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HOMOMORPHISM IN Q-INTUITIONISTIC L-FUZZY SUBRINGS

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Abstract. In this paper, we study the properties of Q-intuitionistic L-fuzzy subrings of a ring. Some new results are obtained based on these properties.

Keywords: (Q, L)-fuzzy subset; Q-intuitionistic L-fuzzy subset; Q-intuitionistic L-fuzzy subring; Q-intuitionistic L-fuzzy normal subring.

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1. Introduction

After the introduction of fuzzy sets by Zadeh [12], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic L-fuzzy subset was introduced by Atanassov [4,5], as a generalization of the notion of fuzzy set.

Rosenfeld [6] defined a fuzzy group. Ray [3] defined a product of fuzzy subgroups and Solairaju and R.Nagarajan [10,11] introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the concept of Q-intuitionistic L-fuzzy subring of a ring and establish some new results.

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2. Preliminaries

Definition 2.1. Let X be a non-empty set, and $L = (L, \leq)$ be a lattice with least element 0, and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function $A : X \times Q \to L$.

Definition 2.2. Let $(L, \leq)$ be a complete lattice with an involutive order reversing operation $N : L \to L$ and Q be a non-empty set. A Q-intuitionistic L-fuzzy subset (QILFS) A in X is defined as an object of the form $A = \{< (x, q), \mu_A(x, q), \nu_A(x, q) > /x$ in X and q in $Q\}$, where $\mu_A : X \times Q \to L$, and $\nu_A : X \times Q \to L$ define the degree of membership, and the degree of non-membership of the element x in X, respectively, and for every x in X and q in Q satisfying $\mu_A(x, q) \leq N(\nu_A(x, q))$.

Definition 2.3. Let $(R, +, .)$ be a ring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subring (QILFSR) of R if it satisfies the following axioms:

(i) $\mu_A(x - y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

(ii) $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

(iii) $\nu_A(x - y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$,

(iv) $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R and q in Q.

Definition 2.4. Let X, and Y be any two sets. Let $f : X \to Y$ be any function and A be a Q-intuitionistic L-fuzzy subset in X, V be a Q-intuitionistic L-fuzzy subset in $f(X) = Y$, defined by $\mu_V(y, q) = \sup_{x \in f^{-1}(y)} \mu_A(x, q)$ and $\nu_V(y, q) = \inf_{x \in f^{-1}(y)} \nu_A(x, q)$, for all x in X and y in Y . A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition 2.5. Let $(R, +, .)$ be a ring. A Q-intuitionistic L-fuzzy subring A of R is said to be a Q-intuitionistic L-fuzzy normal subring (QILFNSR) of R if $\mu_A(xy, q) = \mu_A(yx, q)$ and $\nu_A(xy, q) = \nu_A(yx, q)$, for all x and y in R and q in Q.

3. Some properties of Q-intuitionistic L-fuzzy subrings of a ring

Theorem 3.1. *Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings. The homomorphic image of a Q-intuitionistic L-fuzzy subring of R is a Q-intuitionistic L-fuzzy subring of $f(R) = R_1$.*

Proof. Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings and Q be a non-empty set. Let $f : R \to R_1$ be a homomorphism. Then $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let A be a Q-intuitionistic L-fuzzy subring of R. We have to prove that V is a Q-intuitionistic L-fuzzy subring of $f(R) = R_1$. Now, for $f(x)$, $f(y)$ in R_1 and q in Q,

$$\begin{aligned}\mu_V(f(x) - f(y), q) &= \mu_V(f(x - y), q)\\ &\geq \mu_A(x - y, q)\\ &\geq \mu_A(x, q) \wedge \mu_A(y, q),\end{aligned}$$

which implies that

$$\mu_V(f(x) - f(y), q) \geq \mu_V(f(x), q) \wedge \mu_V(f(y), q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Again,

$$\begin{aligned}\mu_V(f(x)f(y), q) &= \mu_V(f(xy), q)\\ &\geq \mu_A(xy, q)\\ &\geq \mu_A(x, q) \wedge \mu_A(y, q),\end{aligned}$$

which implies that

$$\mu_V(f(x)f(y), q) \geq \mu_V(f(x), q) \wedge \mu_V(f(y), q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Also,

$$\begin{aligned}\nu_V(f(x) - f(y), q) &= \nu_V(f(x - y), q)\\ &\leq \nu_A(x - y, q)\\ &\leq \nu_A(x, q)\nu_A \vee (y, q),\end{aligned}$$

which implies that

$$\nu_V(f(x) - f(y), q) \leq \nu_V(f(x), q) \vee \nu_V(f(y), q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Again,

$$\begin{aligned}\nu_V(f(x)f(y),q) &= \nu_V(f(xy),q)\\ &\leq \nu_A(xy,q)\\ &\leq \nu_A(x,q)\vee\nu_A(y,q),\end{aligned}$$

which implies that

$$\nu_V(f(x)f(y),q) \leq \nu_V(f(x),q)\vee\nu_V(f(y),q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Hence V is a Q-intuitionistic L-fuzzy subring of R_1. This completes the proof.

In view of Theorem 3.1, the following is not hard to derive.

Theorem 3.2. *Let $(R,+,.)$ and $(R_1,+,.)$ be any two rings. The homomorphic preimage of a Q-intuitionistic L-fuzzy subring of $f(R)=R_1$ is a Q-intuitionistic L-fuzzy subring of R.*

Theorem 3.3. *Let $(R,+,.)$ and $(R_1,+,.)$ be any two rings. The anti-homomorphic image of a Q-intuitionistic L-fuzzy subring of R is a Q-intuitionistic L-fuzzy subring of $f(R)=R_1$.*

Proof. Let $(R,+,.)$ and $(R_1,+,.)$ be any two rings and Q be a non-empty set. Let $f: R\to R_1$ be an anti-homomorphism. Then $f(x+y)=f(y)+f(x)$ and $f(xy)=f(y)f(x)$, for all x and y in R. Let A be a Q-intuitionistic L-fuzzy subring of R. We have to prove that V is a Q-intuitionistic L-fuzzy subring of $f(R)=R_1$. Now, for $f(x)$, $f(y)$ in R_1 and q in Q,

$$\begin{aligned}\mu_V(f(x)-f(y),q) &= \mu_V(f(y-x),q)\\ &\geq \mu_A(y-x,q)\\ &\geq \mu_A(y,q)\wedge\mu_A(x,q),\end{aligned}$$

which implies that

$$\mu_V(f(x)-f(y),q) \geq \mu_V(f(x),q)\wedge\mu_V(f(y),q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Again,

$$\begin{aligned}\mu_V(f(x)f(y),q) &= \mu_V(f(yx),q)\\ &\geq \mu_A(yx,q)\\ &\geq \mu_A(y,q)\wedge\mu_A(x,q),\end{aligned}$$

which implies that

$$\mu_V(f(x)f(y),q)\geq\mu_V(f(x),q)\wedge\mu_V(f(y),q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Also,

$$\begin{aligned}\nu_V(f(x)-f(y),q) &= \nu_V(f(y-x),q)\\ &\leq \nu_A(y-x,q)\\ &\leq \nu_A(x,q)\nu_A\vee(y,q),\end{aligned}$$

which implies that

$$\nu_V(f(x)-f(y),q)\leq\nu_V(f(x),q)\vee\nu_V(f(y),q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Again,

$$\begin{aligned}\nu_V(f(x)f(y),q) &= \nu_V(f(yx),q)\\ &\leq \nu_A(yx,q)\\ &\leq \nu_A(y,q)\vee\nu_A(x,q),\end{aligned}$$

which implies that

$$\nu_V(f(x)f(y),q)\leq\nu_V(f(x),q)\vee\nu_V(f(y),q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Hence V is a Q-intuitionistic L-fuzzy subring of R_1. This completes the proof.

In view of Theorem 3.3, the following is not hard to derive.

Theorem 3.4. *Let $(R,+,.)$ and $(R_1,+,.)$ be any two rings. The anti-homomorphic preimage of a Q-intuitionistic L-fuzzy subring of $f(R)=R_1$ is a Q-intuitionistic L-fuzzy subring of R.*

Theorem 3.5. *Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings. The homomorphic image of a Q-intuitionistic L-fuzzy normal subring of R is a Q-intuitionistic L-fuzzy normal subring of $f(R) = R_1$.*

Proof. Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings and Q be a non-empty set. Let $f : R \to R_1$ be a homomorphism. Then $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let A be a Q-intuitionistic L-fuzzy normal subring of R. We have to prove that V is a Q-intuitionistic L-fuzzy normal subring of $f(R) = R_1$. Now, for $f(x)$, $f(y)$ in R_1, clearly V is a Q-intuitionistic L-fuzzy subring of a ring R_1, since A is a Q-intuitionistic L-fuzzy subring of a ring R. Now,

$$\begin{aligned}\mu_V(f(x)f(y), q) &= \mu_V(f(xy), q)\\ &\geq \mu_A(xy, q)\\ &= \mu_A(yx, q)\\ &\geq \mu_V(f(yx), q)\\ &= \mu_V(f(y)f(x), q),\end{aligned}$$

which implies that

$$\mu_V(f(x)f(y), q) = \mu_V(f(y)f(x), q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Also,

$$\begin{aligned}\nu_V(f(x)f(y), q) &= \nu_V(f(xy), q)\\ &\leq \nu_A(xy, q)\\ &= \nu_A(yx, q)\\ &\geq \nu_V(f(yx), q)\\ &= \nu_V(f(y)f(x), q),\end{aligned}$$

which implies that

$$\nu_V(f(x)f(y), q) = \nu_V(f(y)f(x), q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Hence V is a Q-intuitionistic L-fuzzy normal subring of a ring R. This completes the proof.

In view of Theorem 3.5, the following is not hard to derive.

Theorem 3.6. *Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings. The homomorphic preimage of a Q-intuitionistic L-fuzzy normal subring of $f(R) = R_1$ is a Q-intuitionistic L-fuzzy normal subring of R.*

Theorem 3.7. *Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings. The anti-homomorphic image of a*

Q-intuitionistic L-fuzzy normal subring of R is a Q-intuitionistic L-fuzzy normal subring of $f(R) = R_1$.

Proof. Let $(R, +, .)$ and $(R_1, +, .)$ be any two rings and Q be a non-empty set. Let $f : R \to R_1$ be an anti-homomorphism. Then $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let A be a Q-intuitionistic L-fuzzy normal subring of R. We have to prove that V is a Q-intuitionistic L-fuzzy normal subring of $f(R) = R_1$. Now, for $f(x)$, $f(y)$ in R_1, clearly V is a Q-intuitionistic L-fuzzy subring of a ring R_1, since A is a Q-intuitionistic L-fuzzy subring of a ring R. Now,

$$\begin{aligned}\mu_V(f(x)f(y), q) &= \mu_V(f(yx), q)\\ &\geq \mu_A(yx, q)\\ &= \mu_A(xy, q)\\ &\leq \mu_V(f(xy), q)\\ &= \mu_V(f(y)f(x), q),\end{aligned}$$

which implies that

$$\mu_V(f(x)f(y), q) = \mu_V(f(y)f(x), q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Also,

$$\begin{aligned}\nu_V(f(x)f(y),q) &= \nu_V(f(yx),q)\\ &\leq \nu_A(yx,q)\\ &= \nu_A(xy,q)\\ &\geq \nu_V(f(xy),q)\\ &= \nu_V(f(y)f(x),q),\end{aligned}$$

which implies that

$$\nu_V(f(x)f(y),q) = \nu_V(f(y)f(x),q),$$

for all $f(x)$ and $f(y)$ in R_1 and q in Q. Hence V is a Q-intuitionistic L-fuzzy normal subring of a ring $f(R) = R_1$. This completes the proof.

In view of Theorem 3.7, the following is not hard to derive.

Theorem 3.8. *Let $(R,+,.)$ and $(R_1,+,.)$ be any two rings. The anti-homomorphic preimage of a Q-intuitionistic L-fuzzy normal subring of $f(R) = R_1$ is a Q-intuitionistic L-fuzzy normal subring of R.*

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