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ANTI S-FUZZY NORMAL SUBHEMIRINGS AND LOWER LEVEL SUBSETS OF A HEMIRING

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Abstract: In this paper, we made an attempt to study the algebraic nature of an anti S-fuzzy normal subhemiring and lower level subset of a hemiring.

Keywords: fuzzy set, anti S-fuzzy subhemiring, anti S-fuzzy normal subhemiring, lower level subset.

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0. Introduction

There are many concepts of universal algebras generalizing an associative ring (R ; + ; .). Some of them in particular, nearrings and several kinds of semirings have been prove very useful. Semirings (called also halfrings) are algebras (R ; + ; .) share the same properties as a ring except that (R ; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have

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an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[14], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h- ideals in hemirings was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[7]. In this paper, we introduce the some Theorems in anti S-fuzzy normal subhemiring and lower level subset of a hemiring.

1. Preliminaries

1.1 Definition: A S-norm is a binary operation S: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) 0 S x = x, 1 S x = 1 (boundary condition)

(ii) x S y = y S x (commutativity)

(iii) x S (y S z) = (x S y) S z (associativity)

(iv) if $x \leq y$ and $w \leq z$, then $x \text{ S } w \leq y \text{ S } z$ (monotonicity).

1.2 Definition: Let X be a non-empty set. A **fuzzy subset** A of X is a function $A : X \rightarrow [0, 1]$.

1.3 Definition: Let (R , + , .) be a hemiring. A fuzzy subset A of R is said to be an anti S-fuzzy subhemiring(anti fuzzy subhemiring with respect to S-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \leq S(\mu_A(x), \mu_A(y))$,

(ii) $\mu_A(xy) \leq S(\mu_A(x), \mu_A(y))$, for all x and y in R.

1.4 Definition: Let A and B be fuzzy subsets of sets G and H, respectively. Anti-product of A and B, denoted by A×B, is defined as $A\times B = \{\langle (x, y), \mu_{A\times B}(x,y) \rangle /$ for all x in G and y in H $\}$, where $\mu_{A\times B}(x, y) = \max\{ \mu_A(x), \mu_B(y) \}$.

1.5 Definition: Let A be a fuzzy subset in a set S, the anti-strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $\mu_V(x, y) = \max\{ \mu_A(x),$

$\mu_A(y)$ }, for all x and y in S.

1.6 Definition: Let (R, +, .) and (R^I, +, .) be any two hemirings. Let f : R → R^I be any function and A be an anti S-fuzzy subhemiring in R, V be an anti S-fuzzy subhemiring in f (R) = R^I, defined by $\mu_V(y) = \inf_{x \in f^{-1}(y)} \mu_A(x)$, for all x in R and y in R^I. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let (R , + , .) be a hemiring. An anti S-fuzzy subhemiring A of R is said to be an anti S-fuzzy normal subhemiring (ASFNSHR) of R if $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

1.8 Definition: Let A be a fuzzy subset of X. For α in [0, 1], the lower level subset of A is the set $A_\alpha = \{ x \in X : \mu_A(x) \leq \alpha \}$.

2. SOME PROPERTIES:

2.1 Theorem[11]: Union of any two(a family) of anti S-fuzzy subhemirings of a hemiring R is an anti S-fuzzy subhemiring of R.

2.2 Theorem[11]: If A and B are any two anti S-fuzzy subhemirings of the hemirings R_1 and R_2 respectively, then anti-product A×B is an anti S-fuzzy subhemiring of $R_1 \times R_2$.

2.3 Theorem[11]: Let A be a fuzzy subset of a hemiring R and V be the anti-strongest fuzzy relation of R. Then A is an anti S-fuzzy subhemiring of R if and only if V is an anti S-fuzzy subhemiring of R×R.

2.4 Theorem[11]: Let R and R^I be any two hemirings. The homomorphic image (preimage) of an anti S-fuzzy subhemiring of R is an anti S-fuzzy subhemiring of R^I.

2.5 Theorem[11]: Let R and R^I be any two hemirings. The anti-homomorphic image (preimage) of an anti S-fuzzy subhemiring of R is an anti S-fuzzy subhemiring of R^I.

2.6 Theorem: Let (R, +, .) be a hemiring. If any two anti S-fuzzy normal subhemirings of R, then their union is an anti S-fuzzy normal subhemiring of R.

Proof: Let x and y∈R. Let A ={ ⟨ x, $\mu_A(x)$ ⟩/ x∈R } and B = { ⟨ x, $\mu_B(x)$ ⟩ /

$x \in R$ } be anti S-fuzzy normal subhemirings of a hemiring R. Let $C = A \cup B$ and $C = \{ \langle x, \mu_C(x) \rangle / x \in R \}$, where $\mu_C(x) = \max \{\mu_A(x), \mu_B(x)\}$. Then, Clearly C is an anti S-fuzzy subhemiring of a hemiring R, since A and B are two anti S-fuzzy subhemirings of the hemiring R. Then

$\mu_C(xy) = \max \{ \mu_A(xy), \mu_B(xy) \} = \max\{\mu_A(yx), \mu_B(yx)\} = \mu_C(yx)$, for all x and y in R. Hence $A \cup B$ is an anti S-fuzzy normal subhemiring of the hemiring R.

2.7 Theorem: Let (R, +, .) be a hemiring. The union of a family of anti S-fuzzy normal subhemirings of R is an anti S-fuzzy normal subhemiring of R.

Proof: It is trivial.

2.8 Theorem: Let A and B be anti S-fuzzy subhemirings of the hemirings G and H, respectively. If A and B are anti S-fuzzy normal subhemirings, then $A\times B$ is an anti S-fuzzy normal subhemiring of $G\times H$.

Proof: Let A and B be anti S-fuzzy normal subhemirings of the hemirings G and H respectively. Clearly $A\times B$ is an anti S-fuzzy subhemiring of $G\times H$. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1,y_1) and (x_2,y_2) are in $G\times H$. Now, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \max \{\mu_A(x_1x_2), \mu_B(y_1y_2) \}= \max \{\mu_A(x_2x_1), \mu_B(y_2y_1) \}= \mu_{A\times B} (x_2x_1, y_2y_1) = \mu_{A\times B}[(x_2, y_2)(x_1, y_1)]$. Hence $A\times B$ is an anti S-fuzzy normal subhemiring of $G\times H$.

2.9 Theorem: Let A be a fuzzy subset in a hemiring R and V be the anti-strongest fuzzy relation on R. Then A is an anti S-fuzzy normal subhemiring of R if and only if V is an anti S-fuzzy normal subhemiring of $R\times R$.

Proof: It is trivial.

2.10 Theorem: Let (R, +, .) and (R^{I}, +, .) be any two hemirings. The homomorphic image of an anti S-fuzzy normal subhemiring of R is an anti S-fuzzy normal subhemiring of R^{I}.

Proof: Let $f : R \rightarrow R^{I}$ be a homomorphism. Then, $f(x+y)= f(x)+f(y)$, $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where A is an anti S-fuzzy normal subhemiring of a hemiring R. Now, for $f(x)$, $f(y)$ in R^{I}, clearly V is an anti S-fuzzy subhemiring of a hemiring R^{I}, since A is an anti S-fuzzy subhemiring of a hemiring R. Now, $\mu_v(f(x)f(y)) \leq \mu_A(xy) = \mu_A(yx) \geq \mu_v(f(yx)) = \mu_v(f(y)f(x))$, which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y) f(x))$, for all f(x) and f(y) in R^I. Hence V is an anti S-fuzzy normal subhemiring of a hemiring R^I.

2.11 Theorem: Let (R, +, .) and (R^I, +, .) be any two hemirings. The homomorphic preimage of an anti S-fuzzy normal subhemiring of R^I is an anti S-fuzzy normal subhemiring of R.

Proof: Let V = f(A), where V is an anti S-fuzzy normal subhemiring of a hemiring R^I. Let x and y in R. Then, clearly A is an anti S-fuzzy subhemiring of a hemiring R, since V is an anti S-fuzzy subhemiring of a hemiring R^I. Now,

$\mu_A(xy) = \mu_v(f(x)f(y)) = \mu_v(f(y)f(x)) = \mu_v(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence A is an anti S-fuzzy normal subhemiring of a hemiring R.

2.12 Theorem: Let (R, +, .) and (R^I, +, .) be any two hemirings. The anti-homomorphic image of an anti S-fuzzy normal subhemiring of R is an anti S-fuzzy normal subhemiring of R^I.

Proof: Let $f : R \rightarrow R^I$ be an anti-homomorphism. Then, f(x+y) = f(y)+f(x), f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti S-fuzzy normal subhemiring of a hemiring R. Now, for f(x) and f(y) in R^I, clearly V is an anti S-fuzzy subhemiring of a hemiring R^I, since A is an anti S-fuzzy subhemiring of a hemiring R. Now, $\mu_v(f(x)f(y)) \leq \mu_A(yx) = \mu_A(xy) \geq \mu_v(f(xy)) = \mu_v(f(y)f(x))$, which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y)f(x))$, for all f(x) and f(y) in R^I. Hence V is an anti S-fuzzy normal subhemiring of a hemiring R^I.

2.13 Theorem: Let (R, +, .) and (R^I, +, .) be any two hemirings. The anti-homomorphic preimage of an anti S-fuzzy normal subhemiring of R^I is an anti S-fuzzy normal subhemiring of R.

Proof: Let V = f(A), where V is an anti S-fuzzy normal subhemiring of a hemiring R^I. Let x and y in R, then, clearly A is an anti S-fuzzy subhemiring of a hemiring R, since V is an anti S-fuzzy subhemiring of a hemiring R^I. Now,

$\mu_A(xy) = \mu_v(f(y)f(x)) = \mu_v(f(x)f(y)) = \mu_v(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence A is an anti S-fuzzy normal subhemiring of a hemiring R.

In the following Theorem ◦ is the composition operation of functions:

2.14 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. If A is an anti S-fuzzy normal subhemiring of the hemiring H, then A◦f is an anti S-fuzzy normal subhemiring of the hemiring R.

Proof: Let x and y in R. Then we have, clearly A◦f is an anti S-fuzzy subhemiring of a hemiring R. Now, $(\mu_A \circ f)(xy) = \mu_A(f(x)f(y)) = \mu_A(f(y)f(x)) = \mu_A(f(yx)) = (\mu_A \circ f)(yx)$, which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$, for all x and y in R. Hence A◦f is an anti S-fuzzy normal subhemiring of a hemiring R.

2.15 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. If A is an anti S-fuzzy normal subhemiring of the hemiring H, then A◦f is an anti S-fuzzy normal subhemiring of the hemiring R.

Proof: Let x and y in R. Then we have, clearly A◦f is an anti S-fuzzy subhemiring of the hemiring R. Now, $(\mu_A \circ f)(xy) = \mu_A(f(y)f(x)) = \mu_A(f(x)f(y)) = \mu_A(f(yx)) = (\mu_A \circ f)(yx)$, which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$, for all x and y in R. Hence A◦f is an anti S-fuzzy normal subhemiring of the hemiring R.

2.16 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring R. Then for α in [0, 1] such that $\mu_A(0) \leq \alpha$, A_α is a lower level subhemiring of R.

Proof: For all x and y in A_α. Now, $\mu_A(x+y) \leq S(\mu_A(x), \mu_A(y)) \leq \alpha$, which implies that $\mu_A(x+y) \leq \alpha$. And, $\mu_A(xy) \leq S(\mu_A(x), \mu_A(y)) \leq \alpha$, which implies that $\mu_A(xy) \leq \alpha$. Hence A_α is a lower level subhemiring of a hemiring R.

2.17 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring R. Then two lower level subhemiring $A_{\alpha 1}$, $A_{\alpha 2}$ and α_1, α_2 in [0, 1] such that $\mu_A(0) \leq \alpha_1$, $\mu_A(0) \leq \alpha_2$ with $\alpha_1 < \alpha_2$ of A are equal if and only if there is no x in R such that $\alpha_2 > \mu_A(x) > \alpha_1$.

Proof: Assume that $A_{\alpha 1} = A_{\alpha 2}$. Suppose there exists x in R such that $\alpha_2 > \mu_A(x) > \alpha_1$. Then $A_{\alpha 1} \subseteq A_{\alpha 2}$ implies x belongs to $A_{\alpha 2}$, but not in $A_{\alpha 1}$. This is contradiction to $A_{\alpha 1} = A_{\alpha 2}$. Therefore there is no $x \in R$ such that $\alpha_2 > \mu_A(x) > \alpha_1$. Conversely if there is no $x \in R$ such that $\alpha_2 > \mu_A(x) > \alpha_1$. Then $A_{\alpha 1} = A_{\alpha 2}$.

2.18 Theorem: Let R be a hemiring and A be a fuzzy subset of R such that A_α be a subhemiring of R. If α in [0, 1], then A is an anti S-fuzzy subhemiring of R.

Proof: Let x and y in R and $\mu_A(x) = \alpha_1$ and $\mu_A(y) = \alpha_2$. If $\alpha_1 < \alpha_2$, then x, y $\in A_{\alpha 2}$, $\mu_A(x+y) \leq \alpha_2 = \max\{\mu_A(x), \mu_A(y)\} \leq S(\mu_A(x), \mu_A(y))$, which implies that $\mu_A(x+y) \leq S(\mu_A(x), \mu_A(y))$, for all x and y in R and $\mu_A(xy) \leq \alpha_2 = \max\{\mu_A(x), \mu_A(y)\} \leq S(\mu_A(x), \mu_A(y))$, which implies that $\mu_A(xy) \leq S(\mu_A(x), \mu_A(y))$, for all x and y in R. If $\alpha_1 > \alpha_2$, then x and y in $A_{\alpha 1}$, $\mu_A(x+y) \leq \alpha_1 = \max\{\mu_A(y), \mu_A(x)\} \leq S(\mu_A(y), \mu_A(x))$, which implies that $\mu_A(x+y) \leq S(\mu_A(x), \mu_A(y))$, for all x and y in R and $\mu_A(xy) \leq \alpha_2 = \max\{\mu_A(y), \mu_A(x)\} \leq S(\mu_A(y), \mu_A(x))$, which implies that $\mu_A(xy) \leq S(\mu_A(x), \mu_A(y))$, for all x and y in R. Hence A is an anti S-fuzzy subhemiring of the hemiring R.

2.19 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring R. If any two lower level subhemirings of A belongs to R, then their intersection is also lower level subhemiring of A in R.

Proof: Let $\alpha_1, \alpha_2 \in [0, 1]$. If $\alpha_1 < \mu_A(x) < \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a lower level subhemiring of A. If $\alpha_1 > \mu_A(x) > \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a lower level subhemiring of A. If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$. Hence intersection of any two lower level subhemirings is also a lower level subhemiring of A.

2.20 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring R. If $\alpha_i \in [0, 1]$ and $A_{\alpha i}$, $i \in I$ is a collection of lower level subhemirings of A, then their intersection is also a lower level subhemiring of A.

Proof: It is trivial.

2.21 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring R. If any two lower level subhemirings of A belongs to R, then their union is also a lower level subhemiring of A in R.

Proof: It is trivial.

2.22 Theorem: Let A be an anti S-fuzzy subhemiring of a hemiring R. If $\alpha_i \in [0, 1]$ and $A_{\alpha i}$, $i \in I$ is a collection of lower level subhemirings of A, then their union is also a lower level subhemiring of A.

Proof: It is trivial.

2.23 Theorem: The homomorphic image of a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R is a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R^l.

Proof: Let $f : R \to R^l$ be a homomorphism. Then f(x+y)=f(x)+f(y), f(xy)=f(x)f(y), for all x and y in R. Let V = f(A), where A is an anti S-fuzzy subhemiring of a hemiring R. Clearly V is an anti S-fuzzy subhemiring of a hemiring R^l. Let x and y in R, implies f(x) and f(y) in R^l. Let A_α is a lower level subhemiring of A. Now, $\mu_V(f(x)) \leq \mu_A(x) \leq \alpha$, which implies that $\mu_V(f(x)) \leq \alpha$ and $\mu_V(f(y)) \leq \mu_A(y) \leq \alpha$, which implies that $\mu_V(f(y)) \leq \alpha$ and $\mu_V(f(x)+f(y)) \leq \mu_A(x+y) \leq \alpha$, which implies that $\mu_V(f(x)+f(y)) \leq \alpha$. Also, $\mu_V(f(x)f(y)) \leq \mu_A(xy) \leq \alpha$, which implies that $\mu_V(f(x)f(y)) \leq \alpha$. Hence $f(A_\alpha)$ is a lower level subhemiring of an anti S-fuzzy subhemiring V of a hemiring R^l.

2.24 Theorem: The homomorphic pre-image of a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R^l is a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R.

Proof: Let V = f(A), where V is an anti S-fuzzy subhemiring of a hemiring R^l. Clearly A is an anti S-fuzzy subhemiring of a hemiring R. Let f(x) and f(y) in R^l, implies x and y in R. Let $f(A_\alpha)$ is a lower level subhemiring of V. Now,

$\mu_A(x) = \mu_V(f(x)) \leq \alpha$, implies that $\mu_A(x) \leq \alpha$; $\mu_A(y) = \mu_V(f(y)) \leq \alpha$, implies that $\mu_A(y) \leq \alpha$ and $\mu_A(x+y) = \mu_V(f(x)+f(y)) \leq \alpha$, which implies that $\mu_A(x+y) \leq \alpha$. Also, $\mu_A(xy) = \mu_V(f(x)f(y)) \leq \alpha$, which implies that $\mu_A(xy) \leq \alpha$. Hence, A_α is a lower level subhemiring of an anti S-fuzzy subhemiring A of R.

2.25 Theorem: The anti-homomorphic image of a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R is a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R^l.

Proof: Let $f : R \to R^l$ be an anti-homomorphism. Then f(x + y) = f(y) + f(x), f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti S-fuzzy subhemiring of R. Clearly V is an anti S-fuzzy subhemiring of R^l. Let x and y in R, implies f(x) and f(y) in R^l. Let A_α is a lower level subhemiring of A. Now,

$\mu_V(f(x)) \leq \mu_A(x) \leq \alpha$, which implies that $\mu_V(f(x)) \leq \alpha$; $\mu_V(f(y)) \leq \mu_A(y) \leq \alpha$, which implies that $\mu_V(f(y)) \leq \alpha$. Now, $\mu_V(f(x)+f(y)) \leq \mu_A(y+x) \leq \alpha$, which implies that, $\mu_V(f(x)+f(y)) \leq \alpha$. Also, $\mu_V(f(x)f(y)) \leq \mu_A(yx) \leq \alpha$, which implies that $\mu_V(f(x)f(y)) \leq \alpha$. Hence $f(A_\alpha)$ is a lower level subhemiring of an anti S-fuzzy subhemiring V of R^l.

2.26 Theorem: The anti-homomorphic pre-image of a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R^l is a lower level subhemiring of an anti S-fuzzy subhemiring of a hemiring R.

Proof: Let V = f(A), where V is an anti S-fuzzy subhemiring of a hemiring R^l. Clearly A is an anti S-fuzzy subhemiring of a hemiring R. Let f(x) and f(y) in R^l, implies x and y in R. Let $f(A_\alpha)$ is a lower level subhemiring of V. Now,

$\mu_A(x) = \mu_V(f(x)) \leq \alpha$, which implies that $\mu_A(x) \leq \alpha$; $\mu_A(y)= \mu_V(f(y)) \leq \alpha$, which implies that $\mu_A(y) \leq \alpha$. Now, $\mu_A(x+y) =\mu_V(f(y)+f(x)) \leq \alpha$, which implies that $\mu_A(x + y) \leq \alpha$. Also, $\mu_A(xy) = \mu_V(f(y)f(x)) \leq \alpha$, which implies that $\mu_A(xy) \leq \alpha$. Hence A_α is a lower level subhemiring of an anti S-fuzzy subhemiring A of R.

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