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CONTINUATION OF A PARAMETERIZED IMPULSIVE DIFFERENTIAL EQUATION TO AN INITIAL VALUE PROBLEM

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Abstract. In this paper, we are concerned with an initial value problem and a parameterize problem of impulsive differential equation. The existence of solutions are proved. The continuations of the two problems and their solutions will be studied.

Keywords: Impulsive differential equations; Existence of solution; Banach fixed point theorem; differential equations with initial conditions.

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1. Introduction

The impulsive differential equations describe evolution processes which at certain moments change their state rapidly. In the mathematical simulation of such processes it is convenient to assume that this change takes place momentarily and the process changes its state by jump. Processes of such character are observed in numerous fields of science and technology: theoretical physics, mechanics, population dynamics [1], physics, Chemistry [2], engineering [3], ecology, biological systems, biotechnology, industrial robotics, pharmacokinetics, optimal control, impulse technique, chemical technology and so on. The

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wide possibility of applications determines the growing interest in impulsive differential equations. On the basis of numerous results obtained there appeared the first monographs related to this new subject: Samoilenko and Perestyuk (1987), Lakshmikantham, Bainov and Simeonov (1989), Bainov and Simeonov (1989), (1993). According to the way in which the moments of the change by jumps are determined, the impulsive differential equations are classified as follows(see[2],[4],[5]and [6]):

I. Equations with fixed moments of impulse effect (the moments of jump are previously fixed).

II. Equations with unfixed moments of impulse effect (the moments of jump occur when certain space-time relations are satisfied).

In this work, we consider the initial value problem

$$x'(t) = f(t, x(t)),\ t \in (0, T], \tag{1.1}$$

$$x(0) = x_o \tag{1.2}$$

and the parameterized problem of impulsive differential equation

$$x'(t) = f(t, x(t)),\ t \in (0, T] \text{ and } t \neq \tau, \tag{1.3}$$

$$x(\tau^-) = \alpha x(\tau^+),\ \alpha \in (0, 1), \tag{1.4}$$

$$x(0) = x_o. \tag{1.5}$$

Where $f : [0, T] \times R \to R$ is a given function, $x_o \in R$, $x(\tau^+) = \lim_{h \to o^+} x(\tau + h)$ and $x(\tau^-) = \lim_{h \to o^-} x(\tau + h)$ represent the right and left limits of $x(t)$ at $t = \tau$.

Our aim here is to study the continuation of the problem (1.3)-(1.5) and its solution to the problem (1.1)-(1.2) and its solution, as $\alpha \to 1$.

2. Preliminaries

Throughout this paper, we need some basic definitions and properties of impulsive differential equation which are used throughout this paper. By $C[0,T]$ we denote the Banach space of all continuous functions defined on $[0,T]$ with the norm

$$\|x\|_C = \sup\{|x(t)| : t \in [0,T]\},$$

set $PC([0,T],R) = \{\ x : [0,T] \to R$ is continuous everywhere except for $t = \tau$ at which $x(\tau^-)$ and $x(\tau^+)$ exist and $x(\tau^-) = x(\tau)\ \}$ with the norm

$$\|x\|_{PC([0,T],R)} = \sup\{|x(t)| : t \in [0,T]\}.$$

Definition 2.1. ([2,7]) $x(t)$ is said to be the solution of problem (1.3)-(1.5) if it satisfies the following conditions:

(1) $\lim_{t\to o^+} x(t) = x_o = x(0^+)$,

(2) for $(0,+\infty)$, $t \neq \tau$, $x(t)$ is differentiable and $x^{'}(t) = f(t,x(t))$,

(3) $x(t)$ is left continuous in $(0,+\infty)$ and if $t = \tau$, then $x(\tau^-) = \alpha x(\tau^+), \alpha \neq 1$.

Definition 2.2. if $f(t,x(t))$ is differentiable function, then the solution of IVP (1.1)-(1.2) is

$$x(t) = x_o + \int_0^t f(s,x(s))ds. \tag{2.1}$$

3. Main results

3.1. **Impulsive differential equation.**

Definition 3.1. By a solution of problem (1.3)-(1.5), we mean a function $x \in PC([0,T],R)$ that satisfies the problem (1.3)-(1.5).

Theorem 3.1. *Let* $f : [0,T] \times R \to R$, *is continuous function and satisfies the lipschitz condition*

$$|f(t,x(t)) - f(t,\bar{x}(t))| \leq K|x - \bar{x}|,\ \forall (t,x),(t,\bar{x}) \in [0,T] \times R,$$

with lipschitz constant $K > 0$ *. If*

$$KT < 1, \tag{3.1}$$

then the problem (1.3)-(1.5) has a unique solution.

Proof. Integrating equation (1.3) and using (1.4),(1.5) we obtain

$$x_\alpha(t) = \begin{cases} x_o + \int_o^t f(s, x(s))ds & if \quad t \in (o, \tau], \\ \\ x(\tau^+) + \int_\tau^t f(s, x(s))ds & if \quad t \in (\tau, T], \end{cases} \tag{3.2}$$

since, from Eq.(1.4) we obtain

$$x(\tau^+) = \frac{1}{\alpha} x(\tau^-),$$

$$x(\tau^+) = \frac{x_o}{\alpha} + \frac{1}{\alpha} \int_0^\tau .f(s, x(s))ds \tag{3.3}$$

Applying the Banach contraction fixed point theorem, we deduce that there exist a unique solution $x_\alpha \in PC([0, T], R)$ of integral equation (3.2).This solution satisfies the problem (1.3)-(1.5).

3.2. **Continuation theorem.**

Theorem 3.2. *If* $\alpha \to 1$*, then the problems (1.3)-(1.5) and (1.1)-(1.2) are coincide with the same solution.*

Proof. Letting $\alpha \to 1$ in (1.4), then the problem (1.3)-(1.5) coincide with the problem (1.1)-(1.2). Let $x(t)$, $x_\alpha(t)$ are given by (2.1) and (3.2) respectively, then

$$\lim_{\alpha \to 1} x_\alpha(t) = x(t), \ t \in (0, T]. \tag{3.4}$$

And the two problems (1.1)-(1.2),(1.3) and (1.5) have the same solution. □

4. Examples

In this section, we consider some first order impulsive differential equations and the following examples will be helpful to illustrate the main results of this paper.

Example 4.1. *Consider the following impulsive differential equation*

$$x^{'}(t) + 1 = 0;\ t \neq \frac{1}{2}\ , t \in (0, \frac{3}{2}],$$

$$x(\frac{1}{2}^{-}) = \alpha\ x(\frac{1}{2}^{+});\ t = \frac{1}{2},$$

$$x(0) = 0.$$

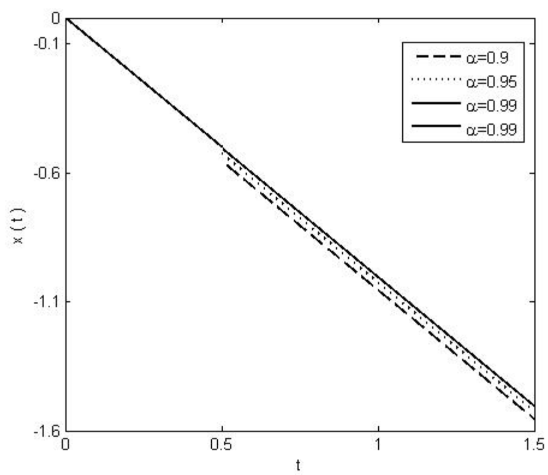

Fig.1. show the continuation of solutions of Ex.(4.1).

Example 4.2. *Consider the following impulsive differential equation*

$$x^{'}(t) = x + \cos(2\pi t);\ t \neq 1,\ t \in (0, 2],$$

$$x(1^{-}) = \alpha\ x(1^{+});\ t = 1,$$

$$x(0) = 0.$$

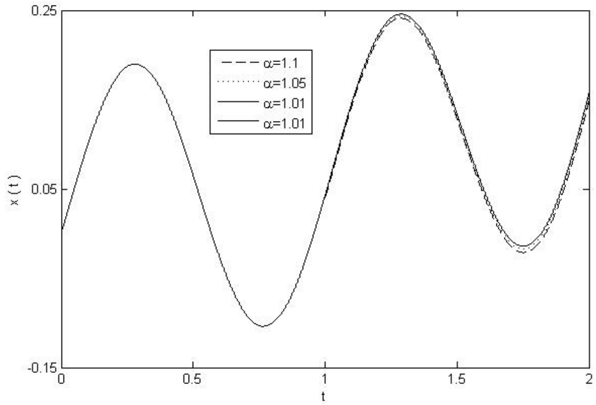

Fig.2. show the continuation of solutions of Ex.(4.2).

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