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SOLITON SOLUTIONS OF DEGASPERIS-PROCESI EQUATION AND A MODIFIED KDV EQUATION BY THE SINE-COSINE METHOD

ISSAM ABU-IRWAQ

Department of Mathematics and Statistics,

Jordan University of Science and Technology, IRBID 22110, Jordan

Abstract. In this paper, we construct solitary wave solutions for a hydrodynamical model called the Degasperis-Procesi equation. The soliton solutions of a modified KdV type is also investigated.

Keywords: Degasperis-Procesi equation; modified KdV type; wave variables, Sine-Cosine Method.

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1. Introduction

In this paper, we study the solutions of the Degasperis-Procesi (DP) equation derived in [21] and given by,

$$u_t + 3k^3 u_x - u_{xxt} + 4uu_x = 3u_x u_{xx} + uu_{xxx}, \tag{1}$$

where subscripts denote partial derivatives and k is a positive parameter. The integrability of the DP equation was proved by constructing a Lax pair, deriving an infinite sequence of conservation laws and the existence of a bi-Hamiltonian structure [2, 3]. In [23], the authors constructed loop soliton solutions and mixed soliton−loop soliton solutions by applying the modified

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$\tau-$function method.

Also, we study the solutions of a modified KdV type derived in [5] and reads

$$uu_{xxt} - u_x u_{xt} - 4u^3 u_t + 4uu_{xxx} - 4u_x u_{xx} - 16u^3 u_x = 0. \quad (2)$$

Wazwaz in [6] obtained a variety of solitary wave solutions that include kinks, solitons, peakons, cuspons, periodic and other solutions of Equation (2) by means of modified Hirota's method. Exact solutions to nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. In the literature, many significant methods have been proposed for obtaining Exact solutions of nonlinear partial differential equations such as the extended tanh-function method [18, 7], the Exp-function method [13], Hirota's method [12], the rational sine-cosine method [11] and many other methods see [7, 10, 17, 19, 20, 21, 22].

The main aim of this paper is to apply the Sine-Cosine function method with the help of symbolic computation to obtain new soliton solutions of (1, 2). By using Sine-Cosine function method, many kinds of nonlinear partial differential equations arising in mathematical physics have been solved successfully [15, 16, 8, 9].

2. The Sine-Cosine Method

Since we restrict our attention to traveling waves, we use the transformation $u(x,t) = u(z)$, where the wave variable $z = x - ct$, converts the the nonlinear PDE to an equivalent ODE. The sine-cosine algorithm admits the use of the ansatz [8, 9]

$$u(x,t) = A\sin^B(\mu z), \quad |z| \le \frac{\pi}{\mu}, \quad (3)$$

and the ansatz [8, 9]

$$u(x,t) = A\cos^B(\mu z), \quad |z| \le \frac{\pi}{2\mu}, \quad (4)$$

where A, μ, c and B are parameters that will be determined. Substituting (3) or (4) into the reduced ODE gives a polynomial equation of cosine or sine terms. Balancing the exponents of the trigonometric functions cosine or sine, collecting all terms with same power in $\cos^B(\mu z)$ or $\sin^B(\mu z)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns λ, μ and β. The problem is now completely reduced to an algebraic one. Having determined A, μ, c and B by algebraic calculations or by using computerized symbolic calculations, the solutions proposed in (3) and in (4) follow immediately.

1. Solving the DP equation by the Sine-Cosine Method

Applying the wave variable $z = x - ct$ to equation (1) yields the following nonlinear ODE

$$(3k^3 - c)f'(z) + (c - f(z))f'''(z) + 4f(z)f'(z) - 3f'(z)f''(z) = 0. \tag{5}$$

First, we apply ansatz (3) to the resulting ODE (5) to obtain an algebraic equation involving powers of the function $\sin(\mu z)$

$$K_1 + K_2\sin^2(\mu z) + K_3\sin^B(\mu z) + K_4\sin^{2+B}(\mu z) = 0, \tag{6}$$

where

$$\begin{aligned} K_1 &= (-4 + 6B - 2B^2)c\mu^2 \\ K_2 &= 2c - 6k^3 + 2B^2c\mu^2 \\ K_3 &= (4 - 12B + 8B^2)A\mu^2 \\ K_4 &= -8A(1 + B^2\mu^2) \end{aligned} \tag{7}$$

Observing (6) and (7), we choose $B = 2$ and accordingly $K_1 = 0$, $K_2 + K_3 = 0$ and $K_4 = 0$. Therefore,

$$A = -2k^3, \quad \mu = \pm\frac{i}{2}. \tag{8}$$

Thus the obtained solution of equation (1) is

$$u(x,t) = 2k^3\sinh^2\left(\frac{x - ct}{2}\right). \tag{9}$$

Figure 1 represents profile solutions of the DP equation by appling the Sine method.

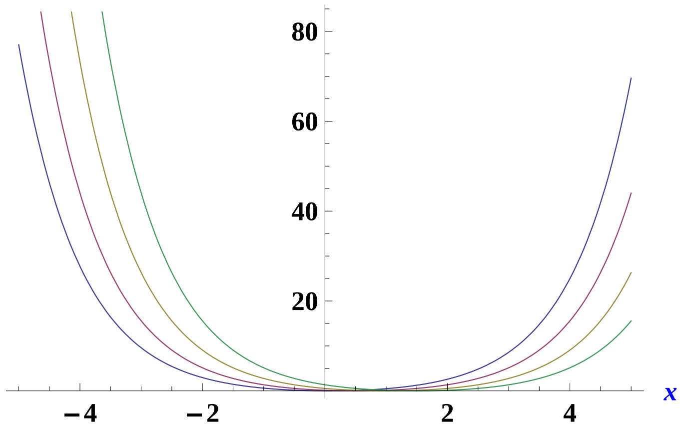

FIGURE 1. Profile solutions of the DP equation when $-5 < x < 5$ and $t = 0.1, 1, 2, 3$ and $k = 1, c = 0.5$

Second, by applying ansatz (4) we obtain the same results as in (8) and therefore the solution of the DP equation by the Cosine method is

$$u(x,t) = -2k^3 \cosh^2(\frac{x-ct}{2}). \tag{10}$$

Figure 2 represents profile solutions of the DP equation by appling the Cosine method

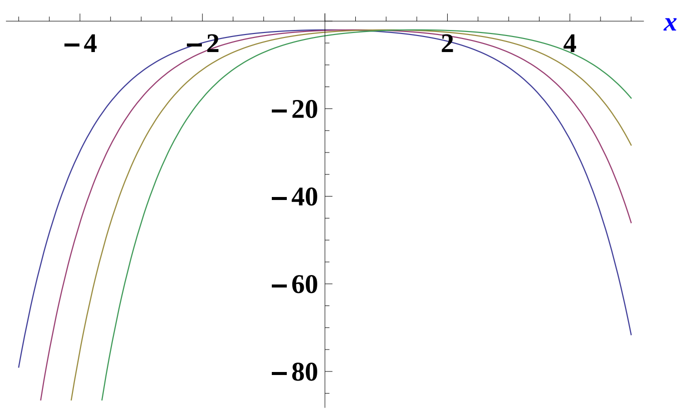

FIGURE 2. Profile solutions of the DP equation when $-5 < x < 5$ and $t = 0.1, 1, 2, 3$ and $k = 1, c = 0.5$

2. MODIFIED KDV EQUATION

Applying the wave variable $z = x - ct$ to Equation (2) yields the following nonlinear ODE

$$(c-4)f'(z)f''(z) - (c-4)f(z)f'''(z) + 4(c-4)f(z)^3 f'(z) = 0. \tag{11}$$

First, we apply ansatz (4) to the resulting ODE (11) to obtain the following algebraic equation,

$$(-1+B)\mu^2 + 2A^2 \sin^{2+2B}(\mu z) = 0. \tag{12}$$

The above equation is satisfied if the following conditions are hold

$$B = -1,\ A = \pm\mu \tag{13}$$

Therefore, the solution of the modified KdV is given by

$$u(x,t) = \mu \csc(\mu(x-ct)). \tag{14}$$

Figure 3, represents the solution obtained in (14).

Second, by applying the cosine method, we get the same algebraic equations and the same conditions. Thus, a second solution of the modified KdV is given by

$$u(x,t) = \mu \sec(\mu(x-ct)). \tag{15}$$

Figure 4, represents the solution of the mKdV obtained in (15).

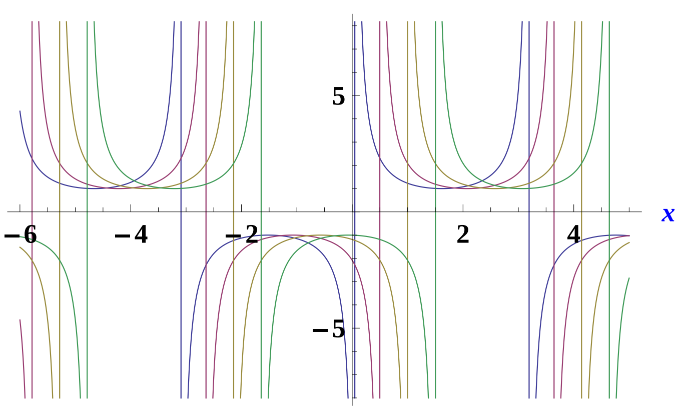

FIGURE 3. Profile solutions of the modified KdV equation when $-5 < x < 5$ and $t = 0.1, 1, 2, 3$ and $\mu = 1, c = 0.5$. The Sine method

3. Conclusion

The Sine-Cosine method has been successfully implemented to establish Periodic and solitary wave solutions of the Degasperis-Procesi equation and the modified KdV equation. The

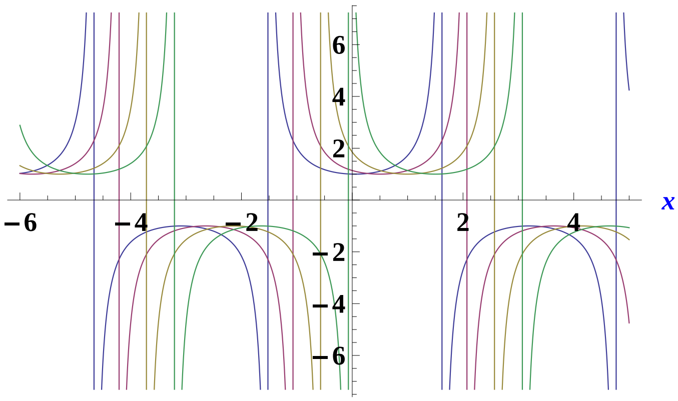

FIGURE 4. Profile solutions of the modified KdV equation when $-5 < x < 5$ and $t =$ 0.1, 1, 2, 3 and $\mu = 1, c = 0.5$. The Cosine method

proposed method is powerful tool that convert the nonlinear partial differential equations into a system of algebraic equation that can be solved by symbolic computation like Mathematica.

Conflict of Interests

The authors declare that there is no conflict of interests.

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