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SOLITONIC STRUCTURES IN A GENERALIZED DISPERSIVE CAMASSA-HOLM MODEL

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Abstract: A generalized Camassa-Holm model is introduced to describe the irrotational incompressible flow for a shallow layer of inviscid fluid moving under the influence of gravity without surface tension when the model has a strong nonlinear dispersion. This physical model also contains a set of nonlinear terms. Rich regular and singular solitons are found when a transaction between nonlinearity and dispersion analysis is performed. Realizations of this model can be made in terms of restrictions on its exponential sequence. Besides compactons, kinks, periodic compactons and multiple compactons that are found, pair compactons which entitled for a simulation that has two coexisting symmetrical humps are taken into account. In a special case when the coefficient of term involving first-order derivative on x satisfies k=0, there occurs blow-up phenomena, as well as usual solitary pattern solutions. In addition, depending on the development of ansatz forms, with some combinations of the parameters, two families of symmetrical and non- symmetrical structures with peak-like wave crests are obtained in exact form.

Key words: Generalized Camassa-Holm equation; multiple soliton; non-symmetrical compacton; pair compacton; blow-up; solitary pattern solution; periodic solitary wave.

2000 AMS Subject Classification: 35J99

1. Introduction

Our model system is based entirely on the completely integrable Hamiltonian Camassa-Holm model which is arising in the context of small amplitude shallow water waves over a flat bottom for inviscid fluid moving under the influence of gravity without surface tension as consistent in this limit as the KdV model, which has been intensively studied for a century. The model is of intrinsic interest in the study of solitonic structures because it is novel in that its solitary waves have a discontinuous

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first derivative in contrast to the great smoothness of most previously known species of soliton forms.

$$u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}$$

The fluid velocity in the x-direction or equivalently the height of the water's free surface is represented by the parameter u, and $4k = c_0$ is the constant critical shallow water wave speed which is proportional to the square root of the wave depth. The underlying theme of this work is the impact of nonlinear dispersion on the formation of sonlitonic patterns. In order to support localized structures, the enhanced spread of waves has to be counteracted by adequately stronger nonlinearities. The nonlinear dispersion presented in this work seems to provide one such mechanism. The degeneracy at the front generates sharp fronts and supports the formation and propagation of robust compact structures. The fact that compact patterns emerge in the parabolic case leads to the conjecture that analogous dispersive models will also support localized patterns. In many cases, though the underlying strongly nonlinear processes are quite complex and thus very hard to model, the emerging patterns are remarkably simple over a wide range of scales. By definition compact patterns are non-analytical entities on the frontlines. In dispersive processes the presence of even a weak singularity poses a formidable numerical challenge, which in recent literature devoted to the numerical aspects of the problem [1].

Unlike the derivations of the KdV equation and BBM equation by asymptotic procedures, the above Camassa-Holm model was derived by approximations in the Hamiltonian that produce unidirectional propagation and preserve the momentum part of the Lie-Poisson structure. The conserved quantities and the initial value problem of the Camassa-Holm model are investigated in [2]. Symmetry properties are discussed in [3]. Integrable perturbation is investigated in [4]. The soliton solution of the Camassa-Holm model is investigated with variation method in [5]. Tian et al. [6] discussed the traveling wave solutions and double soliton solutions for the nonlinear models, and introduced the definitions of concave, convex peaked soliton and smooth soliton solution. Chen et al. [7] proposed the viscous Camassa-Holm equation as a closure approximate for the Reynolds- averaged equation of the incompressible Navier-Stokes fluid. This approximation is tested on turbulent channel flows with steady mean. Dullin et al. [8] studied a class of 1+1 quadratically nonlinear water wave equations that combine the linear dispersion of the KdV equation with the nonlinear/nonlocal dispersion of the C-H equation, yet still preserved integrability via the inverse scatting transform method. And [9] got the integrable equation derived by asymptotic expansion at one order higher approximation than KdV equation. The equation discussed in the work is in this class for unidirectional water waves with fluid velocity u(x,y) as follows,

$$m_t + c_0 u_x + u m_x + 2 m u_x = -\gamma u_{xxx}$$

(1)

where $m = u - \alpha^2 u_{xx}$ is a momentum variable, the constants α^2 and γ / c_0 are squares of length scales, and $c_0 = \sqrt{gh}$ is the linear wave speed for undisturbed water at rest at spatial infinity, where u and m are taken to vanish. Eq. (1) restricts two separately integrable soliton equations for water waves. When $\alpha^2 \to 0$, this equation becomes the KdV equation $u_t + c_0 u_x + 3 u u_x = -\gamma u_{xxx}$ which for $c_0 = 0$ the KdV equation has the famous smooth soliton solution $u(x,t) = u_0 \operatorname{sec} h^2((x-ct)\sqrt{u_0/\gamma}/2)$, where $c = c_0 + u_0$. Instead, taking $\gamma \to 0$ in the Eq. (1) implies the CH equation

$$u_t + c_0 u_x - \alpha^2 u_{xxt} + 3 u u_x = \alpha^2 (2 u_x u_{xx} + u u_{xxx})$$

(2)

which for $c_0 = 0$ has peakon soliton solution u(x,t)=$ce^{-|x-ct|}$. When $\alpha^2 = 1$, the Eq. (2) becomes Eq. (1). Foias et al. [9] reviewed the properties of the nonlinear Navier-Stokes-Alpha (NS-α) model of incompressible fluid turbulence, or called the viscous Camassa-Holm equation in the literature. The NS-α model are derived by filtering the velocity of the fluid loop in Kelvin's circulation theorem for the Navier-Stokes equation. They also found that this filtering causes the wave number spectrum of the translational kinetic energy for the NS-α model to roll off as k^{-3} for $k\alpha > 1$ in three dimensions. Instead of continuing along the slower Kolmogorov Scaling law $k^{-5/3}$, that it follows for $k\alpha < 1$. [9] also explained how the NS-α model is related to large eddy simulation (LES) turbulence modeling and to the stress tensor for second-grade fluids.

Traveling waves are very interesting from the point of view of applications whether their soliton expressions are in explicit or implicit forms. These types of structures will not change their shapes during propagation and are thus easy to detect. The wide variety of solitary waves supported by the Camassa-Holm model should have convinced that nonlinear dispersion opens a window to a whole new class of nonlinear phenomena. It was recently found that the interaction of nonlinear dispersion with nonlinearity convection generates exactly compact structures free of exponential tails. This interaction may also generate many other structures otherwise unattainable.

The purpose of the present paper is try to study the generalized dispersive form of the equation posed above and discuss the possibility of finding solitonic structures, when the underlying system contains transactions between nonlinearity and these strengthened dispersions. At this stage of research, we are concerned with the possibility of obtaining other types of solitonlike structures for the nonlinear dispersive equation (3) given below by taking into consideration some restriction conditions on the parameter domains and the exponential sequence.

$$u_t + ku_x + \beta_1 u_{xxt} + \beta_2 (u^m)_x + \beta_3 u_x (u^n)_{xx} + \beta_4 u(u^p)_{xxx} + (u^l)_{xxx} = 0$$

(3)

We analyze this model by considering two types of parameter conditions. The analysis is done for specific parameter k that determines the term comprising first-order derivative on x. The structures involved in this system, for obvious reasons, will interact with each other only when their exponential sequence satisfies some dependent relations. Additionally, a strong nonlinear dispersive term $\left(u^l\right)_{xxx}$ is added to the generalized Camassa-Holm model with regarding transactions in the equation of motion. With ansatz method, periodic compactons, kinks, multiple compactons and solitary pattern solutions are obtained in exact forms. A typical compacton like structure with two symmetry humps is entitled pair compacton. It is possible to find singular structures by applying the simple reasoning that is common for obtaining classes of regular solitons. In the case of k=0, especially blow up phenomena with one or more wave crests resemble the shape of typical blow up are found. It is worthy of mention that the energy depends on the velocity of each structure. Considering different combinations of the ansatz expressions, two families of symmetrical and non-symmetrical compacton-like structures for special nonlinear dispersion Camassa-Holm models exist. We also take a brief account of the many dimensional cases.

This paper is organized as follows: In section Ⅱ we derive four different forms of solutions for the generalized dispersive Camassa-Holm equation while the parameter satisfies $k \neq 0$, particularly, we give two kinds of kink compacton solutions, and multi-compacton solutions are also found. Similarly, we give the solitary wave solutions for Eq. (3) whiles the parameter $k = 0$. In section Ⅲ we use the combination of ansatz parameters to give pair compactons and non-symmetrical compacton. In sectionⅣ higher-dimensional cases for the generalized model are discussed.

2. Soliton solutions

For the generalized Camassa-Holm equation *C-H (m,n,p,l)* which imposes independent exponential sequence, thus avoid any restrictions in the parameter values and the dispersion models. Such expression reduced us to fix a suitable condition on transactions between nonlinearity and dispersion to produce the structures we aim to find. Let us now present the traveling waves,

$$u(x,t) = u(\xi) = u(x - Dt) \tag{4}$$

By utilizing the above relations in which soliton propagation is represented by constant D, we obtain the following ordinary equation,

$$(k - D)u_{\xi} - \beta_1 D u_{3\xi} + \beta_2 (u^m)_{\xi} + \beta_3 u_{\xi} (u^n)_{2\xi} + \beta_4 u (u^p)_{3\xi} + (u^l)_{3\xi} = 0 \tag{5}$$

Compacton solutions and solitary pattern solutions may be determined by the ansatz expressions below,

Ansatz 1; $$u = A\cos^{\beta} B\xi \tag{6}$$

Ansatz 2: $$u = A\sin^{\beta} B\xi \tag{7}$$

Ansatz 3: $$u = A\cosh^{\beta} B\xi \tag{8}$$

Ansatz 4: $$u = A\sinh^{\beta} B\xi \tag{9}$$

A: Case $k \neq 0$

As a result of applications of the above ansatz forms, in order to hold solitons for the generalized model while choosing the constant coefficient k equals to zero, however, we arrive at three relations for all the values of m, n, p, l at infinity: $l - 1 = m - 1 = n = p$, $l = m \neq n = p$ and a more general case $m \neq n \neq p \neq l$. Only under these situations can we investigate the soliton solutions for nonlinear system (5). We stress that, in these three conditions studied and based on the sine-cosine method, the following analysis are respectively valid for all cases of exponential sequence, and special cases are also investigated. Next we proceed to a detailed description.

Case 1: From the above analysis, we see that on substituting the first ansatz from Eq. (6) into Eq.(5), we get an equation,

$$-A^l B^3 l\beta(l\beta-1)(l\beta-2)\cos^{l\beta-3}B\xi + A^l B^3 l^3\beta^3\cos^{l\beta-1}B\xi + AB^3 D\beta(\beta-1)(\beta-2)\cos^{\beta-3}B\xi$$
$$-AB\beta[(k-D)+B^2D(\beta-1)(\beta-2)+B^2D\beta_1(3\beta-2)]\cos^{\beta-1}B\xi - A^m Bm\beta\beta_2\cos^{m\beta-1}B\xi -$$
$$A^{n+1}B^3 n\beta^2\beta_3(n\beta-1)\cos^{n\beta+\beta-3}B\xi + A^{n+1}B^3n^2\beta^3\beta_3\cos^{n\beta+\beta-1}B\xi$$
$$-A^{p+1}B^3p\beta(p\beta-1)(p\beta-2)\beta_4\cos^{p\beta+\beta-3}B\xi + A^{p+1}B^3p^3\beta^3\beta_4\cos^{p\beta+\beta-1}B\xi = 0$$

In the case of $(\beta-1)(\beta-2)=0$, as the soliton solution cannot have the parameter n exceeds 2, the possible system for the above nonlinear algebraic equation takes the form,

$$\begin{cases} l\beta-3=\beta-1 \\ n\beta+\beta-3=p\beta+\beta-3 \\ l\beta-1=m\beta-1=n\beta+\beta-1=p\beta+\beta-1 \\ -A^lB^3l\beta(l\beta-1)(l\beta-2)-AB\beta[(k-D)+B^2D(\beta-1)(\beta-2)+B^2D\beta_1(3\beta-2)]=0 \\ -A^{n+1}B^3n\beta^2\beta_3(n\beta-1)-A^{p+1}B^3p\beta(p\beta-1)(p\beta-2)\beta_4=0 \\ A^lB^3l^3\beta^3-A^mBm\beta\beta_2+A^{n+1}B^3n^2\beta^3\beta_3+A^{p+1}B^3p^3\beta^3\beta_4=0 \end{cases}$$

We get as $l-1=m-1=n=p=\dfrac{2}{\beta}, B^2=\dfrac{\beta_2(\beta+2)}{\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4}$,

$A^n=\dfrac{(D-k)[\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4]-D\beta_1\beta_2(\beta+2)(3\beta-2)}{\beta_2(\beta+2)^2(\beta+1)}$. It is necessary to determine compacton found as exact solutions for the generalized Camassa-Holm equation, which has infinite amplitude (See Fig.1),

$$u=\begin{cases}\left\{\dfrac{(D-k)[\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4]-D\beta_1\beta_2(\beta+2)(3\beta-2)}{\beta_2(\beta+2)^2(\beta+1)}\cos^2\left(\sqrt{\dfrac{\beta_2(\beta+2)}{\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4}}\xi\right)\right\}^{\frac{1}{n}}, & \left|\sqrt{\dfrac{\beta_2(\beta+2)}{\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4}}\xi\right|\le\dfrac{\pi}{2} \\ 0, otherwise \end{cases}$$

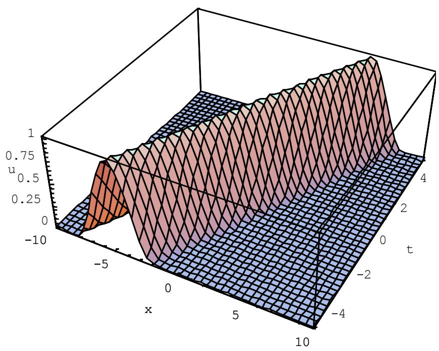


Fig.1 The shape of a compacton for the case when $n\le 2$. Where we plot the above function u. The parameters are $\beta_1=1,\beta_2=2,\beta_3=-2,\beta_4=-1,n=1$ and $k=-3$.

For compacton solutions, indeed, similar considerations can be applied to other exponential sequence conditions. When the relation $l = m \neq n = p$ is satisfied, a compacton solution under the constraints on the exponential sequence for the generalized C-H (m, n, p, l) is given by,

$$u = \begin{cases} \left\{ \dfrac{n^2\beta^3\beta_3 + n^3\beta^3\beta_4}{(\beta+4)(\beta+3)(\beta+2)} \cos^\beta\left(\sqrt{\dfrac{\beta_2}{(2n+1)^2\beta^2}}\,\xi\right)\right\}^{\frac{\beta}{2}}, & \left|\sqrt{\dfrac{\beta_2}{(2n+1)^2\beta^2}}\,\xi\right| \leq \dfrac{\pi}{2}, \\ 0, \textit{otherwise} \end{cases}$$

A general case is finally considered for compactons. While $m \neq n \neq p \neq l$, compacton takes a different form,

$$u = \begin{cases} \left\{ \dfrac{-B^2(n\beta+\beta)(n\beta+\beta-1)(n\beta+\beta-2) - B^2 n\beta^2\beta_3(n\beta-1)}{\beta_2(n\beta+\beta-2)} \cos^\beta\left(\sqrt{\dfrac{D-k}{D\beta_1(3\beta-2)}}\,\xi\right)\right\}^{-\frac{\beta}{2}}, & \left|\sqrt{\dfrac{D-k}{D\beta_1(3\beta-2)}}\,\xi\right| \leq \dfrac{\pi}{2}. \\ 0, \textit{otherwise} \end{cases}$$

Case 2: Compacton type solutions may occur when redefined ansatz u. While putting ansatz 2 into ordinary equation (5), a symmetrical compacton with two wave crests has been found. Indeed, by analogy with the simulation above, we introduce the name pair compacton to designate this soliton that has the form of two humps [Fig. 2(a)]. A similar periodic pair compacton was found [See Fig. 2(b)(c)].When *l-1=m-1=n=p,*

$$u = \begin{cases} \left\{ \dfrac{(D-k)[\beta^3(n+1)^3 + 4\beta\beta_3 + 8\beta\beta_4] - D\beta_1\beta_2(\beta+2)(3\beta-2)}{\beta_2(\beta+2)^2(\beta+1)} \sin^2\left(\sqrt{\dfrac{\beta_2(\beta+2)}{\beta^3(n+1)^3 + 4\beta\beta_3 + 8\beta\beta_4}}\,\xi\right)\right\}^{\frac{1}{n}}, & \left|\sqrt{\dfrac{\beta_2(\beta+2)}{\beta^3(n+1)^3 + 4\beta\beta_3 + 8\beta\beta_4}}\,\xi\right| \leq \pi \\ 0, \textit{otherwise} \end{cases}$$

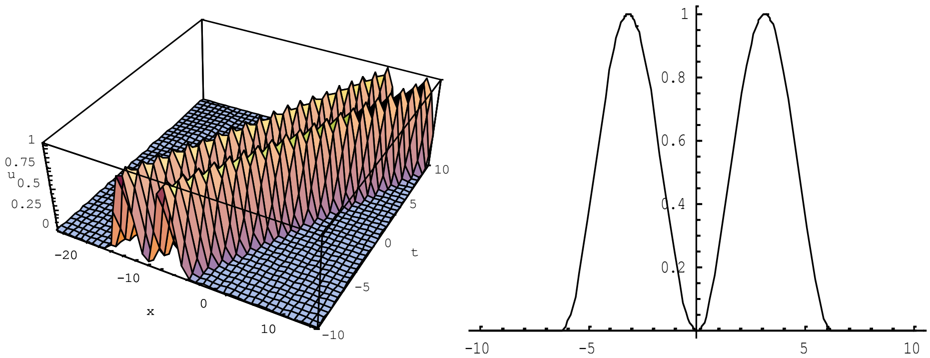


Fig.2 (a) Typical compacton like structure with two symmetry humps near its center which was entitled pair-compacton. The figure corresponds to configuration $\beta_1 = 1, \beta_2 = 2, \beta_3 = -2, \beta_4 = -1, n = 1$ and $k = -1$.

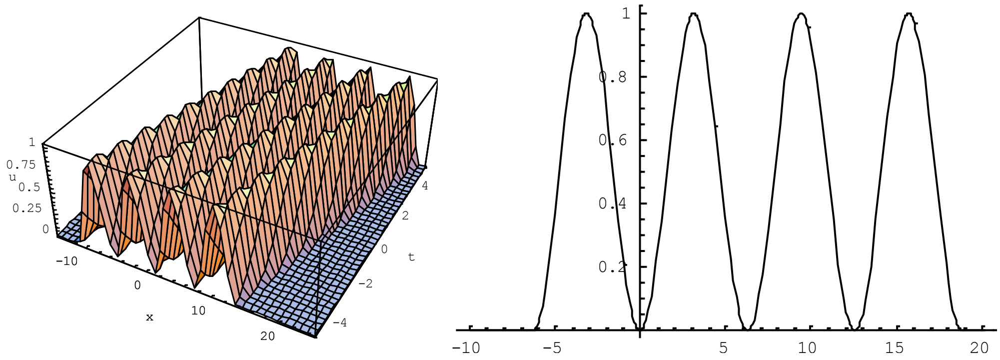


Fig.2 (b) Periodic pair compacton solution in the form of four humps with finite amplitude which represents a more generalized vacuum field for the expression obtained above.

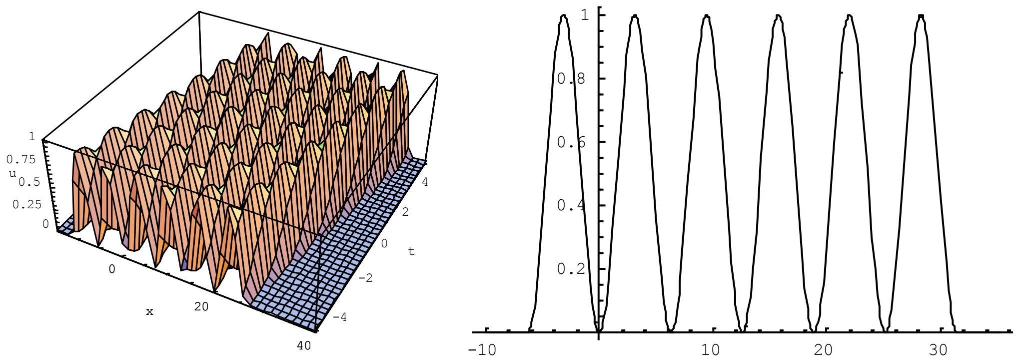


Fig.2(c) Structure similar to Fig.2 (b), indicating that this model can always be extended to multiple humps in infinite spatial-temporal regions.

Next we focus on the case $l = m \neq n = p$ which describes in detail a different compacton which for the generalized Camassa-Holm equation is then

$$u=\begin{cases}\left\{\dfrac{n^2\beta^3\beta_3+n^3\beta^3\beta_4}{(\beta+4)(\beta+3)(\beta+2)}\sin^{\beta}\left(\sqrt{\dfrac{\beta_2}{(2n+1)^2\beta^2}}\xi\right)\right\}^{\frac{\beta}{2}}, & \left|\sqrt{\dfrac{\beta_2}{(2n+1)^2\beta^2}}\xi\right|\le\pi \\ 0, otherwise\end{cases},$$

Utilizing the same approach, we can show below that the compacton solutions of Eq. (5) without any constrictions on its exponential sequence.

$$u=\begin{cases}\left\{\dfrac{-B^2(n\beta+\beta)(n\beta+\beta-1)(n\beta+\beta-2)-B^2n\beta^2\beta_3(n\beta-1)}{\beta_2(n\beta+\beta-2)}\sin^{\beta}\left(\sqrt{\dfrac{D-k}{D\beta_1(3\beta-2)}}\xi\right)\right\}^{-\frac{\beta}{2}}, & \left|\sqrt{\dfrac{D-k}{D\beta_1(3\beta-2)}}\xi\right|\le\pi \\ 0, otherwise\end{cases}.$$

Case 3: As a result of the application of the condition (8) to Eq. (5) we obtain the nonlinear algebraic equation,

$$\begin{aligned}&A^lB^3l\beta(l\beta-1)(l\beta-2)\cosh^{l\beta-3}B\xi-A^lB^3l\beta(l^2\beta^2-6l\beta+4)\cosh^{l\beta-1}B\xi\\&-AB^3D\beta\beta_1(\beta-1)(\beta-2)\cosh^{\beta-3}B\xi+AB\beta(k-D-6B^2D\beta_1\beta+4B^2D\beta_1+B^2D\beta_1\beta^2)\cosh^{\beta-1}B\xi\\&+A^mBm\beta\beta_2\cosh^{m\beta-1}B\xi+A^{n+1}nB^3\beta^2\beta_3(n\beta-1)\cosh^{n\beta+\beta-3}B\xi+A^{n+1}nB^3\beta^2\beta_3(2-n\beta)\cosh^{n\beta+\beta-1}B\xi\\&+A^{P+1}B^3p\beta\beta_4(p\beta-1)(p\beta-2)\cosh^{p\beta+\beta-3}B\xi-A^{p+1}B^3p\beta\beta_4(p^2\beta^2+4)\cosh^{p\beta+\beta-1}B\xi=0\end{aligned}$$

For condition $l-1=m-1=n=p$, the solitary pattern solution of the generalized Camassa-Holm equation by utilizing the same approach mentioned above can be constructed,

$$u=\left\{\frac{(k-D)(\beta^3-8\beta-8+16\beta_4)+D\beta_1\beta_2(\beta^3-4\beta^2+8-8\beta)}{(\beta+2)^2(\beta+1)\beta_2}\cosh^2(\sqrt{\frac{(\beta+2)\beta_2}{\beta^3-8\beta-8+16\beta_4}}\xi)\right\}^{\frac{1}{n}},$$

In what follows, we make the assumption that a suitable relation $l=m\neq n=p$ is defined for determining another possible system. Skipping the details, which is similar to the procedure of obtaining compactons, solitary pattern solution with a different type is presented in the below,

$$u=\left\{\frac{n^3\beta^2\beta_4+4n\beta_4-2n\beta\beta_3+n^2\beta^2\beta_3}{(2n+1)(2n\beta+\beta-1)(2n\beta+\beta-2)}\cosh^2(\sqrt{\frac{\beta_2}{l^2\beta^2-6l\beta+4}}\xi)\right\}^{\frac{1}{n}},$$

Additionally we have the usual solitary pattern solution while the parameter restriction vanishes that is conveniently written as

$$u=\left\{\frac{B^2(n\beta+\beta)(1-n\beta-\beta)(n\beta+\beta-2)-nB^2\beta^2\beta_3(n\beta-1)}{(n\beta+\beta-2)}\cosh^2(\sqrt{\frac{D-k}{D\beta_1\beta^2+4D\beta_1-6D\beta_1\beta}}\xi)\right\}^{-\frac{\beta}{2}}.$$

Case 4: As a result of the application of ansatz Eq. (9), one may recover the solitary pattern waves with different structures in comparison with the already derived ones which satisfy the condition $l-1=m-1=n=p$.

$$u=\left\{\frac{(k-D)(\beta^3-8\beta-8+16\beta_4)+D\beta_1\beta_2(\beta^3-4\beta^2+8-8\beta)}{(\beta+2)^2(\beta+1)\beta_2}\sinh^2(\sqrt{\frac{(\beta+2)\beta_2}{\beta^3-8\beta-8+16\beta_4}}\xi)\right\}^{\frac{1}{n}},$$

Let us analyze the relation $l=m\neq n=p$. It is readily observed that for this case we have a solitary pattern wave according to the definition (9).

$$u=\left\{\frac{n^3\beta^2\beta_4+4n\beta_4-2n\beta\beta_3+n^2\beta^2\beta_3}{(2n+1)(2n\beta+\beta-1)(2n\beta+\beta-2)}\cosh^2(\sqrt{\frac{\beta_2}{l^2\beta^2-6l\beta+4}}\xi)\right\}^{\frac{1}{n}},$$

Below we calculate the general solitary pattern solutions of generalized Camassa-Holm equation that are not imposed on any constrictions.

$$u=\left\{\frac{B^2(n\beta+\beta)(1-n\beta-\beta)(n\beta+\beta-2)-nB^2\beta^2\beta_3(n\beta-1)}{(n\beta+\beta-2)}\sinh^2(\sqrt{\frac{D-k}{D\beta_1\beta^2+4D\beta_1-6D\beta_1\beta}}\xi)\right\}^{-\frac{\beta}{2}}.$$

Remark: (ⅰ) Different parameter restrictions accompanied by the creation of kink structures occur when the boundary conditions in Fig.2 are changed (See Fig.3).

$$u=\begin{cases}\sqrt{\dfrac{(D-k)(27+4\beta_3+8\beta_4)-3D\beta_1\beta_2}{18\beta_2}}\sin\left(\sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi\right), & \left|\sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi\right|\le\dfrac{\pi}{2}\\ -\sqrt{\dfrac{(D-k)(27+4\beta_3+8\beta_4)-3D\beta_1\beta_2}{18\beta_2}}, & \sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi<-\dfrac{\pi}{2}\\ \sqrt{\dfrac{(D-k)(27+4\beta_3+8\beta_4)-3D\beta_1\beta_2}{18\beta_2}}, & \sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi>\dfrac{\pi}{2}\end{cases}$$

$$u=\begin{cases}\sqrt{\dfrac{(D-k)(27+4\beta_3+8\beta_4)-3D\beta_1\beta_2}{18\beta_2}}\sin\left(\sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi\right), & \dfrac{\pi}{2}\le\sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi\le\dfrac{3\pi}{2}\\ -\sqrt{\dfrac{(D-k)(27+4\beta_3+8\beta_4)-3D\beta_1\beta_2}{18\beta_2}}, & \sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi>\dfrac{3\pi}{2}\\ \sqrt{\dfrac{(D-k)(27+4\beta_3+8\beta_4)-3D\beta_1\beta_2}{18\beta_2}}, & \sqrt{\dfrac{3\beta_2}{27+4\beta_3+8\beta_4}}\xi<\dfrac{\pi}{2}\end{cases}$$

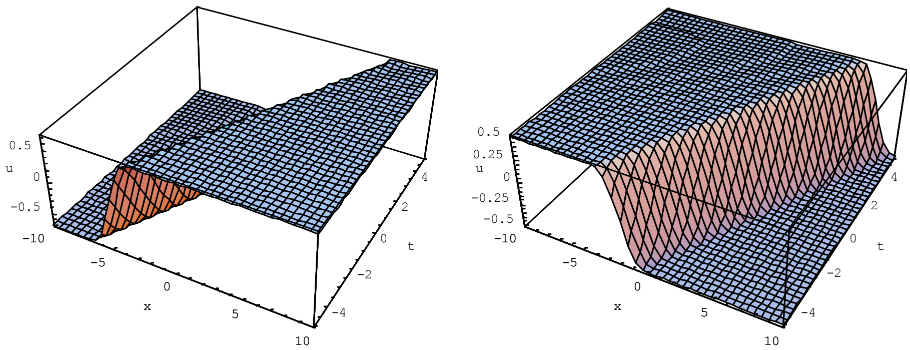


Fig.3 Simulations represent typical kink structures with infinite tails. Exactly at the conditions given above can kinks appear, determining the second state of the generalized system. These solutions correspond to the value $n=2,\beta=1,\beta_1=1,\beta_2=13,\beta_3=\beta_4=1,D=3,k=-3$.

Such structures are also obtained for the generalized Camassa-Holm system when we consider the constrictions of its parameters which are determined by $l=m\ne n=p$,

$$u=\begin{cases}\sqrt{\dfrac{\beta_2}{-B^2n(n+1)-B^2n\beta_3}}\sin\left(\sqrt{\dfrac{D-k}{D\beta_1}}\xi\right), & \left|\sqrt{\dfrac{D-k}{D\beta_1}}\xi\right|\le\dfrac{\pi}{2}\\ -\sqrt{\dfrac{\beta_2}{-B^2n(n+1)-B^2n\beta_3}}, & \sqrt{\dfrac{D-k}{D\beta_1}}\xi<-\dfrac{\pi}{2}\\ \sqrt{\dfrac{\beta_2}{-B^2n(n+1)-B^2n\beta_3}}, & \sqrt{\dfrac{D-k}{D\beta_1}}\xi>\dfrac{\pi}{2}\end{cases}\qquad u=\begin{cases}\sqrt{\dfrac{\beta_2}{-B^2n(n+1)-B^2n\beta_3}}\sin\left(\sqrt{\dfrac{D-k}{D\beta_1}}\xi\right), & \dfrac{\pi}{2}\le\sqrt{\dfrac{D-k}{D\beta_1}}\xi\le\dfrac{3\pi}{2}\\ \sqrt{\dfrac{\beta_2}{-B^2n(n+1)-B^2n\beta_3}}, & \sqrt{\dfrac{D-k}{D\beta_1}}\xi<\dfrac{\pi}{2}\\ -\sqrt{\dfrac{\beta_2}{-B^2n(n+1)-B^2n\beta_3}}, & \sqrt{\dfrac{D-k}{D\beta_1}}\xi>\dfrac{3\pi}{2}\end{cases}$$

Consequently, the special solutions under this constriction could exist for the C-H(5, 2, 2, 5) system.

$$u=\begin{cases}\sqrt{\dfrac{\beta_3+2\beta_4}{15}}\sin(\sqrt{\dfrac{\beta_2}{25}}\xi), \left|\sqrt{\dfrac{\beta_2}{25}}\xi\right|\le\dfrac{\pi}{2}\\ -\sqrt{\dfrac{\beta_3+2\beta_4}{15}}, \sqrt{\dfrac{\beta_2}{25}}\xi<-\dfrac{\pi}{2}\\ \sqrt{\dfrac{\beta_3+2\beta_4}{15}}, \sqrt{\dfrac{\beta_2}{25}}\xi>\dfrac{\pi}{2}\end{cases}\qquad u=\begin{cases}\sqrt{\dfrac{\beta_3+2\beta_4}{15}}\sin(\sqrt{\dfrac{\beta_2}{25}}\xi), \dfrac{\pi}{2}\le\sqrt{\dfrac{\beta_2}{25}}\xi\le\dfrac{3\pi}{2}\\ \sqrt{\dfrac{\beta_3+2\beta_4}{15}}, \sqrt{\dfrac{\beta_2}{25}}\xi<\dfrac{\pi}{2}\\ -\sqrt{\dfrac{\beta_3+2\beta_4}{15}}, \sqrt{\dfrac{\beta_2}{25}}\xi>\dfrac{3\pi}{2}\end{cases}$$

(ii) Recently, important progress has been made to obtain the explicit multiple soliton solutions of the generalized Camassa-Holm models. It is interesting to find that by using the following expressions can we observe the multiple solitons.

$$u=\left\{\frac{(D-k)[\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4]-D\beta_1\beta_2(\beta+2)(3\beta-2)}{\beta_2(\beta+2)^2(\beta+1)}\cos^2(\sqrt{\frac{\beta_2(\beta+2)}{\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4}}\xi)\right\}^{\frac{1}{n}}$$

$$\frac{(4N-1)\pi}{2}\le\sqrt{\frac{\beta_2(\beta+2)}{\beta^3(n+1)^3+4\beta\beta_3+8\beta\beta_4}}(x-Dt)\le\frac{(4N+1)\pi}{2},$$

By evaluating the periodic regions of the usual compactons [See Fig. 4(a)], one arrives at the following results. Multiple compactons can be constructed while the restrictions on x and t imply a constant N. Unlike the case shown in Fig. 1 and 4(a), multi-compacton not only show infinite tails, but also presents a finite trivial field between its waves [See Fig. 4(b) (c)]. All the waves among a multiple compacton travel with the same propagation value. Thus inner interactions may not be happened.

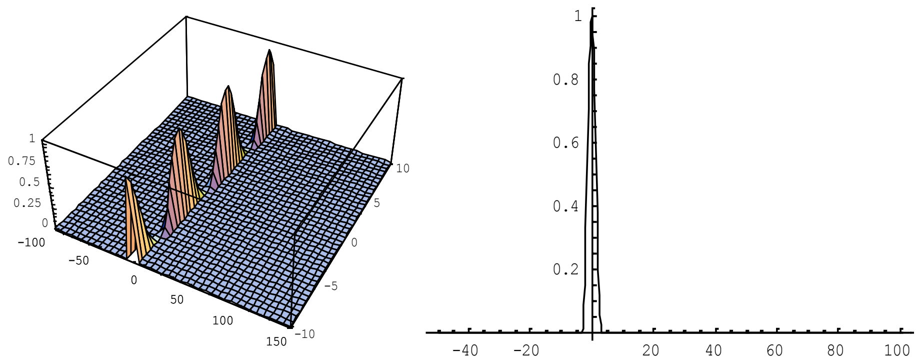


Fig4. (a) A general compacton with smaller amplitude in comparison with Fig.1. This figure represents $\beta_1=1,\beta_2=2,\beta_3=-2,\beta_4=-1,n=1,k=-3$. In this case, N=0.

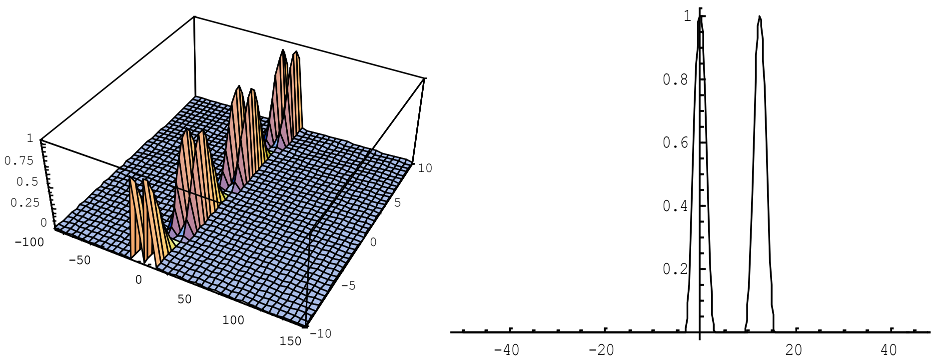


Fig4. (b) This is an example of multiple compacton that has two symmetry wave crests

with the parameter N=1.

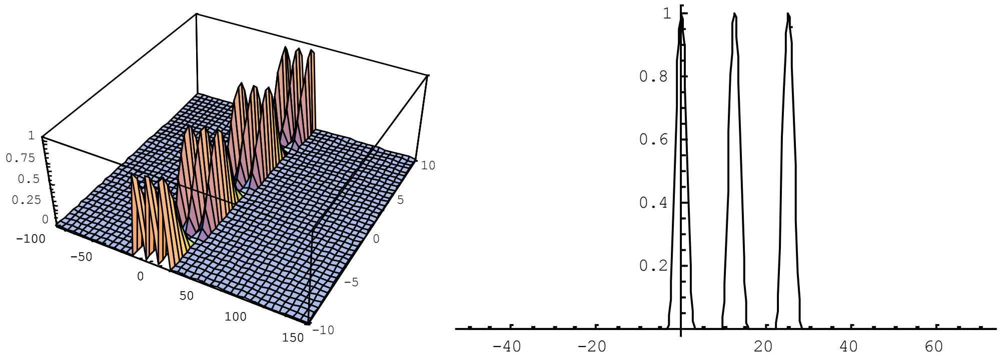


Fig4. (c) Three waves coexist in this traveling solutions for we introduced the variable N=2 to define a more general multiple soliton.

B: Case $k=0$

In this section we present typical examples of the numerical and simulation results for the case when parameter $k=0$ is satisfied. While omit the x-derivative terms, rich singular solitons will be then generated. The systems displayed in the following figures are in some manner having essentially the same characteristics. They all contain infinite wave crests while the evolution region is restricted in finite amplitude. The subsequent analysis is similar to that above.

Case 1: It is now necessary to determine the periodic wave soliton in the case $l \neq m \neq n = -p$ for the generalized Camassa-Holm equation by substituting the first ansatz from Eq. (6) into Eq. (5). Which can give rise to a compacton while the exponential parameter n<0 is satisfied.

$$u=\begin{cases}\{\dfrac{D(1-\beta)(\beta-2)}{l^3\beta^2+n^2\beta^2\beta_3}\sec^2(\sqrt{\dfrac{-A^n\beta_2(2n+1)}{l(l\beta-1)(l\beta-2)+n\beta\beta_3(n\beta-1)}}\xi)\}^{\frac{1}{n}}, & \left|\sqrt{\dfrac{-A^n\beta_2(2n+1)}{l(l\beta-1)(l\beta-2)+n\beta\beta_3(n\beta-1)}}\xi\right|\leq\dfrac{\pi}{2},\\ 0, otherwise\end{cases}$$

Additionally from the relation $l=m\neq n=p$, we observe another form of periodic traveling waves. Compactons may also occur while the condition n<0 holds. Different from the periodic wave expression we obtained above, we have that when the vacuum fields of the solution are restricted in $\sqrt{\dfrac{D}{D(\beta-1)(\beta-2)+D\beta_1(3\beta-2)+A^n n\beta\beta_3(n\beta-1)+A^n p\beta_4(p\beta-1)(p\beta-2)}}\xi\leq\dfrac{\pi}{2}$, thus the periodic wave structure can be determined by

$$u=\{\frac{n^2\beta^2\beta_3+p^3\beta^2\beta_4}{l(l\beta-1)(l\beta-2)}\sec^5\sqrt{\frac{D}{D(\beta-1)(\beta-2)+D\beta_1(3\beta-2)+A^n n\beta\beta_3(n\beta-1)+A^n p\beta_4(p\beta-1)(p\beta-2)}}\xi\}^{\frac{1}{n}},$$

and u=0, otherwise.

Case 2: For the ansatz of Eq. (7), it is clear that the assumption may produce a periodic blow-up (See Fig.5) in the form,

$$u=\begin{cases}\left\{\dfrac{D(1-\beta)(\beta-2)}{l^3\beta^2+n^2\beta^2\beta_3}\csc^2\left(\sqrt{\dfrac{-A^n\beta_2(2n+1)}{l(l\beta-1)(l\beta-2)+n\beta\beta_3(n\beta-1)}}\xi\right)\right\}^{\frac{1}{n}}, & \left|\sqrt{\dfrac{-A^n\beta_2(2n+1)}{l(l\beta-1)(l\beta-2)+n\beta\beta_3(n\beta-1)}}\xi\right|\le\pi, \\ 0, otherwise & \end{cases}$$

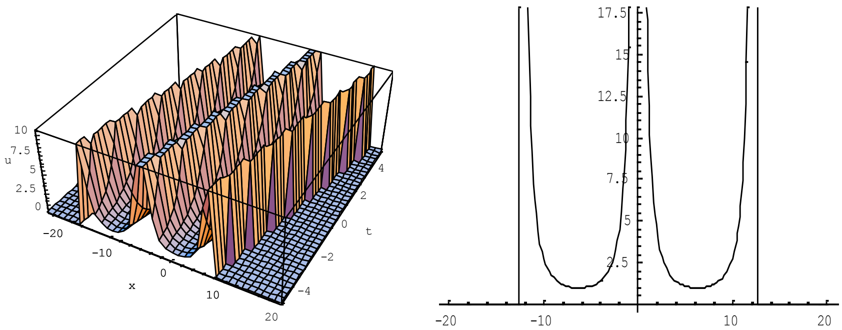


Fig.5 Curves describe periodic blow-up in the case when $l\neq m\neq n=-p$. They are defined in finite space sector. Outside this sector, the field vanishes. For the values $\beta=-2, n=1, p=-1, m=3, l=2, D=-2, \beta_3=-2$ and $\beta_2=-1$.

As we see from Fig. 5, this traveling wave contains three blow-up phenomena that depend on nonlinear excitations. Correspond to the solution, a soliton with the same character but with different shape may be found under the condition $l=m\neq n=p$ (See Fig. 6). While

$$\sqrt{\frac{D}{D(\beta-1)(\beta-2)+D\beta_1(3\beta-2)+A^n n\beta\beta_3(n\beta-1)+A^n p\beta_4(p\beta-1)(p\beta-2)}}\xi\le\pi,$$

$$u=\left\{\frac{n^2\beta^2\beta_3+p^3\beta^2\beta_4}{l(l\beta-1)(l\beta-2)}\csc^5\sqrt{\frac{D}{D(\beta-1)(\beta-2)+D\beta_1(3\beta-2)+A^n n\beta\beta_3(n\beta-1)+A^n p\beta_4(p\beta-1)(p\beta-2)}}\xi\right\}^{\frac{1}{n}},$$

and u=0, otherwise.

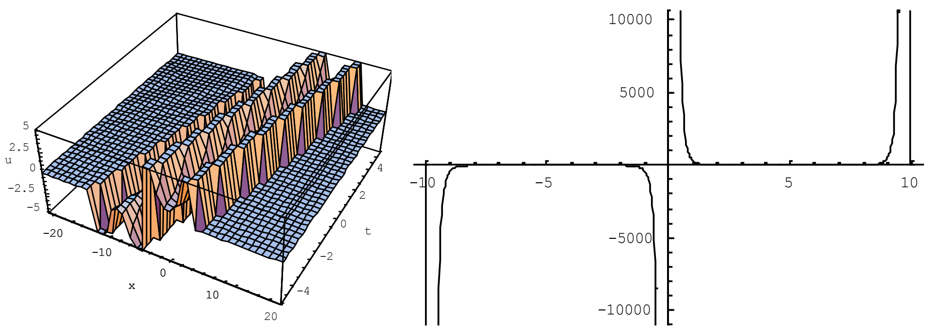


Fig.6 Figures similar to the above description that has four blow-up wave crests in terms of the parameter

satisfies $\beta=-5, n=p=-\dfrac{2}{5}, m=l=\dfrac{1}{5}, D=1, \beta_1=2, \beta_3=1$ and $\beta_4=\dfrac{7}{4}$.

Case 3: From ansatz 3, solitary wave solution is derived. Specifically by making use of the relation n<0 for this solution, solitary pattern solution u (See Fig. 7) takes the form,

$$u=\{\frac{-D\beta_1(\beta-1)(\beta-2)}{l^3\beta^2-6l^2\beta+4l+n\beta\beta_3(n\beta-2)}\cosh^2(\sqrt{\frac{-A^n m\beta_2}{l(l\beta-1)(l\beta-2)+n\beta\beta_3(n\beta-1)}}\xi)\}^{\frac{1}{n}}$$

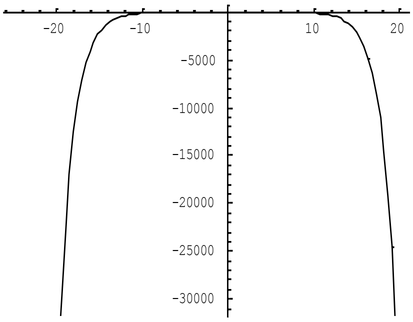


Fig.7 The solitary pattern soliton are strongly localized in space and they do not present any infinite tails. This soliton structure exists when the parameter concerns $\beta=-2, n=1, p=-1, m=3, l=2, D=4, \beta_3=1, \beta_2=2, \beta_1=2$.

For the case $l=m\neq n=p$, we obtained another solitary wave. The solution is

$$u=\{\frac{p\beta_4(p^2\beta^2+4)-n\beta\beta_3(2-n\beta)}{l(l\beta-1)(l\beta-2)}\operatorname{sech}\sqrt{\frac{m\beta_2}{l^3\beta^2-6l^2\beta+4l}}\xi\}^{\frac{1}{n}}.$$

Case 4: We analyze the generalized Camassa-Holm equation by considering two constraints: k=0 and $l\neq m\neq n=-p$. As ususl on putting ansatz 4 into Eq. (5), skipping the details, we find that blow up phenomenon (See Fig. 8) occurs.

$$u=\{\frac{-D\beta_1(\beta-1)(\beta-2)}{l^3\beta^2-6l^2\beta+4l+n\beta\beta_3(n\beta-2)}\operatorname{csch}^2(\sqrt{\frac{-A^n m\beta_2}{l(l\beta-1)(l\beta-2)+n\beta\beta_3(n\beta-1)}}\xi)\}^{\frac{1}{n}}$$

However, when the exponential constant of the solution obtained above holds n<0, a solitary pattern solution may also be defined.

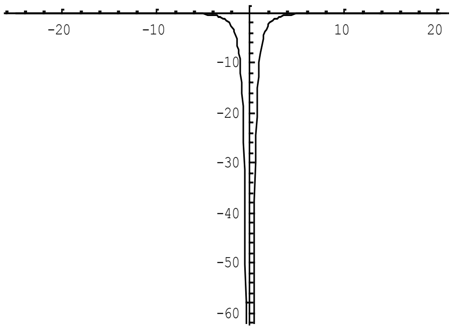


Fig.8 Plot showing the form of a typical blow up phenomenon with an infinite wave crest and a finite amplitude at fixed values $\beta=-2, n=1, p=-1, m=3, l=2, D=4, \beta_3=1, \beta_2=2$ and $\beta_1=2$.

Note that the exponential sequence in Eq. (3), which determines the strength of nonliearity and dispersion, is now in the different case $l = m \neq n = p$ in comparison with the typical blow up induced above. Thus a blow up $u = \{\frac{p\beta_4(p^2\beta^2+4) - n\beta\beta_3(2-n\beta)}{l(l\beta-1)(l\beta-2)} \operatorname{csc}h\sqrt{\frac{m\beta_2}{l^3\beta^2 - 6l^2\beta + 4l}}\xi\}^{\frac{1}{n}}$ which has two wave crests is found (See Fig.9). Comparing Fig. 8 with the simulation given in the following for blow up waves in the limited field, it is readily shown that the two branches of the latter one are nonsymmetrical within its finite amplitudes.

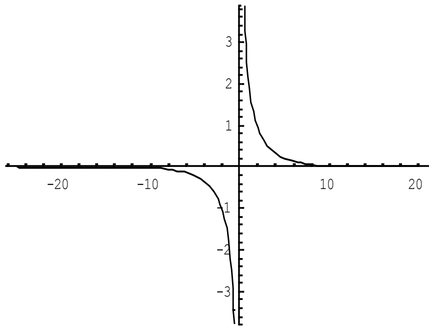


Fig.9 The shape of the blow up for the case when $l = m \neq n = p$, where we plot u. The parameters are $\beta = -1, n = p = -2, m = l = -3, \beta_2 = -1, \beta_4 = \frac{3}{8}$. In this case, $D = 1$.

Let us next suppose that $n < 0$. Then there is again a singular solitary pattern solution (See Fig. 10) given by the following equation $u = \{\frac{p\beta_4(p^2\beta^2+4) - n\beta\beta_3(2-n\beta)}{l(l\beta-1)(l\beta-2)} \sinh\sqrt{\frac{m\beta_2}{l^3\beta^2 - 6l^2\beta + 4l}}\xi\}^{\frac{1}{n}}$.

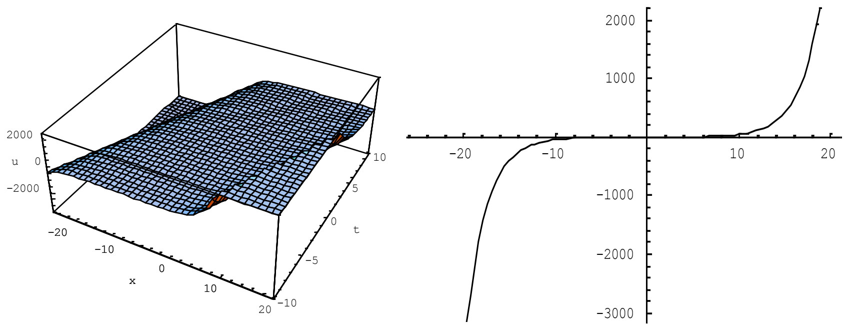


Fig.10 An example of a Singular solitary pattern solution and its plane graph.

3.Singular solitonic structures

As we mentioned in section II, in the case of $k \neq 0$, we have three different generalized Camassa-Holm equations. Solitonlike structures are available if the exponential sequence of the generalized Camassa-Holm model satisfy the relation: $l - 1 = m - 1 = n = p$, $l \neq m \neq n \neq p$ and $l = m \neq n = p$. For the sake of clarity, we now detailed analyze the special cases C-H (2,1,1,2), C- H(1,2,2,3) , and try to find singular solitons by combining two different forms of the ansatz methods.

The nonlinear equation C-H (2,1,1,2) is represented in the form,

$$u_t + ku_x + \beta_1 u_{xxt} + \beta_2 (u^2)_x + \beta_3 u_x (u^1)_{2x} + \beta_4 u(u^1)_{3x} + (u^2)_{3x} = 0$$

(10)

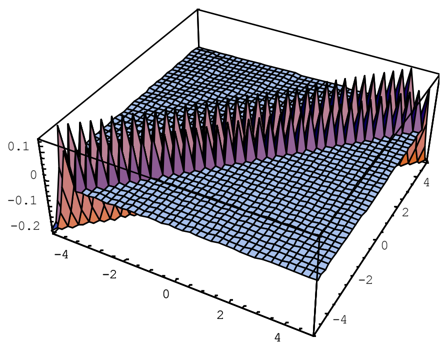
Considering the transformation given as,

$$U = \frac{(D-k)(8+\beta_3+2\beta_4)-2D\beta_1\beta_2}{6\beta_2}[a\cos^2(\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi)+b\sin^2(\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi)]$$

(11)

On substituting the combining ansatz(11) into the differential equation C-H (2,1,1,2), we find that when the relation a=b or a+b=1 are satisfied, expression (11) can be take into consideration. To find singular structures in singular structures in comparison with regular ones simulated before. Solitons are defined now only in a finite sector $a\cos^2(\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi)+b\sin^2(\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi)=0$. Outside this sector, the field vanishes. From here it is easy to find compactonlike structures if the parameters satisfy the restriction $\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi = \arccos\sqrt{\frac{b}{b-a}}$ (i). Thus an unusual solitary wave, named a pair compacton, was found [See Fig.11(a)]. We introduce this name to designate solitons that coexists two symmetry curves resemble cuspons but with finite amplitude from the simulation plot. Multi-pair compactons are also considered [See Fig.11(b)(c)]. When the condition a+b=1 are satisfied,

$$U=\begin{cases}\frac{(D-k)(8+\beta_3+2\beta_4)-2D\beta_1\beta_2}{6\beta_2}[a\cos^2(\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi)+b\sin^2(\sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi)], & \sqrt{\frac{\beta_2}{16+2\beta_3+4\beta_4}}\xi=\arccos\sqrt{\frac{b}{b-a}}\\ 0, otherwise\end{cases}$$



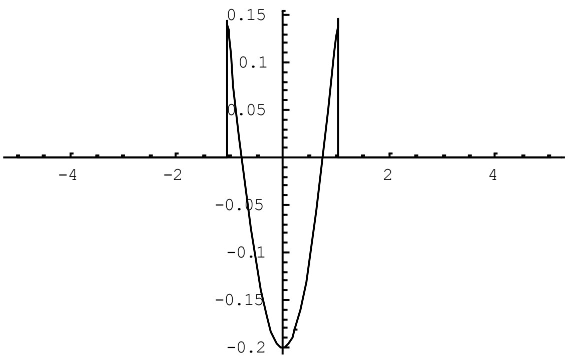


Fig.11 (a) Plot showing the anti-compactonlike structures that have a pair of curves at the wave crest resemble cuspons, which are symmetrical in the finity vacuum. We present this curve at fixed values $\beta_1 = 1, \beta_2 = 2, \beta_3 = -2, \beta_4 = -1, n = 1, k = -3, D = 1, a = 0.2$ and $b = -1.2$

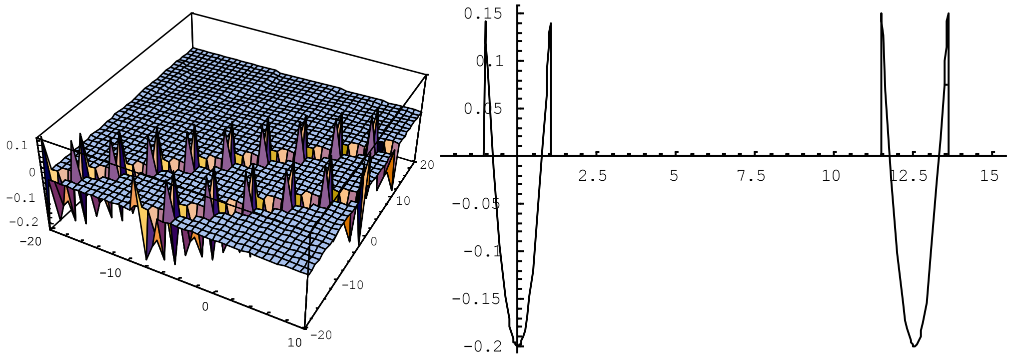


Fig.11 (b) An example of a multi-soliton that two singular structures analyzed above may coexist. Therefore, in this case, we introduced the name multiple pair compacton to define the shape of the compacton like waves.

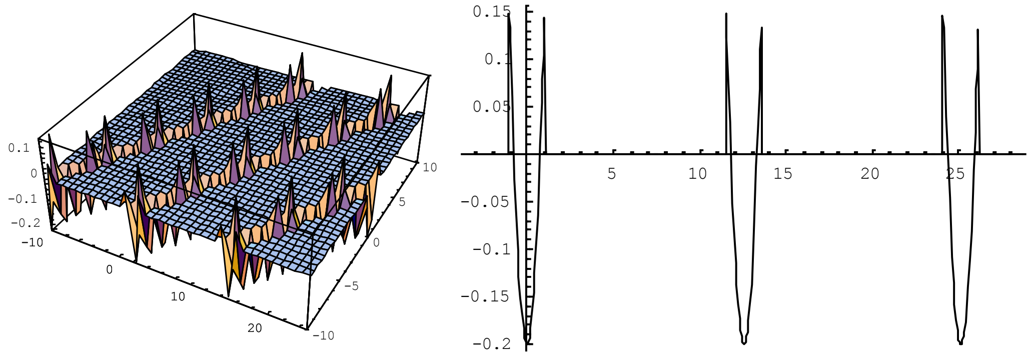


Fig.11(c) In contrast to the two wave system mentioned in (b), we obtained the three wave system which is the general version of multiple pair compacton model.

Next, we turn to the equation *C-H (1, 2, 2, 3),*

$$u_t + ku_x + \beta_1 u_{xxt} + \beta_2 (u^1)_x + \beta_3 u_x (u^2)_{2x} + \beta_4 u (u^2)_{3x} + (u^3)_{3x} = 0$$

(12)

In this case U= $$\frac{D\beta_1\beta_2}{(D-k)(6-2\beta_3)}[a\cos(\sqrt{\frac{D-k}{D\beta_1}}\xi) + b\sin(\sqrt{\frac{D-k}{D\beta_1}}\xi)]$$

(13)

On substituting solution (13) into Eq. (12), one may get that if $a^2 + b^2 = 1$ expression (13) can be the soliton solutions of the C-H (1,2,2,3) system.To obtain a necessary condition for the existence of the compactons, we apply the constraints $a\cos(\sqrt{\frac{D-k}{D\beta_1}}\xi) + b\sin(\sqrt{\frac{D-k}{D\beta_1}}\xi) = 0$, then it is easy to calculate that a general condition on dependent parameter ξ following from this relation

$\sqrt{\frac{D-k}{D\beta_1}}\xi = \arccos\sqrt{\frac{a+b}{a-b}}$ (j).These solutions are strongly localized in space. For their amplitudes are finite, they do not present any infinite tails. We use the nonsymmetry compacton to designate the unusual solution that the two soliton branches impose different properties [See Fig. 12(a)]. When solution (18) satisfied $a^2 + b^2 = 1$, one obtains,

$$U = \begin{cases} \frac{D\beta_1\beta_2}{(D-k)(6-2\beta_3)}[a\cos(\sqrt{\frac{D-k}{D\beta_1}}\xi) + b\sin(\sqrt{\frac{D-k}{D\beta_1}}\xi)], \sqrt{\frac{D-k}{D\beta_1}}\xi = \arccos\sqrt{\frac{a+b}{a-b}} \\ 0, otherwise \end{cases}$$

Multi-solitons similar to that simulated in (a) can also found [See Fig.12 (b)(c)]. Both out soliton structures are restricted in their vacuum spaces.

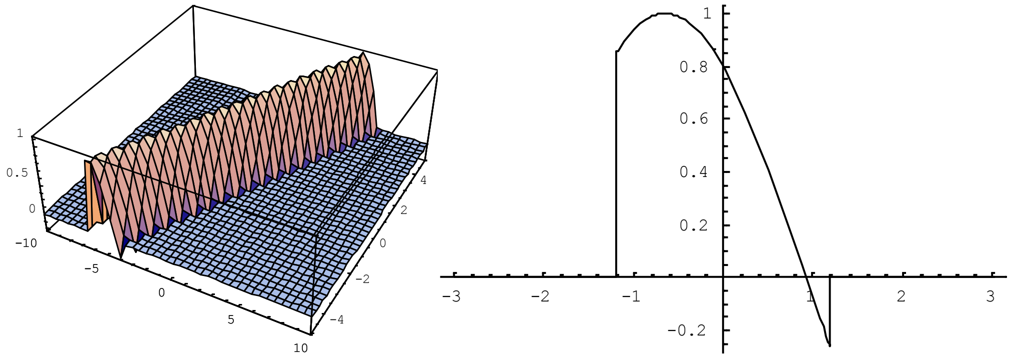


Fig.12 (a) The shape of a non-symmetrical compacton and its plane graph produced by solution U, the amplitude of which is finite. The parameters are

$\beta_1 = 1, \beta_2 = 2, \beta_3 = -2, \beta_4 = -1, n = 1, k = -3, D = 1, a = 0.2$ and $b = -1.2$

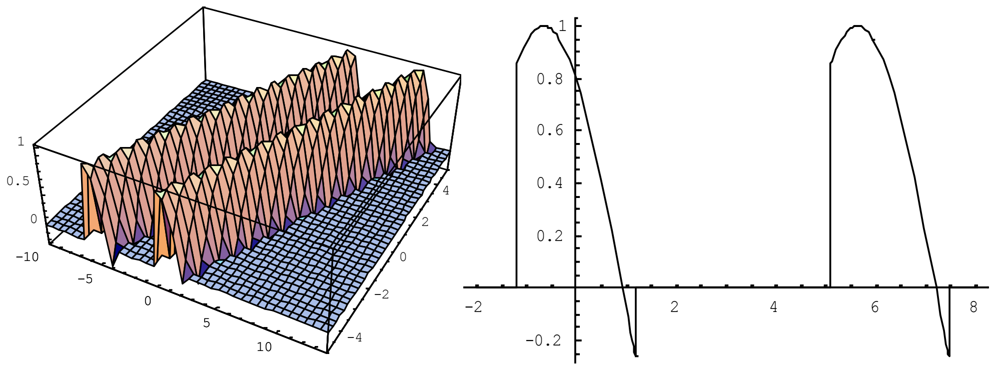


Fig.12 (b) This is an example of a multi-soliton that two nonsymmetry compactons coexist in one structure, we can show from this plot that the curve resembles peaks in the bottom of compacton like waves need infinite energy to be existed above there finite fields.

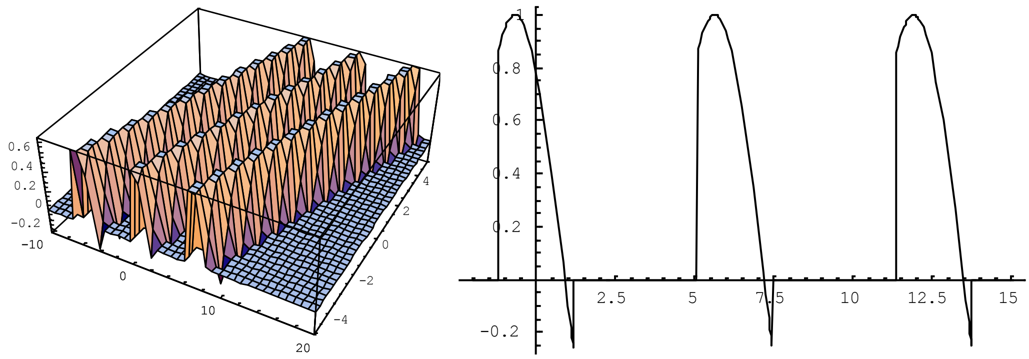


Fig.12(c) Structures that resembles the form of (b), and has three compactons, each of which contains peaklike curves. This is because its energy value diverges.

Remark: Note that solitons U obtained in this section are regular solitary waves before we provide the relations (i) and (j). When applied these relations, the very structures that makes us surprised are the singular wave crests. The two branches of each solitary wave do not show infinite regions for dependent variable ξ .

4.Many dimensions Camassa-Holm model

Considering (2+1) temporal-spatial dimensions generalized Camassa-Holm equations proposed as,

$$u_t + ku_x + ku_y + \beta_1 u_{xxt} + \beta_5 u_{yyt} + \beta_2 (u^m)_x + \beta_6 (u^m)_y + \beta_3 u_x (u^n)_{2x} + \beta_7 u_y (u^n)_{2y} + \beta_4 u(u^p)_{3x}$$
$$+ \beta_8 u(u^p)_{3y} + (u^l)_{3x} + (u^l)_{3y} = 0$$

(14)

By using the transformation $u(x, y, t) = u(\xi), \xi = x + y - \lambda t$, equation (14) can be accordingly transformed into the ordinary differential form,

$$-Du_\xi + ku_\xi + ku_\xi - \beta_1 Du_{3\xi} - \beta_5 Du_{3\xi} + \beta_2 (u^m)_\xi + \beta_6 (u^m)_\xi + \beta_3 u_\xi (u^n)_{2\xi} + \beta_7 u_\xi (u^n)_{2\xi}$$
$$+ \beta_4 u(u^p)_{3\xi} + \beta_8 u(u^p)_{3\xi} + (u^l)_{3\xi} + (u^l)_{3\xi} = 0$$

(15)

Eq. (15) will be simplified into

$$(2k - D)u_\xi - (\beta_1 + \beta_5)Du_{3\xi} + (\beta_2 + \beta_6)(u^m)_\xi + (\beta_3 + \beta_7)u_\xi (u^n)_{2\xi}$$
$$+ (\beta_4 + \beta_8)u(u^p)_{3\xi} + 2(u^l)_{3\xi} = 0$$

(16)

In contrast to Eq.(5) considered in section II, which gives rise to the multiple solitons, kinks, blow-up phenomena, compactons and regular periodic traveling waves, we've found that the relation between nonlinearity and linear dispersion dose not change. Since regular and special solitons exists if nonlinearity and linear dispersion transactions in the C-H model. Thus, the corresponding analysis is similar to that above, and leading to results for the (2+1) temporal-spatial dimensional model that are qualitatively similar to the simulations presented above.

(n+1) temporal-spatial dimensional cases are taking into regard for the generalized Camassa-Holm model. We arrive at a dispersion equation,

$$u_t + k\sum_{i=1}^{n} u_{x_i} + \sum_{i=1}^{n}\beta_i u_{x_i x_i t} + \sum_{i=1}^{n}\beta_i (u^m)_{x_i} + \sum_{i=1}^{n}\beta_i u_{x_i}(u^n)_{2x_i} + \sum_{i=1}^{n}\beta_i u(u^p)_{3x_i} + \sum_{i=1}^{n}(u^l)_{3x_i} = 0 \qquad (17)$$

On putting the transformation $u(x_i,t) = u(\xi_i), \xi_i = \sum_{i=1}^{n} x_i - \lambda t$ into equation (17), we obtain an ordinary differential system,

$$-Du_\xi + k\sum_{i=1}^{n} u_{\xi_i} - D\sum_{i=1}^{n}\beta_i u_{3\xi_i} + \sum_{i=1}^{n}\beta_i (u^m)_{\xi_i} + \sum_{i=1}^{n}\beta_i u_{\xi_i}(u^n)_{2\xi_i} + \sum_{i=1}^{n}\beta_i u(u^p)_{3\xi_i} + \sum_{i=1}^{n}(u^l)_{3\xi_i} = 0$$

Again we notice that the relative coefficients in front of the ξ-derivative terms change, which only accounts for the amplitude and wave velocity of solitonic structures. We can give detailed analysis of these features, however, it is beyond the scope of this paper. Note that in the (n+1) temporal-spatial dimensional cases, we can easily compute the value correspond to the wave structure. This value is available in Table 1 for comparison with the other two cases we discussed above. Even if the constant B verified, it is clear that this parameter may produce a weak effect in solitonic structure.

Tab1e 1 Cases of parameter value (in the case of $l-1 = m-1 = n = p$ which is similar to the other parameter relations)

Dimensions	ξ	B^2
(1+1) dimensional	$\xi = x - Dt$	$\dfrac{\beta_2(\beta+2)}{\beta^3(n+1)^3 + 4\beta\beta_3 + 8\beta\beta_4}$
(2+1) dimensional	$\xi = x + y - Dt$	$\dfrac{(\beta_2+\beta_6)(\beta+2)}{\beta^3(n+1)^3 + 4\beta(\beta_3+\beta_7) + 8\beta(\beta_4+\beta_8)}$
……	……	……
(n+1) dimensional	$\xi = \sum_{i=1}^{n} x_i - Dt$	$\dfrac{\sum_{i=1}^{n}\beta_i(\beta+2)}{\beta^3(n+1)^3 + 4\beta\sum_{i=1}^{n}\beta_i + 8\beta\sum_{i=1}^{n}\beta_i}$

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