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MILDLY FUZZY NORMAL SPACES AND SOME FUNCTIONS

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Abstract. In this paper, Mildly fuzzy normal spaces and some new fuzzy topological functions are introduced. Characterizations and properties of such new notions are studied. Some preservation theorems for mildly fuzzy normal spaces are obtained.

Keywords: mildly fuzzy normal space, rgf-continuous function, completely fuzzy continuous function, almost rgf-continuous function, almost gf-continuous function, fuzzy topological space.

2000 AMS Subject Classification: 57A10, 57A20

1. **Introduction**

The study of fuzzy sets was initiated with the famous paper of Zadeh [20] and thereafter Chang [4] paved the way for subsequent tremendous growth of the numerous fuzzy topological concepts. The theory of fuzzy topological spaces was developed by several authors by considering the basic concepts of general topology. The extensions of functions in fuzzy settings can be carried out using the concepts of quasi-coincidences and q-neighborhoods by Pu and Liu [16]. Generalized fuzzy closed sets and regular generalized fuzzy closed

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sets were defined in [2, 15]. Normality in fuzzy settings was defined by Hutton [6] and many others in terms of fuzzy open and fuzzy closed sets.

In this paper, we introduce and study a new class of spaces called mildly fuzzy normal spaces. Furthermore, we introduce new types of functions called almost rgf-continuous, almost gf-continuous, rgf-open, fuzzy regular open, almost rgf-open, almost gf-open and fuzzy rc-preserving functions in fuzzy topological spaces. Subsequently, the relationships between mildly fuzzy normal spaces and new fuzzy topological functions are investigated. Moreover, we obtain characterizations of mildly fuzzy normal spaces, properties of new fuzzy topological functions and preservation theorems for mildly fuzzy normal spaces in fuzzy topological spaces.

2. **Preliminaries**

Let X be any nonempty set and I be the closed unit interval [0, 1]. A fuzzy set in X is an element of the set of all functions from X into I. The family of all fuzzy sets in X is denoted by I^X. A fuzzy point x_t is a fuzzy set in X defined by $x_t(x) = t$, $x_t(y) = 0$ for all $y \neq x$, $t \in (0, 1]$. The set of all fuzzy points in X is denoted by S(X). For every $x_t \in S(X)$ and $\mu \in I^X$, $x_t \in \mu$ iff $t \leq \mu(x)$. A fuzzy set μ is quasi-coincident with a fuzzy set ν, denoted by $\mu q \nu$, if there exists $x \in X$ such that $\mu(x) + \nu(x) > 1$. If μ is not quasi-coincident with ν, then we write $\mu \bar{q}\, \nu$. It is known that $\mu \leq \nu$ iff $\mu \bar{q} 1-\nu$. For $\mu \in I^X$, the closure, interior and complement of μ are denoted by $cl(\mu)$, $int(\mu)$ and μ^1 or $1 - \mu$ respectively. The constant fuzzy sets taking on the values 0 and 1 on X are denoted by 0_X and 1_X. A family τ of fuzzy sets in X is called a fuzzy topology for X if (1) $0, 1 \in \tau$, (2) $\mu \wedge \rho \in \tau$, whenever μ, $\rho \in \tau$ and (3) $\vee\{\mu_\alpha : \alpha \in I\} \in \tau$, whenever each $\mu_\alpha \in \tau$ ($\alpha \in I$). Moreover, the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a fuzzy open set. The complement of a fuzzy open set is fuzzy closed.

Definition 2.1. [1] *Let (X, τ) be a fuzzy topological space. A fuzzy set μ of X is called*

(1) *fuzzy regular open if $\mu = int(cl(\mu))$;*

(2) *fuzzy regular closed if $\mu = cl(int(\mu))$;*

(3) *fuzzy semi-open if $\mu \leq cl(int(\mu))$.*

The complement of fuzzy regular open set is fuzzy regular closed. The collection of all fuzzy regular open (resp. fuzzy regular closed) sets of X is denoted by FRO(X) (resp. FRC(X)).

Definition 2.2. *Let (X, τ) be a fuzzy topological space. A fuzzy set μ of X is called*

(1) *generalized fuzzy closed (briefly, gf-closed)* [2] *if $cl(\mu) \leq \lambda$ whenever $\mu \leq \lambda$ and λ is fuzzy open in X.*

(2) *regular generalized fuzzy closed (briefly, rgf-closed)*[15] *if $cl(\mu) \leq \lambda$ whenever $\mu \leq \lambda$ and λ is fuzzy regular open in X.*

The complement of a gf-closed (resp. rgf-closed) set is gf-open (resp. rgf-open).

Remark 2.3. [2, 15] *We have the following implications for properties of subsets we stated above.*

fuzzy regular closed $\rightarrow$ fuzzy closed $\rightarrow$ gf-closed $\rightarrow$ rgf-closed.

None of the above implications is reversible.

Theorem 2.4. [15] *A fuzzy set A is rgf-open in X if and only if $F \leq int(A)$ whenever F is fuzzy regular closed and $F \leq A$.*

Definition 2.5. [4] *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Let A be a fuzzy subset in X and B be a fuzzy subset in Y. Then the Zadeh's functions f(A) and $f^{-1}(B)$ are defined by*

(1) *f(A) is a fuzzy subset of Y where*

$$f(A) = \begin{cases} \underset{z \in f^{-1}(y)}{sup} A(z), & if\ f^{-1}(y) \neq \emptyset \\ 0, & otherwise \end{cases}$$

for each $y \in Y$.

(2) *$f^{-1}(B)$ is a fuzzy subset of X where $f^{-1}(B)(x)=B(f(x))$, for each $x \in X$.*

Lemma 2.6. [4] *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. For fuzzy sets A and B of X and Y respectively, the following statements hold:*

(1) $ff^{-1}(B) \leq B$;

(2) $f^{-1}f(A) \geq A$;

(3) $f(A^1) \geq (f(A))^1$;

(4) $f^{-1}(B^1) = (f^{-1}(B))^1$;

(5) *if* f *is injective, then* $f^{-1}(f(A)) = A$;

(6) *if* f *is surjective, then* $ff^{-1}(B) = B$;

(7) *if* f *is bijective, then* $f(A^1) = (f(A))^1$.

Definition 2.7. [4] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called*

(1) *fuzzy continuous if the inverse image of each fuzzy open set of* Y *is a fuzzy open set in* X.

(2) *fuzzy open (resp. fuzzy closed) if the image of each fuzzy open (resp. fuzzy closed) set of* X *is a fuzzy open (resp. fuzzy closed) set in* Y.

Lemma 2.8. [1] *For a fuzzy set* λ *of a fuzzy topological space* (X, τ), *we have*

(1) $1 - int(\lambda) = cl(1 - \lambda)$,

(2) $1 - cl(\lambda) = int(1 - \lambda)$.

3. Characterizations of mildly fuzzy normal spaces

Definition 3.1. *A space* X *is called fuzzy normal if for each pair of fuzzy closed sets* A *and* B *of* X *with* $A\bar{q}B$, *there exist fuzzy open sets* U *and* V *such that* $A \leq U$ *and* $B \leq V$ *and* $U\bar{q}V$.

Definition 3.2. *A space* X *is called fuzzy s-normal if for each pair of fuzzy closed sets* A *and* B *of* X *with* $A\bar{q}B$, *there exist fuzzy semi-open sets* U *and* V *such that* $A \leq U$ *and* $B \leq V$ *and* $U\bar{q}V$.

Remark 3.3. *Every fuzzy normal space is fuzzy s-normal but not conversely.*

Proof. It follows from the fact that every fuzzy open set is fuzzy semi-open.

Example 3.4. *Let* $X=\{a, b\}$ *and* $\lambda_1, \lambda_2 : X \to [0, 1]$ *be defined as* $\lambda_1(a)=5/10$, $\lambda_1(b)=4/10$ *and* $\lambda_2(a)=6/10$, $\lambda_2(b)=6/10$. *Then* (X, τ) *is a fuzzy topological space with* $\tau=\{0_X, 1_X, \lambda_1, \lambda_2\}$ *and* (X, τ) *is a fuzzy s-normal space but not a fuzzy normal.*

Definition 3.5. *A space X is called mildly fuzzy normal if for each pair of fuzzy regular closed sets H and K of X with $H\bar{q}K$, there exist fuzzy open sets U, V such that $H \leq U$ and $K \leq V$ and $U\bar{q}V$.*

Theorem 3.6. *The following are equivalent in a fuzzy topological space (X, τ).*

(1) *X is mildly fuzzy normal;*

(2) *for any H, K $\in$ FRC(X) with $H\bar{q}K$, there exist gf-open sets U, V such that $H \leq U$ and $K \leq V$ and $U\bar{q}V$;*

(3) *for any H, K $\in$ FRC(X) with $H\bar{q}K$, there exist rgf-open sets U, V such that $H \leq U$ and $K \leq V$ and $U\bar{q}V$;*

(4) *for any H $\in$ FRC(X) and any V $\in$ FRO(X) containing H, there exists a rgf-open set U of X such that $H \leq U \leq cl(U) \leq V$.*

Proof. (1) $\Rightarrow$ (2). Proof is immediate from the fact that any fuzzy open set is gf-open.
(2) $\Rightarrow$ (3). Proof is immediate from the fact that any gf-open set is rgf-open.
(3) $\Rightarrow$ (4). Let H$\in$FRC(X) and V$\in$FRO(X) with H $\leq$ V. Then H $\leq$ V $\Rightarrow$ H$\bar{q}$1 $-$ V where 1 $-$ V $\in$FRC(X). By (3) there exist rgf-open sets U and W such that H $\leq$ U and 1 $-$ V $\leq$ W and U$\bar{q}$W. By Theorem 2.4, we have 1 $-$ V $\leq$ int(W) $\Rightarrow$ 1 $-$ int(W) $\leq$ V $\Rightarrow$ cl(1 $-$ W) $\leq$ V. Since U$\bar{q}$W, we have U $\leq$ 1 $-$ W and cl(U) $\leq$ cl(1$-$ W). Therefore, we obtain H $\leq$ U $\leq$ cl(U) $\leq$ V where U is rgf-open.
(4) $\Rightarrow$ (1) Let H and K be fuzzy regular closed sets of X with H$\bar{q}$K. Then H $\leq$ 1 $-$ K where 1 $-$ K $\in$ FRO(X). By (4) there exists a rgf-open set G of X such that H $\leq$ G $\leq$ cl(G) $\leq$ 1 $-$ K. Put U = int(G) and V = 1 $-$ cl(G). Then U and V are fuzzy open sets of X. Since H $\leq$ G where H $\in$ FRC(X) and G is rgf-open, H $\leq$ int(G) = U by Theorem 2.4 and also K $\leq$ 1 $-$ cl(G) $\leq$ V. Further V=1$-$cl(G)$\leq$1$-$int(G)=1$-$U ans hence V$\bar{q}$U. Thus we have the fuzzy open sets U and V as required. This proves (1).

Theorem 3.7. *Every fuzzy normal space is mildly fuzzy normal but not conversely.*

Proof. Let (X, τ) be a fuzzy normal space and A and B be any two fuzzy regular closed sets in X such that A$\bar{q}$B. Since A and B are fuzzy regular closed in X, they are fuzzy closed in X. (X, τ) is fuzzy normal implies there exist fuzzy open sets U and W and A $\leq$

U and B $\leq$ W and U$\bar{q}$W. Hence U and W satisfy the conditions of mildly fuzzy normality and (X, τ) is mildly fuzzy normal.

Example 3.8. *Let X be any nonempty set and the fuzzy topology τ on X be given by τ=$\{C_0, C_{2/3}, C_{3/4}, C_1\}$ where C_a: X$\rightarrow$[0, 1] is defined as $C_a(x)=a\in[0, 1]$ $\forall x\in X$. Then (X, τ) is a mildly fuzzy normal space but not a fuzzy normal space.*

Remark 3.9. *The notions of fuzzy s-normality and mildly fuzzy normality are independent.*

Example 3.10. *Let X be any nonempty set and the fuzzy topology τ on X be given by τ=$\{C_0, C_{7/12}, C_{8/12}, C_1\}$ where C_a: X$\rightarrow$[0, 1] is defined as $C_a(x)=a\in[0, 1]$ $\forall x\in X$. Then (X, τ) is a mildly fuzzy normal space but not a fuzzy s-normal space.*

Example 3.11. *Let X be any nonempty set and the fuzzy topology τ on X be given by τ=$\{C_0, C_{4/12}, C_{6/12}, C_{7/12}, C_1\}$ where C_a: X$\rightarrow$[0, 1] is defined as $C_a(x)=a\in[0, 1]$ $\forall x\in X$. Then (X, τ) is a fuzzy s-normal space but not a mildly fuzzy normal space.*

Remark 3.12. *Fuzzy s-normality + Mildly fuzzy normality $\nrightarrow$ Fuzzy normality.*

In Example 3.4., (X, τ) is a fuzzy s-normal space but not a fuzzy normal space. In this space, there are no non-quasi-coincident fuzzy regular closed sets and therefore (X, τ) is mildly fuzzy normal. This proves that both fuzzy s-normality and mildly fuzzy normality are not sufficient for fuzzy normality.

4. Some fuzzy topological functions

Definition 4.1. *A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be*

(1) *gf-continuous* [2] *if f^{-1}(F) is gf-closed in X for every fuzzy closed set F of Y;*
(2) *rgf-continuous* [15] *if f^{-1}(F) is rgf-closed in X for every fuzzy closed set F of Y;*
(3) *completely fuzzy continuous [19] if f^{-1}(F) $\in$ FRC(X) for every fuzzy closed set F of Y;*
(4) *fuzzy R-continuous* [3] *if f^{-1}(F) $\in$ FRC(X) for every F $\in$ FRC(Y);*
(5) *almost fuzzy continuous if f^{-1}(F) is fuzzy closed in X for every F $\in$ FRC(Y);*

(6) *almost gf-continuous if* $f^{-1}(F)$ *is gf-closed in X for every* $F \in FRC(Y)$;

(7) *almost rgf-continuous if* $f^{-1}(F)$ *is rgf-closed in X for every* $F \in FRC(Y)$.

Example 4.2. *Let X be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on X be given by* $\tau_1=\{C_0,\ C_{5/9},\ C_1\}=\tau_2$ *where* C_a: $X\to[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *is fuzzy continuous but not completely fuzzy continuous.*

Example 4.3. *Let X be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on X be given by* $\tau_1=\{C_0,\ C_{3/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{5/9},\ C_1\}$ *where* C_a: $X\to[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *is gf-continuous but not fuzzy continuous.*

Example 4.4. *Let X be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on X be given by* $\tau_1=\{C_0,\ C_{7/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{5/9},\ C_1\}$ *where* C_a: $X\to[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *is rgf-continuous but not gf-continuous.*

Example 4.5. *The fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *defined in Example 4.2 is fuzzy R-continuous but not completely fuzzy continuous.*

Example 4.6. *The fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *defined in Example 4.3 is almost fuzzy continuity but not fuzzy continuous.*

Example 4.7. *The fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *defined in Example 4.4 is almost gf-continuous but not gf-continuous.*

Example 4.8. *Let X be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on X be given by* $\tau_1=\{C_0,\ C_{4/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{5/9},\ C_1\}$ *where* C_a: $X\to[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\ \tau_1)\to(X,\ \tau_2)$ *is almost rgf-continuous but not rgf-continuous.*

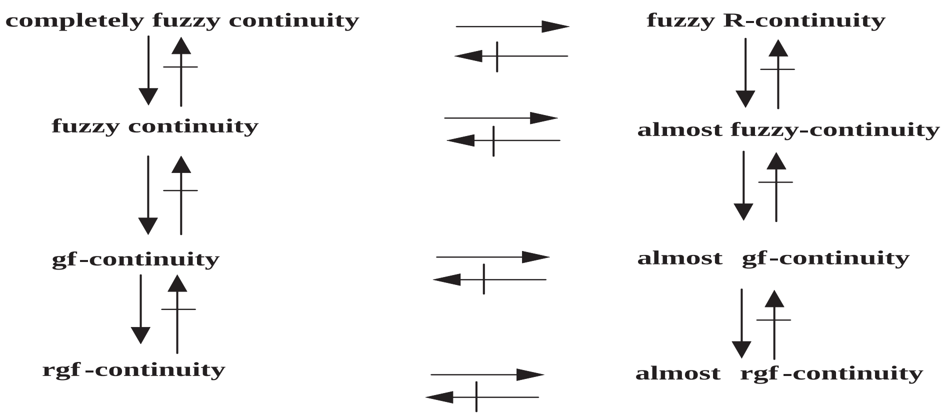
Example 4.9. *Let X be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on X be given by* $\tau_1=\{C_0,\ C_{5/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{4/9},\ C_1\}$ *where* C_a: $X\to[0,\ 1]$ *is defined as*

$C_a(x)=a\in[0,\ 1]\ \forall x\in X$. Then the fuzzy identity function $f:(X,\tau_1)\rightarrow(X,\tau_2)$ is almost rgf-continuous but not almost gf-continuous.

Example 4.10. *Let X be any nonempty set and the fuzzy topologies τ_1 and τ_2 on X be given by $\tau_1=\{C_0,\ C_{3/9},\ C_1\}$ and $\tau_2=\{C_0,\ C_{4/9},\ C_1\}$where C_a: $X\rightarrow[0,\ 1]$ is defined as $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. Then the fuzzy identity function $f:(X,\tau_1)\rightarrow(X,\tau_2)$ is almost gf-continuous but not almost fuzzy continuous.*

Example 4.11. *Let X be any nonempty set and the fuzzy topologies τ_1 and τ_2 on X be given by $\tau_1=\{C_0,\ C_{4/9},\ C_{1/2},\ C_1\}$ and $\tau_2=\{C_0,\ C_{4/9},\ C_1\}$where C_a: $X\rightarrow[0,\ 1]$ is defined as $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. Then the fuzzy identity function $f:(X,\tau_1)\rightarrow(X,\tau_2)$ is almost fuzzy continuous but not fuzzy R-continuous.*

Remark 4.12. *From the definitions stated above and the examples given above, we obtain the following diagram.*



Definition 4.13. [15] *A space X is said to be fuzzy regular-$T_{1/2}$ if every rgf-closed set in X is fuzzy regular closed in X.*

Proposition 4.14. *If a function $f:X\rightarrow Y$ is rgf-continuous and X is fuzzy regular-$T_{1/2}$, then f is completely fuzzy continuous.*

Proof. Let F be any fuzzy closed set of Y. Since f is rgf-continuous, $f^{-1}(F)$ is rgf-closed in X and hence $f^{-1}(F)\in FRC(X)$. Therefore f is completely fuzzy continuous.

Definition 4.15. [15] *A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be fuzzy rgc-irresolute if f^{-1}(F) is rgf-closed in X for every rgf-closed set F of Y.*

Remark 4.16. [15] *Every fuzzy rgc-irresolute function is rgf-continuous but not converse-ly.*

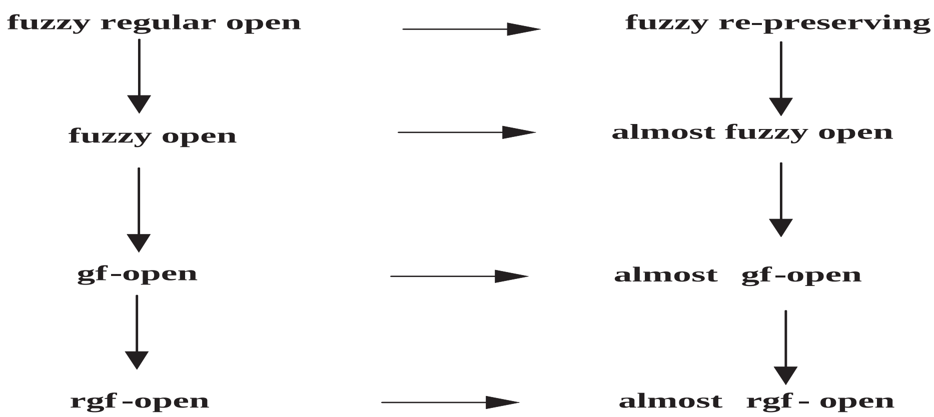
Proposition 4.17. *If f : X $\rightarrow$ Y is almost rgf-continuous and X is fuzzy regular-$T_{1/2}$, then f is a fuzzy R-continuous.*

Proof. Let V $\in$ FRC(X). Since f is almost rgf-continuous, f^{-1}(V) is rgf-closed in X. But X is fuzzy regular-$T_{1/2}$. Therefore f^{-1}(V) $\in$ FRC(X). Hence f is a fuzzy R-continuous.

Definition 4.18. *A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be*

(1) *fuzzy regular open if f(F) is fuzzy regular open in Y for every fuzzy open set F of X;*
(2) *gf-open if f(F) is gf-open in Y for every fuzzy open set F of X;*
(3) *rgf-open if f(F) is rgf-open in Y for every fuzzy open set F of X;*
(4) *fuzzy rc-preserving if f(F) is fuzzy regular closed in Y for every F $\in$ FRC(X);*
(5) *almost fuzzy open if f(F) is fuzzy open in Y for every F $\in$ FRO(X);*
(6) *almost gf-open if f(F) is gf-open in Y for every F $\in$ FRO(X);*
(7) *almost rgf-open if f(F) is rgf-open in Y for every F $\in$ FRO(X).*

Remark 4.19. *From the definitions we stated above, we obtain the following diagram.*



Remark 4.20. *The following examples enable us to realize that none of the implications in the above diagram is reversible.*

Example 4.21. *The fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *defined in Example 4.2 is fuzzy open but not fuzzy regular open.*

Example 4.22. *Let* X *be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on* X *be given by* $\tau_1=\{C_0,\ C_{5/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{3/9},\ C_1\}$ *where* C_a: $X\rightarrow[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *is gf-open but not fuzzy open.*

Example 4.23. *Let* X *be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on* X *be given by* $\tau_1=\{C_0,\ C_{5/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{7/9},\ C_1\}$ *where* C_a: $X\rightarrow[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *is rgf-open but not gf-open.*

Example 4.24. *The fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *defined in Example 4.2 is fuzzy rc-preserving but not fuzzy regular open.*

Example 4.25. *The fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *defined in Example 4.22 is almost fuzzy open but not fuzzy open.*

Example 4.26. *The fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *defined in Example 4.23 is almost gf-open but not gf-open.*

Example 4.27. *The fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *defined in Example 4.9 is almost rgf-open but not rgf-open.*

Example 4.28. *The fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *defined in Example 4.8 is almost rgf-open but not almost gf-open.*

Example 4.29. *Let* X *be any nonempty set and the fuzzy topologies* τ_1 *and* τ_2 *on* X *be given by* $\tau_1=\{C_0,\ C_{4/9},\ C_1\}$ *and* $\tau_2=\{C_0,\ C_{3/9},\ C_1\}$ *where* C_a: $X\rightarrow[0,\ 1]$ *is defined as* $C_a(x)=a\in[0,\ 1]\ \forall x\in X$. *Then the fuzzy identity function* $f:(X,\tau_1)\rightarrow(X,\tau_2)$ *is almost gf-open but not almost fuzzy open.*

Example 4.30. *Let X be any nonempty set and the fuzzy topologies τ_1 and τ_2 on X be given by τ_1=$\{C_0, C_{4/9}, C_1\}$ and τ_2=$\{C_0, C_{4/9}, C_{1/2}, C_1\}$ where C_a: X→[0, 1] is defined as $C_a(x)$=a∈[0, 1] ∀x∈X. Then the fuzzy identity function f : (X, τ_1) → (X, τ_2) is almost fuzzy open but not fuzzy rc-preserving.*

Proposition 4.31. *Let f : (X, τ) → (Y, σ) be a function. Then*

(1) *if f is rgf-continuous, fuzzy rc-preserving then it is fuzzy rgc-irresolute;*
(2) *if f is fuzzy R-continuous and rgf-closed then f(A) is rgf-closed in Y for every rgf-closed set A of X.*

Proof. (1) Let B be any rgf-closed set of Y and let U ∈ FRO(X) contain $f^{-1}(B)$. Put V = $(f(U^1))^1$, then we have B ≤ V, $f^{-1}(V)$ ≤ U and V ∈ FRO(Y) since f is fuzzy rc-preserving. Hence we obtain cl(B) ≤ V and hence $f^{-1}(cl(B))$ ≤ U. By the rgf-continuity of f we have $cl(f^{-1}(B)) \leq cl(f^{-1}(cl(B))) \leq U$. This shows that $f^{-1}(B)$ is rgf-closed in X. Therefore f is fuzzy rgc-irresolute.
(2) Let A be any rgf-closed set of X and let V ∈ FRO(Y) contain f(A). Since f is a fuzzy R-continuous, $f^{-1}(V)$ ∈ FRO(X) and A ≤ $f^{-1}(V)$. Therefore, we have cl(A) ≤ $f^{-1}(V)$ and hence f(cl(A)) ≤ V. Since f is rgf-closed, f(cl(A)) is rgf-closed in Y and hence we obtain cl(f(A)) ≤ cl(f(cl(A))) ≤ V. This shows that f(A) is rgf-closed in Y.

Corollary 4.32. *Let f : X → Y be a function. Then*

(1) *if f is fuzzy continuous, fuzzy regular open, then $f^{-1}(B)$ is rgf-closed in X for every rgf-closed set B of Y.*
(2) *if f is a fuzzy R-continuous and rgf-closed, then f(A) is rgf-closed in Y for every rgf-closed set A of X.*

Proof. This is an immediate consequence of Proposition 4.31

Proposition 4.33. *A surjection f : X → Y is almost rgf-closed (or) almost gf-closed if and only if for each fuzzy subset S of Y and each fuzzy U ∈ FRO(X) containing $f^{-1}(S)$ there exists respectively a rgf-open (or) gf-open set V of Y such that S ≤ V and $f^{-1}(V)$ ≤ U.*

Proof. We prove only the first case, the proof of the other being entirely analogus. Necessity. Suppose that f is almost rgf-closed. Let S be a subset of Y and let U $\in$ FRO(X) contain $f^{-1}(S)$. Put $V = [f(U^1)]^1$, then V is a rgf-open set of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

Sufficiency. Let F be any fuzzy regular closed set of X. Then $f^{-1}(f(F)^1) \leq F^1$ and $F^1 \in$ FRO(X). There exists a rgf-open set V of Y such that $f(F)^1 \leq V$ and $f^{-1}(V) \leq F^1$. Therefore, we have $f(F) \geq V^1$ and $F \leq f^{-1}(V^1)$. Hence we obtain $f(F) = V^1$ and f(F) is rgf-closed in Y. This shows that f is almost rgf-closed .

5. **Preservation Theorems**

In this section we investigate preservation theorems concerning mildly fuzzy normal spaces in fuzzy topological spaces.

Theorem 5.1. *If $f : (X, \tau) \to (Y, \sigma)$ is an almost rgf-continuous fuzzy rc-preserving (almost fuzzy closed) injection and Y is mildly fuzzy normal (fuzzy normal) respectively, then X is mildly fuzzy normal.*

Proof. Let A and B be fuzzy regular closed sets of X with $A\bar{q}B$. Since f is a fuzzy rc-preserving (almost fuzzy closed) injection, f(A) and f(B) are fuzzy regular closed (fuzzy closed) sets of Y with $f(A)\bar{q}f(B)$. By the mild fuzzy normality (fuzzy normality) of Y, there exist fuzzy open sets U and V of Y such that $f(A) \leq U$ and $f(B) \leq V$ and $U\bar{q}V$. Now put $G = int(cl(U))$ and $H = int(cl(V))$, then G and H are fuzzy regular open sets such that $f(A) \leq G$ and $f(B) \leq H$ and $G\bar{q}H$. Since f is almost rgf-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are rgf-open sets containing A and B, respectively, and $f^{-1}(G)\bar{q}f^{-1}(H)$. It follows from Theorem 3.6 that X is mildly fuzzy normal.

Theorem 5.2. *If $f : (X, \tau) \to (Y, \sigma)$ is a completely fuzzy continuous almost gf-closed surjection and X is mildly fuzzy normal then Y is fuzzy normal.*

Proof. Let A and B be fuzzy closed sets of Y with $A\bar{q}B$. Then $f^{-1}(A)$ and $f^{-1}(B)$ are fuzzy regular closed sets of X. Since X is mildly fuzzy normal, there exist fuzzy open sets U and V such that $f^{-1}(A) \leq U$ and $f^{-1}(B) \leq V$ and $U\bar{q}V$. Let $G = int(cl(U))$ and H =

int(cl(V)), then G and H are fuzzy regular open sets such that $f^{-1}(A) \leq G$ and $f^{-1}(B) \leq H$ and $G\bar{q}H$. By Proposition 4.33, there exist gf-open sets K and L of Y such that $A \leq K$, $B \leq L$, $f^{-1}(K) \leq G$ and $f^{-1}(L) \leq H$. Since $G\bar{q}H$ and f is a surjection, $K\bar{q}L$. Since K and L are gf-open, we obtain $A \leq \text{int}(K)$, $B \leq \text{int}(L)$ and $\text{int}(K)\bar{q}\text{int}(L)$. This shows that Y is fuzzy normal.

Corollary 5.3. *If $f : (X, \tau) \to (Y, \sigma)$ is a completely fuzzy continuous fuzzy closed surjection and X is mildly fuzzy normal, then Y is fuzzy normal.*

Theorem 5.4. *Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy R-continuous (resp. almost fuzzy continuous) and almost rgf-open surjection. If X is mildly fuzzy normal (resp. fuzzy normal), then Y is mildly fuzzy normal.*

Proof. Let A and B be fuzzy regular closed sets of Y with $A\bar{q}B$. Then $f^{-1}(A)$ and $f^{-1}(B)$ are fuzzy regular closed sets (or) fuzzy closed sets of X and $f^{-1}(A)\bar{q}f^{-1}(B)$. Since X is respectively mildly fuzzy normal (or) fuzzy normal, there exist fuzzy open sets U and V of X such that $f^{-1}(A) \leq U$ and $f^{-1}(B) \leq V$ and $U\bar{q}V$. Put $G = \text{int}(\text{cl}(U))$ and $H = \text{int}(\text{cl}(V))$, then G and H are fuzzy regular open sets of X such that $f^{-1}(A) \leq G$ and $f^{-1}(B) \leq H$ and $G\bar{q}H$. By Proposition 4.33, there exist rgf-open sets K and L of Y such that $A \leq K$, $B \leq L$, $f^{-1}(K) \leq G$ and $f^{-1}(L) \leq H$. Since $G\bar{q}H$, so $K\bar{q}L$. It follows from Theorem 3.6 that Y is mildly fuzzy normal.

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