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A TWO-UNIT STANDBY SYSTEM WITH REGULAR REPAIRMAN AND WAITING TIME OF SKILLED REPAIRMAN

RAKESH GUPTA* AND PARUL BHARDWAJ
Department of Statistics, Ch. Charan Singh University,Meerut-250004, India

ABSTRACT: The paper deals with a stochastic behavior of a two-identical unit cold standby system model assuming the two modes of a unit. Two repairmen are considered to repair a failed unit. One is regular repairman who is always available with the system and other is skilled repairman who is arranged from the outside. The skilled repairman takes some significant time to reach at the system. As soon as a unit fails, it is attended immediately by regular repairman, who is able to complete the repair of the failed unit within a specified time limit with probability p, otherwise with probability q the skilled repairman is intimated to reach at the system in order to complete the repair of the failed unit. The system model is analyzed by using regenerative point technique in order to obtainvarious measures of systemeffectiveness.

Keywords: Regenerative point, reliability, MTSF, availability, busy period of repairman, net expected profit.

2000 AMS Subject Classification: 90B25

1. INTRODUCTION

It is obvious that a unit/system is composed of a number of components/elements and to achieve high reliability of the unit/system; we have to use high reliable components/elements. In many cases when it is not possible to produce such type of components/elements, we can increase the unit/system reliability by incorporating the redundancies of the corresponding components/elements. The another way to increase the reliability of the system is repair

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*Corresponding author
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maintenance. Various authors [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] have analyzed the reparable redundant system models by using the various concepts of repair such as two-phase repair, repair and post-repair, repair machine failure, poor and good physical conditions of repairman, random appearance and disappearance of repairman, rest period of repairman, preparation time for repair, waiting time of repairman, patience time of repairman etc. In most of the studies it is considered that only one repairman remains busy at a time. V. Goyal and K. Murari [2] analyzed a two-identical unit standby system model considering two types of repairmen- regular and expert. The regular repairman is always available with the system whereas expert repairman can be made available from the outside instantaneously. They further assumed that the failed unit is immediately attended by the regular repairman and if he is unable to repair it during prescribe time limit (patience time), the failed unit is attended by the expert repairman who is instantaneously available. They have assumed that once the expert repairman starts the repair of a failed unit, he repairs the all the units that fail during his stay at the system and then no failure unit is attended by the regular repairman. This assumption of Goyal and Murari seems to be unrealistic.

In present paper we analyze a two-identical unit standby system model considering two types of repairmen- regular and skilled. Regular repairman is always available with the system and skilled repairman is arranged from the outside who takes some significant time to reach at the system.The following economic related measures of system effectiveness have been obtained by using regenerative point technique-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and Mean time to system failure.
iii. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. Expected busy period of each repairmen during time interval (0, t).
v. Net expected profit in time interval (0, t) and in steady-state.

2. MODEL DESCRIPTION AND ASSUMPTIONS

i. The system comprises of two identical units. Initially, one unitis operative and other is kept into cold standby.

ii. Each unit of the system has two modes- normal (N) and total failure (F).

iii. Two repairmen are considered to repair a failed unit. One is regular repairman who is always available with the system and other is skilled repairman who is arranged from the outside who takes some significant time to reach at the system.

iv. As soon as an unit fails, it is attended immediately by regular repairman who is able to complete the repair of the failed unit within specified time limit with probability p. Otherwise, with probability q the skilled repairman is intimated to reach at the system in order to complete the repair of failed unit. The failed unit waits for skilled repairman till he arrives at the system.

v. The skilled repairman completes the repair of the failed unit with certainty.

vi. The failure time distribution of an operating unit, waiting and repair time of skilled repairman are taken exponential with different parameters whereas the repair time distribution of regular repairman is taken general.

vii. Each repaired unit works as good as new.

3. NOTATIONS AND STATES OF THE SYSTEM

a) Notations:

E	:	Set of regenerative states i.e. S_0 to S_2 and S_4 to S_8
α	:	Constant failure rate of an operating unit.
β	:	Constant repair rate of a skilled repairman.
λ	:	Constant rate of arrived of a skilled repairman.
$G(\cdot), g(\cdot)$	:	Cdf and pdf of repair time of aregular repairman within specified limit.
$p = (1-q)$	:	The probability that the regular repairman is able to repair of a failed unit during his prescribe limit.
$q_{ij}(\cdot)$	:	Pdf of transition time from state S_i to S_j.
p_{ij}	:	Steady state probability that the system transits from state S_i to S_j.
ψ_i	:	Mean sojourn time in state S_i
n	:	Mean repair time of totally failed unit $= \int_0^\infty t\,dG(t)$

*,~ : Symbols for Laplace and Laplace-Stieltjes transforms.

b) Symbols for the states of the systems:

N_o, N_s : Unit is in normal (N) mode and operative/standby.

F_{r1}, F_{r2} : Unit is in total failure (F) mode and under repair for regular/skilled repairman.

F_{w1}, F_{w2} : Unit is in total failure mode and waiting for regular/skilled repairman.

A, NA : Skilled repairman is available/not available.

TRANSITION DIAGRAM

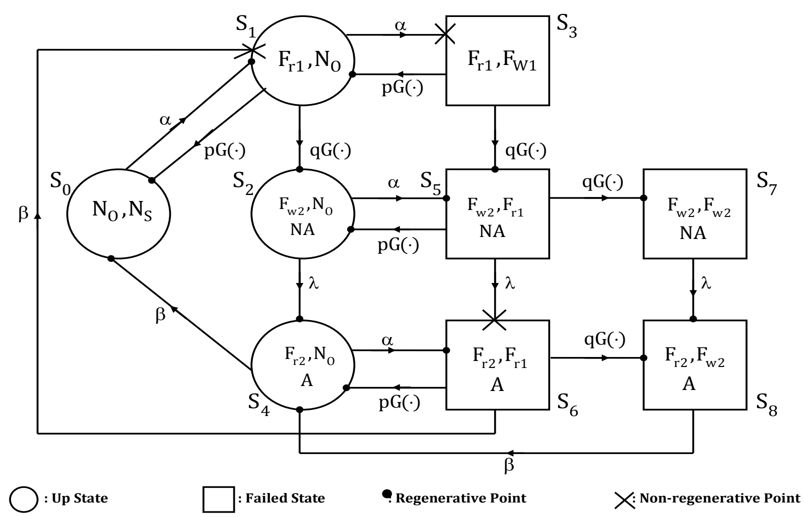

Fig.1

Using these symbols and keeping in view the assumptions stated in section-2, the possible states of the system are shown in transition diagram (Fig. 1). The epochs of transitions into the states S_3 from S_1, S_6 from S_5 and S_1 from S_6 are non-regenerative while all the other entrance epochs into the states are regenerative.

4. TRANSITION PROBABILITIES

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space E. The transition probability matrix of the embedded Markov-Chain is

$$\mathbf{P}=\begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{04} & p_{05} & p_{06} & p_{07} & p_{08} \\ p_{10} & p_{11}^{(3)} & p_{12} & p_{14} & p_{15}^{(3)} & p_{16} & p_{17} & p_{18} \\ p_{20} & p_{21} & p_{22} & p_{24} & p_{25} & p_{26} & p_{27} & p_{28} \\ p_{40} & p_{41} & p_{42} & p_{44} & p_{45} & p_{46} & p_{47} & p_{48} \\ p_{50}^{(6,1)} & p_{51}^{(6,1,3)} & p_{52}+p_{52}^{(6,1)} & p_{54}^{(6)} & p_{55}^{(6,1,3)} & p_{56} & p_{57} & p_{58}^{(6)} \\ p_{60}^{(1)} & p_{61}^{(1,3)} & p_{62}^{(1)} & p_{64} & p_{65}^{(1,3)} & p_{66} & p_{67} & p_{68} \\ p_{70} & p_{71} & p_{72} & p_{74} & p_{75} & p_{76} & p_{77} & p_{78} \\ p_{80} & p_{81} & p_{82} & p_{84} & p_{85} & p_{86} & p_{87} & p_{88} \end{bmatrix}$$

With non-zero elements-

$$p_{01}=1,\quad p_{10}=p\tilde{G}(\alpha),\quad p_{12}=q\tilde{G}(\alpha),\quad p_{11}^{(3)}=p\left\{1-\tilde{G}(\alpha)\right\},\quad p_{15}^{(3)}=q\left\{1-\tilde{G}(\alpha)\right\}$$

$$p_{24}=\frac{\lambda}{(\lambda+\alpha)},\quad p_{25}=\frac{\alpha}{(\lambda+\alpha)},\quad p_{40}=\frac{\beta}{(\alpha+\beta)},\quad p_{46}=\frac{\alpha}{(\alpha+\beta)},\quad p_{52}=p\tilde{G}(\lambda)$$

$$p_{57}=q\tilde{G}(\lambda),\quad p_{54}^{(6)}=\frac{p\lambda}{\lambda-\beta}\left\{\tilde{G}(\beta)-\tilde{G}(\lambda)\right\},\quad p_{58}^{(6)}=\frac{q\lambda}{\lambda-\beta}\left\{\tilde{G}(\beta)-\tilde{G}(\lambda)\right\}$$

$$p_{50}^{(6,1)}=\frac{p\lambda\beta}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\left\{(\lambda-\beta)\tilde{G}(\alpha)-(\lambda-\alpha)\tilde{G}(\beta)+(\beta-\alpha)\tilde{G}(\lambda)\right\}$$

$$p_{52}^{(6,1)}=\frac{q\lambda\beta}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\left\{(\lambda-\beta)\tilde{G}(\alpha)-(\lambda-\alpha)\tilde{G}(\beta)+(\beta-\alpha)\tilde{G}(\lambda)\right\}$$

$$p_{51}^{(6,1,3)}=\frac{p\lambda\beta}{(\lambda-\alpha)(\beta-\alpha)}\left\{1-\tilde{G}(\alpha)\right\}-\frac{p\lambda\alpha}{(\lambda-\beta)(\beta-\alpha)}\left\{1-\tilde{G}(\beta)\right\}+\frac{p\alpha\beta}{(\lambda-\alpha)(\lambda-\beta)}\left\{1-\tilde{G}(\lambda)\right\}$$

$$p_{55}^{(6,1,3)}=\frac{q\lambda\beta}{(\lambda-\alpha)(\beta-\alpha)}\left\{1-\tilde{G}(\alpha)\right\}-\frac{q\lambda\alpha}{(\lambda-\beta)(\beta-\alpha)}\left\{1-\tilde{G}(\beta)\right\}+\frac{q\alpha\beta}{(\lambda-\alpha)(\lambda-\beta)}\left\{1-\tilde{G}(\lambda)\right\}$$

$$p_{64}=p\tilde{G}(\beta),\quad p_{68}=q\tilde{G}(\beta),\quad p_{60}^{(1)}=\frac{p\beta}{\beta-\alpha}\left\{\tilde{G}(\alpha)-\tilde{G}(\beta)\right\},\quad p_{62}^{(1)}=\frac{q\beta}{\beta-\alpha}\left\{\tilde{G}(\alpha)-\tilde{G}(\beta)\right\}$$

$$p_{61}^{(1,3)}=\frac{p}{\beta-\alpha}\left[\beta\left\{1-\tilde{G}(\alpha)\right\}-\alpha\left\{1-\tilde{G}(\beta)\right\}\right],\quad p_{65}^{(1,3)}=\frac{q}{\beta-\alpha}\left[\beta\left\{1-\tilde{G}(\alpha)\right\}-\alpha\left\{1-\tilde{G}(\beta)\right\}\right]$$

$$p_{78}=1,\quad p_{84}=1 \tag{1-25}$$

The other elements of t. p. m. will be zero.

It can be easily verified that

$$p_{01}=p_{78}=p_{84}=1,\quad p_{10}+p_{12}+p_{11}^{(3)}+p_{15}^{(3)}=1,\quad p_{24}+p_{25}=1$$

$$p_{40}+p_{46}=1,\quad p_{52}+p_{57}+p_{54}^{(6)}+p_{58}^{(6)}+p_{50}^{(6,1)}+p_{52}^{(6,1)}+p_{51}^{(6,1,3)}+p_{55}^{(6,1,3)}=1$$

$$p_{64} + p_{68} + p_{60}^{(1)} + p_{62}^{(1)} + p_{61}^{(1,3)} + p_{65}^{(1,3)} = 1 \qquad (26\text{-}31)$$

5. MEAN SOJOURN TIMES

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then

$$\psi_i = \int_0^\infty P\left[U_i > t\right]dt$$

Therefore, its values for various regenerative states are as follows:

$$\psi_0 = \frac{1}{\alpha}, \qquad \psi_1 = \frac{\left\{1-\tilde{G}(\alpha)\right\}}{\alpha}, \qquad \psi_2 = \frac{1}{(\alpha+\lambda)}, \qquad \psi_4 = \frac{1}{(\alpha+\beta)}$$

$$\psi_5 = \frac{\left\{1-\tilde{G}(\lambda)\right\}}{\lambda}, \qquad \psi_6 = \frac{\left\{1-\tilde{G}(\beta)\right\}}{\beta}, \qquad \psi_7 = \frac{1}{\lambda}, \qquad \psi_8 = \frac{1}{\beta} \qquad (32\text{-}39)$$

6. ANALYSIS OF CHARACTERISTICS

a) Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_5, S_6, S_7 and S_8 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad (40)$$

Where,

$$N_1(s) = Z_0^* + q_{01}^* Z_1^* + q_{01}^* q_{12}^* Z_2^* + q_{01}^* q_{12}^* q_{24}^* Z_4^*$$

$$D_1(s) = 1 - q_{01}^* q_{10}^* - q_{01}^* q_{12}^* q_{24}^* q_{40}^*$$

and

$Z_i^*(i = 0,1,2,4)$ are the L. T. of

$$Z_0(t) = e^{-\alpha t}, \qquad Z_1(t) = e^{-\alpha t}\bar{G}(t), \qquad Z_2(t) = e^{-(\alpha+\lambda)t}, \qquad Z_4(t) = e^{-(\alpha+\beta)t}$$

Taking the Inverse Laplace Transform of (40), one can get the reliability of the system when it starts from state S_0.

The MTSF is given by

$$E(T_0) = \int_0^\infty R_0(t)\,dt = \lim_{s\to 0} R_0^*(s) = N_1/D_1 \tag{41}$$

Where,

$$N_1 = \psi_0 + \psi_1 + p_{12}\psi_2 + p_{12}p_{24}\psi_4$$

$$D_1 = 1 - p_{10} - p_{12}p_{24}p_{40}$$

b) Availability Analysis

Let $A_i(t)$ be the probability that the system is operative at epoch t, when it initially starts from $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the value $A_0(t)$ in terms of its L.T. $A_0^*(s)$.

The steady-state availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = N_2/D_2 \tag{42}$$

Where,

$$\begin{aligned}
N_2 = \Big[& \big(1 - p_{11}^{(3)}\big)\Big\langle \big(1 - p_{55}^{(6,1,3)}\big)\big\{1 - p_{46}(p_{64}+p_{68}) - p_{24}p_{46}p_{62}^{(1)}\big\} - \big(p_{52} + p_{52}^{(6,1)}\big)\big\{p_{24}p_{46}p_{65}^{(1,3)} \\
& + p_{25}\big(1 - p_{46}(p_{64}+p_{68})\big)\big\} - p_{46}\big(p_{57} + p_{54}^{(6)} + p_{58}^{(6)}\big)\big(p_{65}^{(1,3)} + p_{62}^{(1)}p_{25}\big)\Big\rangle - \big(p_{12}p_{25} + p_{15}^{(3)}\big) \\
& \big\{p_{46}p_{61}^{(1,3)}\big(p_{57} + p_{54}^{(6)} + p_{58}^{(6)}\big) + p_{51}^{(6,1,3)}\big(1 - p_{46}(p_{64}+p_{68})\big)\big\} - p_{24}p_{46}\Big\langle p_{51}^{(6,1,3)}\big(p_{12}p_{65}^{(1,3)} - p_{15}^{(3)}p_{62}^{(1)}\big) \\
& + p_{61}^{(1,3)}\big\{p_{12}\big(1 - p_{55}^{(6,1,3)}\big) + p_{15}^{(3)}\big(p_{52} + p_{52}^{(6,1)}\big)\big\}\Big\rangle\Big]\psi_0 + \Big[\big(1 - p_{55}^{(6,1,3)}\big)\big\{1 - p_{46}(p_{64}+p_{68}) \\
& - p_{24}p_{46}p_{62}^{(1)}\big\} - \big(p_{52} + p_{52}^{(6,1)}\big)\big\{p_{24}p_{46}p_{65}^{(1,3)} + p_{25}\big(1 - p_{46}(p_{64}+p_{68})\big)\big\} - p_{46}\big(p_{57} + p_{54}^{(6)} + p_{58}^{(6)}\big) \\
& \big(p_{65}^{(1,3)} + p_{62}^{(1)}p_{25}\big)\Big]\psi_1 + \Big[\big(1 - p_{46}(p_{64}+p_{68})\big)\big\{p_{12}\big(1 - p_{55}^{(6,1,3)}\big) + p_{15}^{(3)}\big(p_{52} + p_{52}^{(6,1)}\big)\big\} \\
& - p_{46}\big(p_{57} + p_{54}^{(6)} + p_{58}^{(6)}\big)\big(p_{12}p_{65}^{(1,3)} - p_{15}^{(3)}p_{62}^{(1)}\big)\Big]\psi_2 + \Big[p_{24}\big\{p_{12}\big(1 - p_{55}^{(6,1,3)}\big) + p_{15}^{(3)}\big(p_{52} + p_{52}^{(6,1)}\big)\big\} \\
& + \big(p_{57} + p_{54}^{(6)} + p_{58}^{(6)}\big)\big(p_{12}p_{25} + p_{15}^{(3)}\big)\Big]\psi_4 + \Big[p_{15}^{(3)} + p_{12}p_{25} - p_{46}(p_{64}+p_{68})\big(p_{12}p_{25} + p_{15}^{(3)}\big)
\end{aligned}$$

$$+p_{24}p_{46}\left(p_{12}p_{65}^{(1,3)}+p_{15}^{(3)}p_{62}^{(1)}\right)\Big]\left\{\frac{\lambda\beta\{(\lambda-\beta)\psi_1-(\lambda-\alpha)\psi_6+(\beta-\alpha)\psi_5\}}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\right\}+\frac{\beta}{(\beta-\alpha)}$$

$$\left[p_{24}p_{46}\left\{p_{12}\left(1-p_{55}^{(6,1,3)}\right)+p_{15}^{(3)}\left(p_{52}+p_{52}^{(6,1)}\right)\right\}+p_{46}\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)\left(p_{12}p_{25}+p_{15}^{(3)}\right)\right](\psi_1-\psi_6)$$

and

$$D_2=\Big[\left(1-p_{11}^{(3)}\right)\Big\langle\left(1-p_{55}^{(6,1,3)}\right)\left\{1-p_{46}\left(p_{64}+p_{68}\right)-p_{24}p_{46}p_{62}^{(1)}\right\}-\left(p_{52}+p_{52}^{(6,1)}\right)\Big\{p_{24}p_{46}p_{65}^{(1,3)}$$

$$+p_{25}\left(1-p_{46}\left(p_{64}+p_{68}\right)\right)\Big\}-p_{46}\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)\left(p_{65}^{(1,3)}+p_{62}^{(1)}p_{25}\right)\Big\rangle-\left(p_{12}p_{25}+p_{15}^{(3)}\right)$$

$$\left\{p_{46}p_{61}^{(1,3)}\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)+p_{51}^{(6,1,3)}\left(1-p_{46}\left(p_{64}+p_{68}\right)\right)\right\}-p_{24}p_{46}\Big\langle p_{51}^{(6,1,3)}\left(p_{12}p_{65}^{(1,3)}-p_{15}^{(3)}p_{62}^{(1)}\right)$$

$$+p_{61}^{(1,3)}\left\{p_{12}\left(1-p_{55}^{(6,1,3)}\right)+p_{15}^{(3)}\left(p_{52}+p_{52}^{(6,1)}\right)\right\}\Big\rangle\Big]\psi_0+\Big[\left(1-p_{55}^{(6,1,3)}\right)\Big\{1-p_{46}\left(p_{64}+p_{68}\right)$$

$$-p_{24}p_{46}p_{62}^{(1)}\Big\}-\left(p_{52}+p_{52}^{(6,1)}\right)\left\{p_{24}p_{46}p_{65}^{(1,3)}+p_{25}\left(1-p_{46}\left(p_{64}+p_{68}\right)\right)\right\}-p_{46}\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)$$

$$\left(p_{65}^{(1,3)}+p_{62}^{(1)}p_{25}\right)\Big]n+\Big[\left(1-p_{46}\left(p_{64}+p_{68}\right)\right)\left\{p_{12}\left(1-p_{55}^{(6,1,3)}\right)+p_{15}^{(3)}\left(p_{52}+p_{52}^{(6,1)}\right)\right\}$$

$$-p_{46}\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)\left(p_{12}p_{65}^{(1,3)}-p_{15}^{(3)}p_{62}^{(1)}\right)\Big]\psi_2+\Big[p_{24}\left\{p_{12}\left(1-p_{55}^{(6,1,3)}\right)+p_{15}^{(3)}\left(p_{52}+p_{52}^{(6,1)}\right)\right\}$$

$$+\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)\left(p_{12}p_{25}+p_{15}^{(3)}\right)\Big]\psi_4+\left(1+2p_{57}+p_{58}^{(6)}\right)\Big[p_{24}p_{46}\left(p_{12}p_{65}^{(1,3)}+p_{15}^{(3)}p_{62}^{(1)}\right)$$

$$p_{15}^{(3)}+p_{12}p_{25}-p_{46}\left(p_{64}+p_{68}\right)\left(p_{12}p_{25}+p_{15}^{(3)}\right)\Big]n+p_{46}\left(1+p_{64}\right)\Big[p_{24}\Big\{p_{12}\left(1-p_{55}^{(6,1,3)}\right)$$

$$+p_{15}^{(3)}\left(p_{52}+p_{52}^{(6,1)}\right)\Big\}+\left(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\right)\left(p_{12}p_{25}+p_{15}^{(3)}\right)\Big]n \tag{43}$$

The expected up (operative) time of the systemduring (0, t) is given by

$$\mu_{up}(t)=\int_0^t A_0(u)\,du \tag{44}$$

So that

$$\mu_{up}^*(s)=A_0^*(s)/s \tag{45}$$

c) Busy Period Analysis

Let $B_i^r(t)$ and $B_i^s(t)$ be the respective probabilities that the regular and skilled repairmen are busy in the repair of a totally failed unitat epoch t, when the system initially starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the values of above two probabilities in terms of their L.T. i.e. $B_i^{r*}(s)$ and $B_i^{s*}(s)$.

The steady state results for the above two probabilities are given by

$$B_o^r = \lim_{s \to 0} sB_0^{r*}(s) = N_3/D_2 \tag{46}$$

Similarly,

$$B_0^s = \lim_{s \to 0} sB_0^{s*}(s) = N_4/D_2 \tag{47}$$

Where,

$$N_3 = n\Big[\Big(1-p_{55}^{(6,1,3)}\Big)\Big\{1-p_{46}\left(p_{64}+p_{68}\right)-p_{24}p_{46}p_{62}^{(1)}\Big\}-\Big(p_{52}+p_{52}^{(6,1)}\Big)\Big\{p_{24}p_{46}p_{65}^{(1,3)}$$
$$+p_{25}\left(1-p_{46}\left(p_{64}+p_{68}\right)\right)\Big\}-p_{46}\Big(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\Big)\Big(p_{65}^{(1,3)}+p_{62}^{(1)}p_{25}\Big)$$
$$+p_{15}^{(3)}+p_{12}p_{25}+p_{24}p_{46}\Big(p_{12}p_{65}^{(1,3)}+p_{15}^{(3)}p_{62}^{(1)}\Big)-p_{46}\left(p_{64}+p_{68}\right)\Big(p_{12}p_{25}+p_{15}^{(3)}\Big)$$
$$+p_{24}p_{46}\Big\{p_{12}\Big(1-p_{55}^{(6,1,3)}\Big)+p_{15}^{(3)}\Big(p_{52}+p_{52}^{(6,1)}\Big)\Big\}+p_{46}\Big(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\Big)\Big(p_{12}p_{25}+p_{15}^{(3)}\Big)\Big]$$

$$N_4 = \Big[p_{24}p_{46}\Big(p_{12}p_{65}^{(1,3)}-p_{15}^{(3)}p_{62}^{(1)}\Big)+\Big(p_{12}p_{25}+p_{15}^{(3)}\Big)\Big\{1-p_{46}\left(p_{64}+p_{68}\right)\Big\}\Big]\left(n-\psi_5\right)+p_{46}\big[p_{24}$$
$$\Big\{p_{12}\Big(1-p_{55}^{(6,1,3)}\Big)+p_{15}^{(3)}\Big(p_{52}+p_{52}^{(6,1)}\Big)\Big\}+\Big(p_{57}+p_{54}^{(6)}+p_{58}^{(6)}\Big)\Big(p_{12}p_{25}+p_{15}^{(3)}\Big)\Big]\left(n-\psi_6\right)$$

and D_2 is same as expressed by equation (28).

The expected busy period of regular and skilled repairmen in repair of a totally failed unit during time interval (0,t) are respectively given by

$$\mu_b^r(t) = \int_0^t B_0^r(u)\,du \quad \text{and} \quad \mu_b^s(t) = \int_0^t B_0^s(u)\,du \tag{48-49}$$

So that,

$$\mu_b^{r*}(s) = B_0^{r*}(s)/s \quad \text{and} \quad \mu_b^{s*}(s) = B_0^{s*}(s)/s \tag{50-51}$$

7. COST BENEFIT ANALYSIS

We are now in the position to obtain the profit function by considering mean up time of the system during (0, t), expected busy period of regular and skilled repairmen in repair of a totally failed unit during (0, t).

Let us suppose

K_0 =revenue per-unit time by the system when it is operative.

K_1 =cost per-unit time when the regular repairman is busy in repair of a totallyfailed unit.

K_2 =cost per-unit time when the skilled repairman is busy in repair ofa totally failed unit.

Now, the net expected profit incurred in time interval (0, t) is given by-

$$P_0(t) = K_0\mu_{up}(t) - K_1\mu_b^r(t) - K_2\mu_b^s(t) \quad (52)$$

The expected profit per-unit time in steady state is

$$P_0 = \lim_{t\to\infty} P_0(t)/t = \lim_{s\to 0} s^2 P_0^*(s)$$

$$= K_0A_0 - K_1B_0^r - K_2B_0^s \quad (53)$$

8. CASE STUDIES

The system model has wide applicability for various form of p.d.f.s of repair time of a regular repairman. As an illustration, we consider the following two cases to obtain the measures of system effectiveness obtained in earlier sections.

Case I: When the repair time of a regular repairman followsInverse Gaussian distribution with p.d.f. as given below-

$$g(t) = \frac{1}{\sqrt{2\pi}t^{3/2}} e^{-(t-\theta)^2/2\theta^2 t} \qquad ; t \geq p$$

The Laplace Transform of above density function and Laplace Stieltjes Transform of corresponding c.d.f.'s is given by-

$$g^*(s) = \tilde{G}(s) = e^{\left\{1-\sqrt{\left(1+2s\theta^2\right)}\right\}/\theta}$$

Here $\tilde{G}(s)$ is the Laplace Stieltjes Transform of the c.d.f. $G(t)$corresponding to the p.d.f. $g(t)$.

In view of above we have the following changesin results (2-5), (10-23), (33), and (36-37)-

$$p_{10} = pe^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}, \qquad p_{12} = qe^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}, \quad p_{11}^{(3)} = p\left[1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right]$$

$$p_{15}^{(3)} = q\left[1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right], \qquad p_{52} = pe^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}, \quad p_{57} = qe^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}$$

$$p_{54}^{(6)} = \frac{\lambda}{(\lambda-\beta)}p\left[e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta} - e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right], \qquad p_{58}^{(6)} = \frac{\lambda}{(\lambda-\beta)}q\left[e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta} - e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right]$$

$$p_{50}^{(6,1)} = \frac{p\lambda\beta}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\left[(\lambda-\beta)e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta} - (\lambda-\alpha)e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta} + (\beta-\alpha)e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right]$$

$$p_{52}^{(6,1)} = \frac{q\lambda\beta}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\left[(\lambda-\beta)e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta} - (\lambda-\alpha)e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta} + (\beta-\alpha)e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right]$$

$$p_{51}^{(6,1,3)} = \frac{p\lambda\beta}{(\lambda-\alpha)(\beta-\alpha)}\left[1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right] - \frac{p\lambda\alpha}{(\lambda-\beta)(\beta-\alpha)}\left[1-e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right]$$
$$+\frac{p\alpha\beta}{(\lambda-\alpha)(\lambda-\beta)}\left[1-e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right]$$

$$p_{55}^{(6,1,3)} = \frac{q\lambda\beta}{(\lambda-\alpha)(\beta-\alpha)}\left[1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right] - \frac{q\lambda\alpha}{(\lambda-\beta)(\beta-\alpha)}\left[1-e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right]$$
$$+\frac{q\alpha\beta}{(\lambda-\alpha)(\lambda-\beta)}\left[1-e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right]$$

$$p_{68} = pe^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}, \qquad p_{64} = qe^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}$$

$$p_{60}^{(1)} = \frac{\beta}{(\beta-\alpha)}p\left[e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta} - e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right], \qquad p_{62}^{(1)} = \frac{\beta}{(\beta-\alpha)}q\left[e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta} - e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right]$$

$$p_{61}^{(1,3)} = \frac{p}{(\beta-\alpha)}\left[\beta\left\{1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right\} - \alpha\left\{1-e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right\}\right]$$

$$p_{65}^{(1,3)} = \frac{q}{(\beta-\alpha)}\left[\beta\left\{1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right\} - \alpha\left\{1-e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right\}\right]$$

$$\psi_1 = \frac{1}{\alpha}\left[1-e^{\left\{1-\sqrt{(1+2\alpha\theta^2)}\right\}/\theta}\right], \qquad \psi_5 = \frac{1}{\lambda}\left[1-e^{\left\{1-\sqrt{(1+2\lambda\theta^2)}\right\}/\theta}\right], \qquad \psi_6 = \frac{1}{\beta}\left[1-e^{\left\{1-\sqrt{(1+2\beta\theta^2)}\right\}/\theta}\right]$$

The values of n will be as follows-

$$n = \theta$$

Case II: When the repair time of a regular repairman follows exponential distribution with p.d.f.s as follows-

$$g(t) = \theta e^{-\theta t}$$

The Laplace Transform of above density function is as given below.

$$g^*(s) = \tilde{G}(s) = \theta/(s+\theta)$$

Here $\tilde{G}(s)$ is the Laplace-Stieltjes Transform of the c.d.f. $G(t)$ corresponding to the p.d.f. $g(t)$.

In view of above the changed values of transition probabilities, mean sojourn times andthe value of n are given below-

$$p_{10} = p\frac{\theta}{\theta+\alpha}, \qquad p_{12} = q\frac{\theta}{\theta+\alpha}, \qquad p_{11}^{(3)} = p\left\{1-\frac{\theta}{\theta+\alpha}\right\}$$

$$p_{15}^{(3)} = q\left\{1-\frac{\theta}{\theta+\alpha}\right\}, \qquad p_{52} = p\frac{\theta}{\theta+\lambda}, \qquad p_{57} = q\frac{\theta}{\theta+\lambda},$$

$$p_{54}^{(6)} = p\frac{\lambda}{(\lambda-\beta)}\left[\frac{\theta}{\theta+\beta}-\frac{\theta}{\theta+\lambda}\right], \qquad p_{58}^{(6)} = q\frac{\lambda}{(\lambda-\beta)}\left[\frac{\theta}{\theta+\beta}-\frac{\theta}{\theta+\lambda}\right]$$

$$p_{50}^{(6,1)} = \frac{p\lambda\beta}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\left[(\lambda-\beta)\frac{\theta}{\theta+\alpha}-(\lambda-\alpha)\frac{\theta}{\theta+\beta}+(\beta-\alpha)\frac{\theta}{\theta+\lambda}\right]$$

$$p_{52}^{(6,1)} = \frac{q\lambda\beta}{(\lambda-\alpha)(\lambda-\beta)(\beta-\alpha)}\left[(\lambda-\beta)\frac{\theta}{\theta+\alpha}-(\lambda-\alpha)\frac{\theta}{\theta+\beta}+(\beta-\alpha)\frac{\theta}{\theta+\lambda}\right]$$

$$p_{51}^{(6,1,3)} = \frac{p\lambda\beta}{(\lambda-\alpha)(\beta-\alpha)}\left[1-\frac{\theta}{\theta+\alpha}\right]-\frac{p\lambda\alpha}{(\lambda-\beta)(\beta-\alpha)}\left[1-\frac{\theta}{\theta+\beta}\right]+\frac{p\alpha\beta}{(\lambda-\alpha)(\lambda-\beta)}\left[1-\frac{\theta}{\theta+\lambda}\right]$$

$$p_{55}^{(6,1,3)} = \frac{q\lambda\beta}{(\lambda-\alpha)(\beta-\alpha)}\left[1-\frac{\theta}{\theta+\alpha}\right]-\frac{q\lambda\alpha}{(\lambda-\beta)(\beta-\alpha)}\left[1-\frac{\theta}{\theta+\beta}\right]+\frac{q\alpha\beta}{(\lambda-\alpha)(\lambda-\beta)}\left[1-\frac{\theta}{\theta+\lambda}\right]$$

$$p_{68} = p\frac{\theta}{\theta+\beta}, \qquad p_{64} = q\frac{\theta}{\theta+\beta}, \qquad p_{60}^{(1)} = p\frac{\beta}{(\beta-\alpha)}\left[\frac{\theta}{\theta+\alpha}-\frac{\theta}{\theta+\beta}\right]$$

$$p_{62}^{(1)} = q\frac{\beta}{(\beta-\alpha)}\left[\frac{\theta}{\theta+\alpha}-\frac{\theta}{\theta+\beta}\right], \qquad p_{61}^{(1,3)} = \frac{p}{(\beta-\alpha)}\left[\beta\left\{1-\frac{\theta}{\theta+\alpha}\right\}-\alpha\left\{1-\frac{\theta}{\theta+\beta}\right\}\right]$$

$$p_{65}^{(1,3)} = \frac{q}{(\beta-\alpha)}\left[\beta\left\{1-\frac{\theta}{\theta+\alpha}\right\}-\alpha\left\{1-\frac{\theta}{\theta+\beta}\right\}\right]$$

$$\psi_1 = \frac{1}{\alpha}\left[1-\frac{\theta}{\theta+\alpha}\right], \qquad \psi_5 = \frac{1}{\lambda}\left[1-\frac{\theta}{\theta+\lambda}\right], \qquad \psi_6 = \frac{1}{\beta}\left[1-\frac{\theta}{\theta+\beta}\right]$$

$$n = 1/\theta$$

9. GRAPHICAL REPRESENTATION AND CONCLUSIONS

The curves for MTSF and profit function are drawn for two particular cases I and II in respect of different parameters. **In case-I, when repair time of regular repairman follows Inverse GaussianDistribution,** Figs. 2 and 3 depict the variations in MTSF and profit function with respect to failure parameter of an operating unit α for different value of waiting time of skilled repairman λ and repair time of a regular repairman θ. We may clearly observe from Fig.2 that MTSF decreases uniformly as the values of α increase. It also reveals that MTSF increases with the increase in λ and decreases with the increase in θ.

Similarly, Fig. 3, reveals the variation in profit with respect to α for varying values of λ and θ when the values of other parameters are kept fix as β=5, K_0=300, K_1=200 and K_2=300. From this Figure it is clearly observed from dotted curves that system is profitable only if α is less than 0.235, 0.242 and 0.255 for λ=0.02, 0.04 and 0.06respectively for fixed value of θ=5 and from smooth curves we conclude that system is profitable only if α is less than 0.14, 0.145 and 0.15 for λ=0.02, 0.04 and 0.06 respectively for fixed value of θ=8.

In case-II, when repair time of regular repairman follows exponential distribution, Figs. 4 and 5 depict the variations in MTSF and profit function with respect to failure parameter α for different values of λ and θ. We may clearly reveal from Fig.4 that MTSF decreases as the values of α increase. It is also pointed out that MTSF increases with the increase in λ and increases with the increase in θ.

Similarly, Fig. 5, shows the variation in profit with respect to α for varying values of λ and θ when the values of other parameters are kept fix as β=5, K_0=400, K_1=150 and K_2=300. From Fig. 4 it is observed from dotted curves that system is profitable only if α is less than 0.12, 0.122 and 0.13 for λ =0.1, 0.2 and 0.3respectively for fixed value of θ=0.075 and from smooth curves we conclude that system is profitable only if α is less than 0.205, 0.21 and 0.23 for λ =0.1, 0.2 and 0.3 respectively for fixed value of θ=0.125.

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Behavior of MTSF for particular case-1 with respect to α, λ and θ

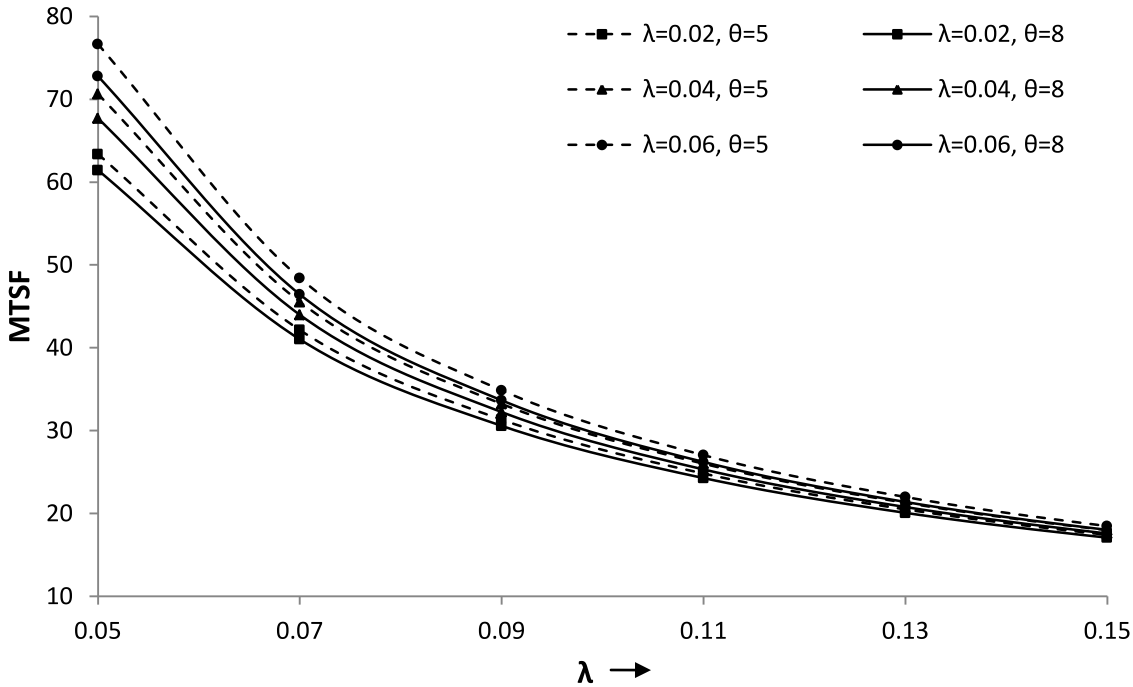

Fig.2

Behavior of profit for particular case-1 with respect to α, λ and θ

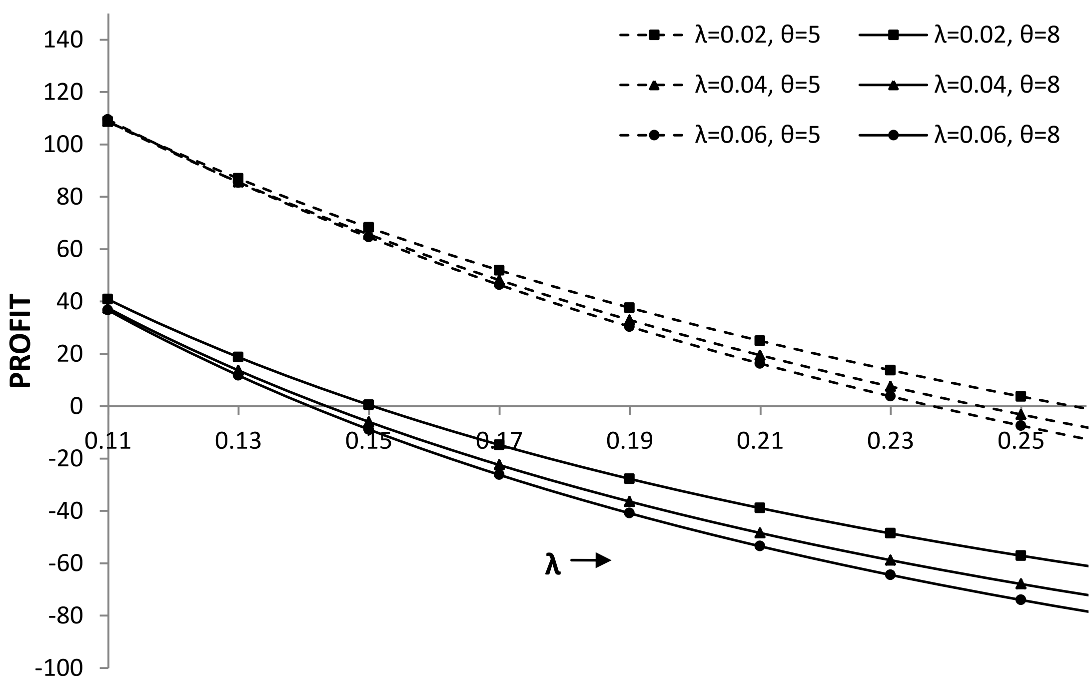

Fig.3

Behavior of MTSF for particular case-2 with respect to α, λ and θ

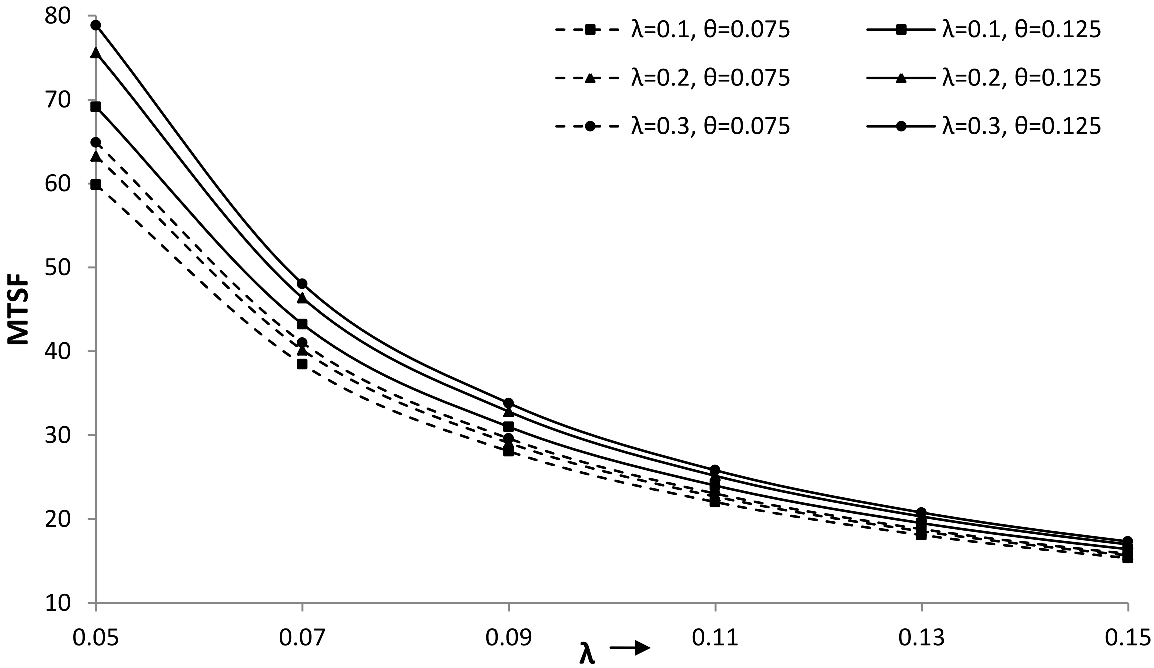

Fig. 4

Behavior of profit for particular case-2 with respect to α, λ and θ

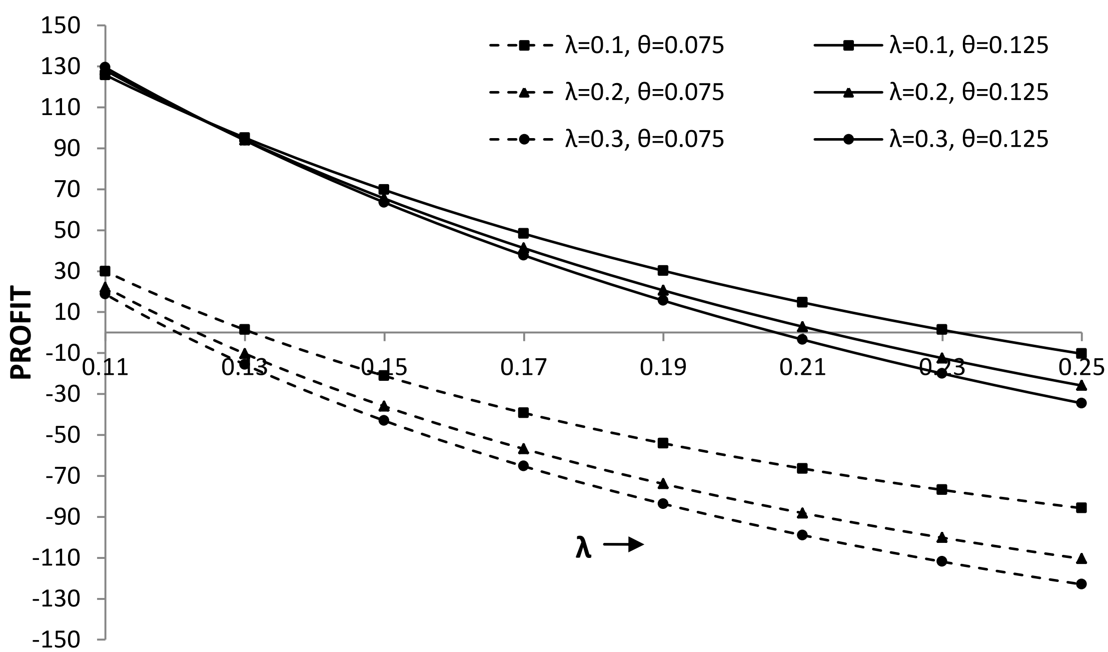

Fig.5