[5] Chuan Ding and Jing Quan, A Strong Convergence Theorem for Total Asymptotically Pseudocontractive Mappings in Hilbert Spaces, Abstract and Applied Analysis, Vol. 2012, Article ID 127851, 8 pages, 2012.

[6] G. Marino and H.-K. Xu, Weak and strong convergence theorems for strict pseudo-contractions in Hilbert spaces, Journal of Mathematical Analysis and Applications, 329 (2007) 336-346.

[7] Carlos Martinez-Yanes, Hong-Kun Xu, Strong convergence of the CQ method for fixed point iteration processes, Nonlinear Anal. 64 (2006) 2400-2411.