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SOME RESULTS ON IF GENERALIZED MINIMAL Λ SET

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Abstract: The purpose of this paper is to introduce the new concept of IF generalized minimal Λ set. Then another concept of IF Λclosed set is introduced which is a stronger form of IF generalized minimal Λ set and a weaker form of IF pre open set It is to be noticed that the collection of IF generalized minimal closed set forms an Alexandroff space if $1_{\sim}$ is included but the collection of IF Λclosed set forms a supra topological space . Also the connection of this set with some other sets is studied in this paper.

Keywords: IF m_X structure, IF dense set, IF minimal open set, IF generalized * closed set etc.

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1: **Introduction & Preliminaries**

The concept of fuzzy sets was introduced by Zadeh [28] and later Atanassov [2] generalized this idea to intuitionistic fuzzy sets. But a contradiction arose with the concept of Intuitionistic Logic. The concept of Intuitionistic Logic is not similar with Intuitionistic fuzzy logic and hence to avoid this contradiction various Researchers suggested various nomenclatures. Following some of their suggestion we are using the nomenclature as IF Set in place of Intuitionistic fuzzy set.

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Coker [13] introduced the notions of IF topological space and other related concepts. The initiation of the study of generalized closed sets was done by N. Levine in 1970[17] as he considered sets whose closure belongs to every open super set. He called them generalized closed sets and studied their most fundamental properties. Later on the concept of fuzzy generalized closed set has been introduced by S.S.Thakur and R.Malviya in 1995[26].Lots of researchers [4], [5], [6], [12], [14], [15], [17],[20],[22],[23],[24],[25] has studied various concepts of generalized closed set in ordinary topology and in fuzzy topological space .

The concept of minimal open set has been introduced by F Nakaoka and N Oda [21] in 2001 The concept of IF generalized minimal open set has been introduced by the author [9]. The concept of generalized * closed set has been introduced by the author in 2008[7] and the concept of dense m_X set has been introduced by the author [8]. The aim of this paper is to study IF generalized* minimal closed set. It can be shown that IF generalized * minimal closed set is not coinciding with any IF dense set . It is an independent concept. It can also be shown that if a set is not a rare set then it is not an independent concept but it coincides with the concept of IF minimal open set.

In section 2 some preliminaries related to the topic is given.

In section 3 the concept of IF generalized minimal Λ set is studied .Some properties of this set is studied and also connection of this set with some other set is introduced in this section of the paper. The topological space obtained by the collection of this set is also studied in this section of the paper

Now let us recall some of the definitions and theorems related to intuitionistic fuzzy topology and ordinary topological space.

Definition 1.1[1] Let X be a nonempty set and I the unit interval [0,1]. An IF set U is an object having the form $U=\{<x, \mu_u(x), \gamma_u(x)>:x\in X\}$where the functions $\mu_u:X\to I$ and $\gamma_u:X\to I$ denote respectively the degree of membership and degree of non-membership of each element $x\in X$ to the set U, and $0\le \mu_u(x)+\gamma_u(x) \le 1$ for each $x\in X$.

Lemma 1.2[1] Let X be a nonempty set and let IF Set's U and V be in the form $U = \{<x,\mu_u(x),\gamma_u(x)>:x\in X\}$and

$$V = \{<x,\mu_v(x),\gamma_v(x)>:x\in X\}.\text{Then}$$

(1) $U^C =\{<x,\gamma_U(x), \mu_U(x)>: x\in X\}$

(2) $U \cap V = \{<x, \wedge(\mu_U(x), \mu_V(x)), \vee(\gamma_U(x), \gamma_V(x))>, x \in X\}$

(3) $U \cup V = \{<x, \vee(\mu_U(x), \mu_V(x)), \wedge(\gamma_U(x), \gamma_V(x))>: x \in X\}$

(4) $1_{\sim} = \{<x, 1,0>: x \in X\}$, $0_{\sim} = \{<x, 0,1>: x \in X\}$

(5) $(\dot{U}^C)^C = \dot{U}$, $0_{\sim}{}^C = 1_{\sim}$, $1_{\sim}{}^C = 0_{\sim}$.

Definition 1.3[3] An IF topology on a nonempty set X is a family τ of IF Sets in X containing $0_{\sim}, 1_{\sim}$ and closed under arbitrary infimum and finite supremum.

In this case the pair (X, τ) is called an IF Topological Space and each IF Set in τ is known as an IF Open Set .The compliment U^C of an IF Open Set U in an IF Topological Space (X, τ) is called an IF Closed Set in X.

Definition 1.4[15] Let (X,τ) is a topological space. A family τ of IF sets on X is called an IF supra-topological space on X if $0_{\sim} \in \tau$, $1_{\sim} \in \tau$ and τ is closed under arbitrary supremum. Each member of τ is called an IF supra-open set and compliment of an IF supra-open set is an IF supra-closed set.

Definition 1.5[3] Let (X, τ) is an IF Topological Space and A an IF Set in X. Then closure of A is defined by $Cl(A) = \cap\{F: A \subseteq F, F^C \in \tau\}$.

and the fuzzy interior of A is defined by $Int(A) = \cup\{G: A \supseteq G, G \in \tau\}$

Definition 1.6[12] A fuzzy subset A of X is a fuzzy generalized closed set if $Cl(A) \subseteq H$ whenever $A \subseteq H$,H being a fuzzy open subset of X.

Definition 1.7[12] A fuzzy subset A of X is a fuzzy dense set if $Cl(A)=1_{\sim}$.

Definition 1.8[13] A subset A of a family τ of IF sets on X is called an IF minimal open set in X if an IF open set which is contained in A is either $0_{\sim}$ or A.

Definition 1.9[13]An IF set is said to be an IF Maximal open set of IFTS (X,τ) iff it is not contained in any other open set of τ.

Definition 1.10[12]A fuzzy subset A of X is a fuzzy generalized closed set if $Cl(A) \subseteq H$ whenever $A \subseteq H$,H being a fuzzy open set of X.

Definition1.11 [14] Let m_X an IF m_X-structure on X. An IF m_X open set is said to be an open m_X if m_X –Int (A) = A.

Definition 1.12[5] An IF set is said to be an IF semi open set of IF Topological Space (X,τ) iff $A \subseteq Int(Cl(IntA)$.

Definition 1.13[3] Let f be a map from set X to set Y. Let A = $\{<x, \mu_A(x),\gamma_A(x)>: x\in X\}$ be an IF Open Set in X and B=$\{<y, \mu_B(y),\gamma_B(y)>: y\in Y\}$ be an IF Open Set in Y. Then $f^{-1}(B)$ is an IF Open Set in X defined by $f^{-1}(B)=\{<x, f^{-1}(\mu_B(x)), f^{-1}(\gamma_B(x))>: x\in X\}$ and f(A) is an IFOS in Y defined by $f(A)=\{<y,f(\mu_A(y)),1-f(1-\gamma_A(y))>:y\in Y\}$.

Definition 1.14[3] A map $f : (X,\tau) \to (Y,\sigma)$ is said to be an IF continuous function from IF topological space (X,τ) to IF topological space (Y,σ) iff $f^{-1}(V)$ is an IF open set in X for every open set V of Y.

Throughout this paper IF Topological Space's are denoted by (X,τ) and (Y,σ) and complement of a set A is denoted by A^C.

2. Some results on IF generalized minimal Λ set

In this section the concept of IF generalized minimal Λ set is introduced and some of its properties are discussed. Lastly the IF topological structure obtained by the collection of this set is studied.

Definition 2.1: An IF set A is said to be an IF generalized minimal Λ set, if there exist at least one IF Minimal closed Set U containing A such that $\Lambda(A) \subseteq U$.

Theorem 2.2: *An IF set A is an IF generalized minimal Λ set iff $\Lambda(A) \subseteq ClA$, A is contained in an IF minimal closed set*

Proof: Let if possible A be an IF generalized minimal Λ set, then from definition 2.1, there exist a minimal closed set U containing A such that $\Lambda(A) \subseteq U$ Now arbitrary infimum of closed set is closed and hence U = ClA. Thus $\Lambda(A) \subseteq ClA$

Conversely, Let $\Lambda(A) \subseteq ClA$. Then ClA is the infimum of all closed set containing A. Thus any minimal closed set U containing A is ClA. Thus $\Lambda(A) \subseteq ClA = U$. From definition 2.1 , A is an IF generalized minimal Λ set.

Example 2.3: Let $A=\{<x, 0.3,0.2>,x\in X\}$ and $B=\{<x,0.5,0.3>,x\in X\}$ be two IF subsets of X

Let the corresponding topological space be $\tau = \{0_\sim,1_\sim,A,B,A\cup B,A\cap B\}$. Here $A\cap B$ is an IF Minimal Open Set of τ.

Consider a set $C=\{<x, 0.1,0.6>: x\in X\}$,then $C \subseteq A\cap B$ and $\Lambda(C)= \{<x,0.3,0.3>: x \in X\}\subseteq ClA$.

Hence C is an IF generalized minimal Λ set

Let us consider another IF set D = {<x, 0.3, 0.6>: x ∈X} . Here Λ(D) ⊆ Cl(D) but D is not contained in any IF minimal closed set thus D is not an IF minimal Λ set

Theorem 2.4:

(1) *Let A ⊆ B .If B is an IF generalized minimal Λ set, then A is also so.*

(2) If A ⊆ B⊆ Λ(A) and A is an IF generalized minimal Λ set then B is also so if B is contained in an IF minimal closed set.

Proof :(1) Let B ⊆ U , where U is an IF minimal closed set

i.e. A ⊆ B ⊆ U. From definition as B is an IF generalized minimal Λ set Λ(B) ⊆ U implies Λ(A) ⊆ Λ(B) ⊆ U

i.e. A is also an IF generalized minimal Λ set.

(2) Since A is an IF generalized minimal closed set

i.e. A ⊆ U where U is an IF minimal closed set and from definition as A is an IF generalized minimal Λ set Λ(A) ⊆ U = ClA . Since A ⊆ B⊆ Λ(A), Λ(A) = Λ(B) ⊆ ClA ⊆ ClB

i.e. A is also an IF generalized minimal closed set.

Theorem 2.5: *If an IF Λ set is contained in an IF minimal closed set then it is an IF generalized minimal Λ set*

Proof : Let if possible A be an IF Λ set which is contained in an IF minimal closed set U. Then Λ(A) =A ⊆ Cl(A) . Hence from theorem 2.2 A is an IF generalized minimal Λ set

Remark 2.6: Converse of the above theorem need not be true which follows from the example 2.3, where C is an IF generalized minimal Λ set but not an IF Λ set

Theorem 2.7: *If an IF set is an IF closed set and an IF generalized minimal Λ set then it is an IF Λ set*

Proof: From theorem 2.2, if a Set A is an IF generalized minimal Λ set then Λ(A) ⊆ ClA. A being IF closed set Λ(A) ⊆ ClA = A. Thus A is an IF Λ set.

Theorem 2.8: *If an IF dense set is contained in an IF minimal closed set then it is an IF generalized minimal Λ set*

Proof: Since A is an IF dense set ClA= $1_{\sim}$, Therefore $\Lambda(A) \subseteq Cl(A)$. A is an IF generalized minimal Λ set

Remark 2.9: Converse of the above theorem need not be true which follows from the example 2.3, where C is an IF generalized minimal Λ set but not an IF dense set

Theorem 2.10:

(1) *$0_{\sim}$ is an IF generalized minimal Λ set but $1_{\sim}$ is not an IF generalized minimal Λ set.*

(2) *Arbitrary union of IF generalized minimal Λ set is an IF generalized minimal Λ set if the arbitrary union is contained in an IF minimal closed set*

(3) *Arbitrary intersection of IF generalized minimal Λ set is an IF generalized minimal Λ set.*

Proof: (1) is obvious.

To prove (2)

Let $(A_i : i\in I\}$ be an arbitrary collection of IF generalized minimal Λ set. Let $\{U_i : i\in I\}$ be the corresponding IF minimal closed set. i.e. $A_i \subseteq U_i$, where U_i is an IF minimal closed set, implies $\Lambda(A_i) \subseteq U_i$.

Let $\cup(A_i : i\in I \}\subseteq U$, an IF minimal closed set which implies $\cup\Lambda(A_i : i\in I \}\subseteq U$. But we know that $\Lambda\cup(A_i : i\in I)= \cup\Lambda (A_i : i\in I\} \subseteq U$.

Thus arbitrary union of IF generalized minimal Λ set is an IF generalized minimal Λ set if the arbitrary union is contained in an IF minimal closed set

To Prove (3)

Let $(A_i : i\in I \}$ be an arbitrary collection of IF generalized minimal closed set. Since Let U be the corresponding IF minimal open set. i.e. $A_i \subseteq U_i$., where U_i. is an IF minimal closed set Now $\cap(A_i : i\in I \} \subseteq \cap(U_i : i\in I \} = 0_{\sim}$ implies $\cap(A_i : i\in I \} = 0_{\sim}$. From (1) $0_{\sim}$ is anIF generalized minimal Λ set. Thus arbitrary intersection of IF generalized minimal Λ set is an IF generalized minimal Λ set

Remark 2.11: The arbitrary collection of IF generalized minimal Λ set forms a conditional IF Alexandroff space if $1_{\sim}$ is included in the set. Let us denote the space as$(X, G^{T}_{M\Lambda})$ where T is the IF topological space

Definition 2.12: An IF set A is said to be an IF Λ Closed set iff $\Lambda(A) \leq ClA$

Example 2.13: From example 2.3 C and D are both IF Λ Closed set .But C is also IF generalized minimal Λ set. But D is not

Remark 2.14 : From the above example it is clear that every IF generalized minimal Λ set is an IF Λ closed set but not conversely

Let us construct a diagram indicating the connection between the above sets

IF generalized minimal Λ set ⇒ IF Λ closed sets ⇒ IF pre open set

IF Minimal Λ set ⇒ IF Λ set IF dense set

⇑

IF open set

Theorem 2.15*: An IF Λ closed set is an IF semi open set*

Proof: Let A be an IF Λ closed set then from the definition 2.12, $\Lambda(A) \leq ClA$. Now $\Lambda(A) = Int\, \Lambda(A) \leq IntClA$. $A \leq \Lambda(A) \leq IntClA$

Thus A is an IF semi open set

Theorem 2.16 : (1*) $0_\sim$, $1_\sim$ are IF Λ closed set*

(2) *Arbitrary union of IF Λ closed set is an IF Λ closed set .*

Proof: (1) is obviously true.

To prove (2)

Let $\{A_i : i \in I\}$be a collection of IF Λ closed set then $\Lambda(A_i) \leq ClA_i$. $\Lambda(\vee A_i) = \vee\Lambda(A_i) \leq \vee(ClA_i) \leq (Cl\vee(A_i))$

Thus arbitrary union of IF Λ closed set is an IF Λ closed set

Remark 2.17 : Finite intersection of IF Λ closed set need not be so. This follows from the example 2.3, Let E = {<x, 0.4, 0.5>: $x \in X$}. Clearly Λ(E) = {<x, 0.5, 0.3>: $x \in X$} , $ClE = 1_\sim$. Thus E is an IF Λ closed set. Also D is an IF Λ closed set

Now $D \wedge E$ = {<x, 0.2, 0.5>: $x \in X$}, $\Lambda(D\wedge E)$ = {<x. 0.3, 0.3>: $x \in X$} and $Cl\,(D\wedge E)$ = {<x, 0.2, 0.5>: $x\in X$}. Here $D\wedge E$ need not be an IF Λ closed set.

Remark 2.18: The collection of all IF Λ closed set forms an IF supra topological space

Definition 2.19: Let f : (X, τ_1) →(Y, τ_2) be a mapping such that the inverse image of an IF Λ set in Y is an IF generalized minimal Λ set in X . Then this mapping is said to be an IF generalized minimal Λ continuous mapping .

Example 2.15: Let A={<x, 0.3,0.2>,x∈X} and B={<x,0.5,0.3>,x∈X}, C= {<x, 0.5, 0.3>:x∈X} , D={<x,0.3,0.2>: x ∈ X} be some IF subsets of X

Let the corresponding topological space be τ_1 = {$0_\sim$,$1_\sim$,A,B,A∪B,A∩B}. Here A∩B is an IF Minimal Open Set of τ_1. τ_2 = {$0_\sim$,$1_\sim$,C,D,C∪D,C∩D} The IF minimal open set is itself IF generalized minimal closed set. Therefore {U: U≤ the IF minimal open set in τ_1} is the collection of IF generalized minimal closed set in τ_1. And {U: U≤ the IF minimal open set in τ_2} is the collection of IF generalized minimal closed set in τ_2. Let f : (X, τ_1)→ (Y, τ_2) be a mapping such that f(x) = x, then f is an IF generalized minimal Λ continuous mapping

Definition 2.16: . Let f : (X, τ_1) →(Y, τ_2) be a mapping such that the inverse image of an IF Λ set in Y is an IF ΛClosed set in X . Then this mapping is said to be an IF Λ closed continuous mapping

Theorem 2.17: *Every IF generalized minimal Λ continuous mapping is an IF Λ closed continuous mapping*

Proof : Let A be a IF Λ set in Y .Since f is IF generalized minimal Λ continuous mapping, f^{-1}(A) is an IF generalized minimal Λ set in X . Since every IF generalized minimal Λ set is an IF Λ closed set f^{-1}(A) is an IF Λ closed set. Thus f is an IF Λ closed continuous mapping.

Remark 2.18: Converse of the above theorem need not be true which follows from the following example

Let A = {<x, 0.3, 0.2>: x ∈X} and B = {<x, 0.5, 0.3>:x ∈X} be two IF subsets of X . Let the corresponding IF topological space be τ_X = {0~, 1~, A, B, A∨B, A∧B}. Again C = {<y, 0.1, 0.6> :y ∈Y}, D= {<y, 0.3, 0.6>: y ∈Y}be two IF subsets of Y . The corresponding IF topological space be τ_Y = {$0_\sim$, $1_\sim$, C, D}. Let f : (X, τ_X) → (Y, τ_Y) be a mapping such that f(x)=y , f -1(0~) = 0~, f -1(1~) = 1~, f -1© = C, f -1(D) = D . Here D is an IF Λ closed set but not IF generalized minimal Λ set. Thus f is an IF Λ closed continuous mapping but not IF generalized minimal Λ continuous mapping

We can thus construct the following figure which indicates the connection of the newly constructed function with the previous one.

IF generalized minimal Λ continuous function ⇒ IF Λ closed continuous function ⇒ IF Pre

⇑ Continuous function

IF Λ continuous function

Theorem 2.19: (1) *Let f : (X, τ_X) →(Y, τ_Y) be an IF Λ closed continuous function and g : (Y, τ_Y) →(Z, τ_Z) be an IF Λ continuous mapping then gof : (X, τ_X) →(Z, τ_Z) is an IF Λ closed continuous function*

***(2)** Let f : (X, τ_X) →(Y, τ_Y) be an IF generalized Λ continuous function and g : (Y, τ_Y) →(Z, τ_Z) be an IF Λ continuous mapping then gof : (X, τ_X) →(Z, τ_Z) is an IF generalized minimal Λ continuous function*

Proof is obvious

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