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JORDAN GENERALIZED TRIPLE DERIVATIONS OF PRIME Γ-RINGS

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Abstract. In this article, we develop some important results relating to the concepts of generalized triple derivation and Jordan generalized triple derivation of gamma rings. Through every generalized triple derivation of a gamma ring M is obviously a Jordan genaralized triple derivation of M, but the converse statement is in general not true. Here we prove that every Jordan generalized triple derivation of a 2-torsion free prime gamma ring is a generalized derivation.

Keywords: Derivation and generalized triple derivation, Jordan generalized triple derivation, gamma rings and prime gamma rings.

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1. Introduction

Let M and Γ be additive abelian groups. M is said to be a Γ-ring if there exists a mapping $M \times \Gamma \times M \to M$ (sending (x, α, y) into $x\alpha y$) such that

$(a)(x + y)\alpha z = x\alpha z + y\alpha z,$

$x(\alpha + \beta)y = x\alpha y + x\beta y,$

$x\alpha(y + z) = x\alpha y + x\alpha z,$

$(b)(x\alpha y)\beta z = x\alpha(y\beta z),$

for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

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A subset A of a Γ-ring M is a left(right) ideal of M if A is an additive subgroup of M and $M\Gamma A = \{m\alpha a : m \in M, \alpha \in \Gamma \text{ and } a \in A\}$, $(A\Gamma M)$ is contained in A. An ideal P of a Γ-ring M is prime if $P \neq M$ and for any ideals A and B of $M, A\Gamma B \subseteq P$, then $A \subseteq P$ or$B \subseteq P$. M is prime if $a\Gamma M\Gamma b = 0$ with $a, b \in M$, then $a = 0$ or $b = 0$. M is 2-torsion free if $2m = 0$, for $m \in M$ implies $m = 0$.

N. Nobusawa [5] was first introduced the notion of a gamma ring. The gamma ring due to N. Nobusawa is now denoted by Γ_N-ring. Next Barnes [1] generalized it and gave the above defination. Now a day we mean the gamma ring which is given by Barnes. It is clear that every ring is a gamma ring.

M. Bresar [3] worked on Jordan triple derivations of semiprime rings and he proved that R is a two torsion free semiprime ring, then every Jordan derivation is a derivation.

Wu Jing and Shijie [6] defined generalized Jordan triple derivation. They showed that every generalized Jordan triple derivation is a generalized derivation.

M. Sapanci and A. Nakajima [4] worked on Jordan derivation on completely prime gamma rings. They proved that every Jordan derivation on a two torsion free completely prime gamma rings is a derivation.

In this paper, we define generalized triple derivation and Jordan generalized triple derivation of a gamma ring. We give an example of a generalized triple derivation and an example of a Jordan generalized triple derivation for gamma rings. We also prove that every Jordan generalized triple derivation is a generalized derivation if it is a two torsion free prime Γ-ring.

2. Jordan generalized triple derivation.

Let R be an associative ring. An additive mapping $d : R \to R$ is called a Triple derivation if

d(abc)=d(a)bc+ad(b)c+abd(c).

and a Jordan Triple derivation if

d(aba)=d(a)ba+ad(b)a+abd(a).

Let M be Γ-ring. An additive mapping $f : M \to M$ is called a generalized Triple derivation if

$f(a\alpha b\beta c) = f(a)\alpha b\beta c + a\alpha d(b)\beta c + a\alpha b\beta d(c)$. For all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$

and a Jordan generalized Triple derivation if

$f(a\alpha b\beta a) = f(a)\alpha b\beta a + a\alpha d(b)\beta a + a\alpha b\beta d(a)$. For all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$

It is clear that every generalized triple derivation is a Jordan generalized triple derivation but the converse is not ingeneral true.

Now we give the following examples:

2.1 Example

Let R be an associative ring with unity element 1. Let $M = M_{1,2}(R)$ and $\Gamma = \left\{ \begin{pmatrix} n.1 \\ 0 \end{pmatrix}, n \in Z \right\}$.

Then M is a Γ-ring. Let $f : R \to R$ be a generalized triple derivation with associated derivation $d : R \to R$. Now define

F((x,y))=(f(x),f(y)) and D((x,y))=(d(x),d(y)). Then F is a generalized triple derivation associated to the derivation d.

To show it consider $a = (x_1, y_1)$, $b = (x_2, y_2)$, $c = (x_3, y_3)$ $\alpha = \begin{pmatrix} n_1.1 \\ 0 \end{pmatrix}$, $\beta = \begin{pmatrix} n_2.1 \\ 0 \end{pmatrix}$,

then $a\alpha b\beta c = (x_1 n_1 x_2 n_2 x_3, x_1 n_1 x_2 n_2 y_3)$

And finally we get $F(a\alpha b\beta c) = F(a)\alpha b\beta c + a\alpha D(b)\beta c + a\alpha b\beta D(c)$. F is a generalized triple derivation associated to the derivation D.

2.2 Example

Let M be a Γ-ring defined as in example 2.1. Let $N = \{(x, x) : x \in M\}$.

Then N is a Γ-ring containd in M. Let d be a triple derivation given in example 2.1. Define

$D : N \to N$ is a Jordan triple derivation. Define

F((x,x))=(f(x),f(x)).

Then F is a Jordan generalized triple derivation.

To show it consider $a = (x, x)$, $b = (y, y)$ and $\alpha = \begin{pmatrix} n_1.1 \\ 0 \end{pmatrix}$, $\beta = \begin{pmatrix} n_2.1 \\ 0 \end{pmatrix}$ then $a\alpha b\beta a = (xn_1yn_2x, xn_1yn_2x)$, $F(a\alpha b\beta a) = F(a)\alpha b\beta a + a\alpha D(b)\beta a + a\alpha b\beta D(a)$. So F is a Jordan generalized triple derivation associated to the derivation D.

Note that it is not a generalized Jordan triple derivation.

Lemma 2.1. *Let M be a Γ ring and d be a Jordan triple derivation of a Γ ring M. Then for all $a, b, c \in M$, we have*

$d(a\alpha b\beta c + c\alpha b\beta a) = d(a)\alpha b\beta c + d(c)\alpha b\beta a + a\alpha d(b)\beta c + c\alpha d(b)\beta a + c\alpha b\beta d(a) + a\alpha b\beta d(c).$

proof: Computing $d((a + c)\alpha b\beta(a + c))$ and cancelling the like terms from both sides, we prove the lemma.

Lemma 2.2. *Let M be a Γ ring and d be a Jordan generalized triple derivation on a Γ ring M. Then for all $a, b, c \in M$, we have*

$f(a\alpha b\beta c + c\alpha b\beta a) = f(a)\alpha b\beta c + f(c)\alpha b\beta a + a\alpha d(b)\beta c + c\alpha d(b)\beta a + c\alpha b\beta d(a) + a\alpha b\beta d(c).$

proof: Computing $f((a + c)\alpha b\beta(a + c))$ and cancelling the like terms from both sides, we prove the lemma.

Definition 1. *Let M be a Γ-ring. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ we define*

$[a, b, c]_{\alpha,\beta} = a\alpha b\beta c - c\alpha b\beta a.$

Lemma 2.3. *If M is a Γ-ring, then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$*

(1) $[a, b, c]_{\alpha,\beta} + [c, b, a]_{\alpha,\beta} = 0$

(2) $[a + c, b, d]_{\alpha,\beta} = [a, b, d]_{\alpha,\beta} + [c, b, d]_{\alpha,\beta}$

(3) $[a, b, c + d]_{\alpha,\beta} = [a, b, c]_{\alpha,\beta} + [a, b, d]_{\alpha,\beta}$

(4)$[a, b + d, c]_{\alpha,\beta} = [a, b, c]_{\alpha,\beta} + [a, d, c]_{\alpha,\beta}$

(5) $[a, b, c]_{\alpha+\beta,\gamma} = [a, b, c]_{\alpha,\gamma} + [a, b, c]_{\beta,\gamma}$

(6) $[a, b, c]_{\alpha,\beta+\gamma} = [a, b, c]_{\alpha,\beta} + [a, b, c]_{\alpha,\gamma}$

proof: Obvious

Definition 2. *Let d be a Jordan triple derivation of a Γ-ring M. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ we define*

$G_{\alpha,\beta}(a, b, c) = d(a\alpha b\beta c) - d(a)\alpha b\beta c - a\alpha d(b)\beta c - a\alpha b\beta d(c).$

Lemma 2.4. *Let d be a Jordan triple derivation of a Γ-ring M. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ we have*

(1) $G_{\alpha,\beta}(a, b, c) + G_{\alpha,\beta}(c, b, a) = 0$

(2) $G_{\alpha,\beta}(a + c, b, e) = G_{\alpha,\beta}(a, b, e) + G_{\alpha,\beta}(c, b, e)$

(3) $G_{\alpha,\beta}(a, b, c + e) = G_{\alpha,\beta}(a, b, c) + G_{\alpha,\beta}(a, b, e)$

(4) $G_{\alpha,\beta}(a, b + c, e) = G_{\alpha,\beta}(a, b, e) + G_{\alpha,\beta}(a, c, e)$

(5) $G_{\alpha+\gamma,\beta}(a, b, c) = G_{\alpha,\beta}(a, b, c) + G_{\gamma,\beta}(a, b, c)$

(6) $G_{\alpha,\beta+\gamma}(a, b, c) = G_{\alpha,\beta}(a, b, c) + G_{\alpha,\gamma}(a, b, c).$

proof: Obvious

Lemma 2.5. *If M is a Γ-ring , then*

$G_{\alpha,\beta}(a, b, c)\gamma x\delta[a, b, c]_{\alpha,\beta} + [a, b, c]_{\alpha,\beta}\gamma x\delta G_{\alpha,\beta}(a, b, c) = 0$

for all $x \in M$ and $\gamma, \delta \in \Gamma$.

proof: First , we compute $d(a\alpha(b\beta c\gamma x\delta c\alpha b)\beta a+c\alpha(b\beta a\gamma x\delta a\alpha b)\beta c)$ by using the definition of Jordan triple derivation we get $d(a)\alpha b\beta c\gamma x\delta c\alpha b\beta a+a\alpha d(b)\beta c\gamma x\delta c\alpha b\beta a+a\alpha b\beta d(c)\gamma x\delta c\alpha b\beta a+a\alpha b\beta c\gamma d(x)\delta c\alpha b\beta a+a\alpha b\beta c\gamma x\delta d(c)\alpha b\beta a+a\alpha b\beta c\gamma x\delta c\alpha d(b)\beta a+a\alpha b\beta c\gamma x\delta c\alpha b\beta d(a)+d(c)\alpha b\beta a\gamma x\delta a\alpha b\beta c+c\alpha d(b)\beta a\gamma x\delta a\alpha b\beta c+c\alpha b\beta d(a)\gamma x\delta a\alpha b\beta c+c\alpha b\beta a\gamma d(x)\delta a\alpha b\beta c+c\alpha b\beta a\gamma x\delta d(a)\alpha b\beta c+c\alpha b\beta a\gamma x\delta a\alpha d(b)\beta c+c\alpha b\beta a\gamma x\delta a\alpha b\beta d(c)$. On the other hand, we $d((a\alpha b\beta c)\gamma x\delta(c\alpha b\beta a) + (c\alpha b\beta a)\gamma x\delta(a\alpha b\beta c))$ and using lemma 2.1, then we get $d(a\alpha b\beta c)\gamma x\delta c\alpha b\beta a+d(c\alpha b\beta a)\gamma x\delta a\alpha b\beta c+a\alpha b\beta c\gamma d(x)\delta c\alpha b\beta a+c\alpha b\beta a\gamma d(x)\delta a\alpha b\beta c + a\alpha b\beta c\gamma x\delta d(c\alpha b\beta a) + c\alpha b\beta a\gamma x\delta d(a\alpha b\beta c)$ Since these two are equal, cancelling the like terms from both sides of this equality and then rearranging them, we get

$G_{\alpha,\beta}(a, b, c)\gamma x\delta[a, b, c]_{\alpha,\beta} + [a, b, c]_{\alpha,\beta}\gamma x\delta G_{\alpha,\beta}(a, b, c) = 0$

Lemma 2.6. *Let M be a 2-torsion free semi prime Γ-ring and suppose that* $a, b \in M$. *If* $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ *for all* $m \in M$, *then* $a\Gamma m\Gamma b = b\Gamma m\Gamma a = 0$

proof: Let m and m' be two arbitrary elements of M. Then by hypothesis, we have $(a\Gamma m\Gamma b)\Gamma m'\Gamma(a\Gamma m\Gamma b) = -(b\Gamma m\Gamma a)\Gamma m'\Gamma(a\Gamma m\Gamma b) = -(b\Gamma(m\Gamma a)\Gamma m')\Gamma a)\Gamma m\Gamma b) = (a\Gamma(m\Gamma a\Gamma m')\Gamma b)\Gamma m\Gamma$ $a\Gamma m\Gamma(a\Gamma m'\Gamma b)\Gamma m\Gamma b) = -a\Gamma m\Gamma(b\Gamma m'\Gamma a)\Gamma m\Gamma b) = -(a\Gamma m\Gamma b)\Gamma m'\Gamma(a\Gamma m\Gamma b)$. This implies , $2(a\Gamma m\Gamma b)\Gamma m'\Gamma(a\Gamma m\Gamma b) = 0$. Since M is a 2-torsion free, $(a\Gamma m\Gamma b)\Gamma m'\Gamma(a\Gamma m\Gamma b) = 0$

By the semiprimeness of M, $a\Gamma m\Gamma b = 0$ for all $m \in M$. Hence we get ,$a\Gamma m\Gamma b = a\Gamma m\Gamma b = 0$ for all $m \in M$.

Lemma 2.7. *Let M is a 2-torsion free prime Γ-ring. Then for all* $a, b, x \in M$ *and* $\alpha, \beta, \gamma, \delta \in \Gamma$, *then* $G_{\alpha,\beta}(a, b, c)\gamma x\delta[a, b, c]_{\alpha,\beta} = [a, b, c]_{\alpha,\beta}\gamma x\delta G_{\alpha,\beta}(a, b, c) = 0$.

proof: The lemma is semiler to the proof of [7] corollary 3.11

Definition 3. *Let f be a Jordan generalized teiple derivation of a Γ-ring M . Then for all* $a, b, c \in M$ *and* $\alpha, \beta \in \Gamma$ *we define*
$F_{\alpha,\beta}(a, b, c) = f(a\alpha b\beta c) - f(a)\alpha b\gamma c - a\alpha d(b)\beta c - a\alpha b\beta d(c)$.

Lemma 2.8. *Let f be a Jordan generalized teiple derivation of a Γ-ring M . Then for all* $a, b, c \in M$ *and* $\alpha, \beta \in \Gamma$

(1) $F_{\alpha,\beta}(a, b, c) + F_{\alpha,\beta}(c, b, a) = 0$

(2) $F_{\alpha,\beta}(a + c, b, e) = F_{\alpha,\beta}(a, b, e) + F_{\alpha,\beta}(c, b, e)$

(3) $F_{\alpha,\beta}(a, b, c + e) = F_{\alpha,\beta}(a, b, c) + F_{\alpha,\beta}(a, b, e)$

(4)$F_{\alpha,\beta}(a, b + c, e) = F_{\alpha,\beta}(a, b, e) + F_{\alpha,\beta}(a, c, e)$

(5) $F_{\alpha+\beta,\gamma}(a, b, c) = F_{\alpha,\gamma}(a, b, c) + F_{\beta,\gamma}(a, b, c)$

(6) $F_{\alpha,\beta+\gamma}(a, b, c) = F_{\alpha,\beta}(a, b, c) + F_{\alpha,\gamma}(a, b, c)$

proof: Obvious

Lemma 2.9. *If M is a Prime Γ-ring , then*
$F_{\alpha,\beta}(a,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta}+[a,b,c]_{\alpha,\beta}\gamma x\delta G_{\alpha,\beta}(a,b,c)=0$
for all $x\in M$ and $\gamma,\delta\in\Gamma$.

proof: First , we compute $f(a\alpha(b\beta c\gamma x\delta c\alpha b)\beta a+c\alpha(b\beta a\gamma x\delta a\alpha b)\beta c)$ by using the definition of Jordan generalized triple derivation we get $f(a)\alpha b\beta c\gamma x\delta c\alpha b\beta a+a\alpha d(b)\beta c\gamma x\delta c\alpha b\beta a+a\alpha b\beta d(c)\gamma x\delta c\alpha b\beta a+a\alpha b\beta c\gamma d(x)\delta c\alpha b\beta a+a\alpha b\beta c\gamma x\delta d(c)\alpha b\beta a+a\alpha b\beta c\gamma x\delta c\alpha d(b)\beta a+a\alpha b\beta c\gamma x\delta c\alpha b\beta d(a)+f(c)\alpha b\beta a\gamma x\delta a\alpha b\beta c+c\alpha d(b)\beta a\gamma x\delta a\alpha b\beta c+c\alpha b\beta d(a)\gamma x\delta a\alpha b\beta c+c\alpha b\beta a\gamma d(x)\delta a\alpha b\beta c+c\alpha b\beta a\gamma x\delta d(a)\alpha b\beta c+c\alpha b\beta a\gamma x\delta a\alpha d(b)\beta c+c\alpha b\beta a\gamma x\delta a\alpha b\beta d(c)$.
On the other hand, we compute $f((a\alpha b\beta c)\gamma x\delta(c\alpha b\beta a)+f(c\alpha b\beta a)\gamma x\delta(a\alpha b\beta c)+(a\alpha b\beta c)\gamma d(x)\delta(a\alpha b\beta c)+(c\alpha b\beta a)\gamma d(x)\delta(a\alpha b\beta c)+(a\alpha b\beta c)\gamma x\delta d(c\alpha b\beta a)+(c\alpha b\beta a)\gamma x\delta d(a\alpha b\beta c)$ we get $d(a\alpha b\beta c)\gamma x\delta c\alpha b\beta a+d(c\alpha b\beta a)\gamma x\delta a\alpha b\beta c+a\alpha b\beta c\gamma d(x)\delta c\alpha b\beta a+c\alpha b\beta a\gamma d(x)\delta a\alpha b\beta c+a\alpha b\beta c\gamma x\delta d(c\alpha b\beta a)+c\alpha b\beta a\gamma x\delta d(a\alpha b\beta c)$
Since these two are equal, cancelling the like terms from both sides of this equality and then rearranging them, we get
$F_{\alpha,\beta}(a,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta}+[a,b,c]_{\alpha,\beta}\gamma x\delta G_{\alpha,\beta}(a,b,c)=0.$

Lemma 2.10. *If M is a Prime Γ-ring , then*
$F_{\alpha,\beta}(a,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta}=0$
for all $x\in M$ and $\gamma,\delta\in\Gamma$.

proof: From lemma 2.9 we get $F_{\alpha,\beta}(a,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta}+[a,b,c]_{\alpha,\beta}\gamma x\delta G_{\alpha,\beta}(a,b,c)=0$ and using lemma 2.5 we get
$F_{\alpha,\beta}(a,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta}=0.$

Lemma 2.11. *If M is a semi Prime Γ-ring , then*
$[a,b,c]_{\alpha,\beta}\gamma x\delta F_{\alpha,\beta}(a,b,c)=0$
for all $x\in M$ and $\gamma,\delta\in\Gamma$.

proof: Since $[a,b,c]_{\alpha,\beta}\gamma x\delta F_{\alpha,\beta}(a,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta}\gamma x\delta F_{\alpha,\beta}(a,b,c)$=0, then by semiprimeness of Γ ring M we get $[a,b,c]_{\alpha,\beta}\gamma x\delta F_{\alpha,\beta}(a,b,c)=0$

Lemma 2.12. *Let M is a 2-torsion free semi prime Γ-ring. Then for all $a,b,c,u,v,w,x\in M$ and $\alpha,\beta,\gamma,\delta\in\Gamma$, then $F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,v,w]_{\alpha,\beta}=0$.*

Proof: Replacing a by a+u in the lemma 2.10 we get $F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,b,c]_{\alpha,\beta}+F_{\alpha,\beta}(u,b,c)\gamma x\delta[a,b,c]_{\alpha,\beta} = 0$. Now $F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,b,c]_{\alpha,\beta}\gamma x\delta F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,b,c]_{\alpha,\beta}=$
$-F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,b,c]_{\alpha,\beta}\gamma x\delta F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,b,c]_{\alpha,\beta} = 0$ by using lemma 2.6. Since M is a 2-torsion free semiprime Γ-ring , then $F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,b,c]_{\alpha,\beta} = 0$. Similarly, replacing b by b+v and c by c+w we get $F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,v,w]_{\alpha,\beta} = 0$

Lemma 2.13. *Let M is a 2-torsion free prime Γ-ring. Then for all $a,b,c,x \in M$ and $\alpha,\beta,\gamma,\delta \in \Gamma$. Then $F_{\alpha,\beta}(a,b,c) = 0$ or $[u,v,w]_{\alpha,\beta} = 0$*

proof: From lemma 2.12 we get $F_{\alpha,\beta}(a,b,c)\gamma x\delta[u,v,w]_{\alpha,\beta} = 0$.
Since M is a prime Γ-ring, then either $F_{\alpha,\beta}(a,b,c) = 0$ or $[u,v,w]_{\alpha,\beta} = 0$

Theorem 2.1. *Let M is prime Γ-ring, then every generalized Jordan triple derivation is a generalized triple derivation.*

proof: By lemma 2.13, we have $F_{\alpha,\beta}(a,b,c) = 0$ or $[u,v,w]_{\alpha,\beta} = 0$.
case 1: Suppose $[u,v,w]_{\alpha,\beta} = 0$, then $u\alpha v\beta w = w\alpha v\beta u$ .Therefore, we have from lemma 2.2, $f(u\alpha v\beta w) = f(u)\alpha v\beta w + u\alpha d(v)\beta w + u\alpha v\beta d(w)$ i.e Jordan generalized triple derivation is a generalized triple derivation.
case 2: Suppose $F_{\alpha,\beta}(a,b,c) = 0$ then $f(a\alpha b\beta c) = f(a)\alpha b\beta c + a\alpha d(b)\beta c + a\alpha b\beta d(c)$. Hence Jordan generalized triple derivation is a generalized triple derivation.

Theorem 2.2. *Any Jordan triple derivation of a 2-torsion free prime Γ-ring is a derivation.*

proof: Consider $w = f(a\alpha(b\gamma x\delta a)\alpha b)$
$= f(a)\alpha b\gamma x\delta a\alpha b + a\alpha d(b\gamma x\delta a)\alpha b + a\alpha b\gamma x\delta a\alpha d(b)$
$= f(a)\alpha b\gamma x\delta a\alpha b + a\alpha d(b)\gamma x\delta a\alpha b + a\alpha b\gamma d(x)\delta a\alpha b + a\alpha b\gamma x\delta d(a)\alpha b + a\alpha b\gamma x\delta a\alpha d(b)$
Again,$W = f((a\alpha b)\gamma x\delta(a\alpha b)) = f(a\alpha b)\gamma x\delta a\alpha b + a\alpha b\gamma d(x)\delta a\alpha b + a\alpha b\gamma x\delta d(a\alpha b)$
Comparing the two exprations so obtained for W we obtain $(f(a\alpha b)-f(a)\alpha b-a\alpha d(b))\gamma x\delta a\alpha b+$
$a\alpha b\gamma x\delta(d(a\alpha b) - d(a)\alpha b - a\alpha d(b)) = 0$
Since d is a derivation , so $(f(a\alpha b) - f(a)\alpha b - a\alpha d(b))\gamma x\delta a\alpha b = 0$, Again by primeness of M, $f(a\alpha b) - f(a)\alpha b - a\alpha d(b) = 0$, i.e. f is generalized derivation.

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