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INELASTIC FLOWS OF CURVES IN 4D GALILEAN SPACE

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Abstract: The flow of a curve is said to be inelastic, if in the former case, the arc-length is preserved, and in the latter case, if the intrinsic curvature is preserved. Physically, inelastic curve flow is characterized by the absence of any strain energy induced from the motion. In this paper, we investigate inextensible flows of curves in Galilean 4-space. Also, we give necessary and sufficient conditions for an inelastic curve flow are first expressed as a partial differential equation involving the curvature and torsion.

Keywords: Galilean space, Inelastic flow, Frenet-Serret frame.

2000 AMS Subject Classification: 53A35

1. Introduction

The flow of a curve is called to be inelastic if the arc-length of a curve is preserved. Inelastic curve flows have growing importance in many applications such engineering, computer vision, structural mechanics and computer animation [2]. Physically, inelastic curve flows give rise to motion which no strain energy is induced. The swinging motion of a cord of fixed length can be described by inelastic curve flows.

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There are elastic and inelastic collisions, both the kinetic energy and momentum are conserved. In elastic collision, the kinetic energy is not conserved in the collision. However the momentum is conserved.

Curves are a natural shape that many users often wish to use in many different areas such as mathematicians, physicist and engineers. Recently, the study of the motion of inelastic curves has arisen in a number of diverse engineering applications.

Chirikjian G.S, Burdick J.W [1] studied applications of inelastic curve flows. Gage M, Hamilton R.S [4] and Grayson M.A [5] investigated shrinking of closed plane curves to a circle via the heat equation. Also, Kwon D.Y, Park F.C [8] derived the evolution equation for an inelastic plane and space curve. Recently, Yoon DW [14] studied inelastic flows according to equiform in Galilean space. Kiziltuğ, S [6] investigated flow curves in according to type-2 Bishop frame in Euclidean 3-space.

A Galilean space may be considered as the limit case of a pseudo-Euclidean space in which the isotropic cone degenerates to a plane. The limit transition corresponds to the limit transition from the special theory of relatively to classical mechanics [10].

In this paper, we derived inelastic flows of curve Galilean 4-space. Conditions for an inelastic curve flow were expressed as a partial differential equation involving the curvature and torsion. We hope that these result will be helpful to mathematicians who are specialized on this area.

2. Preliminaries

The Galilean space is a 3D complex projective space P3 in which the absolute figure $\{w, f, I_1, I_2\}$ consists of a real plane w (the absolute plane), a real line $f \subset w$ (the absolute plane) and two complex conjugate points $I_1, I_2 \in f$ (the absolute points).The study of mechanics of plane-parallel motions reduces to the study of a geometry of 3D space with coordinates $\{x, y, t\}$ are given by the motion formula [13]. This geometry is called 3D Galilean geometry. In [13], is explained that 4D Galilean geometry, which studies all properties invariant under motions of objects in space, is even more complex.

In addition it is stated that this geometry can be described more precisely as the study of those properties of 4D space with coordinates which are invariant under the general Galilean transformations as follows:

$$
\begin{aligned}
x' &= (\cos\beta\cos\alpha - \cos\gamma\sin\beta\sin\alpha) + (\sin\beta\cos\alpha - \cos\gamma\cos\beta\sin\alpha)y \\
&+(\sin\gamma\sin\alpha)z + (v\cos\delta_1)t + a, \\
y' &= -(\cos\beta\sin\alpha + \cos\gamma\sin\beta\cos\alpha)x + (-\sin\beta\sin\alpha + \cos\gamma\cos\beta\cos\alpha)y \\
&+(\sin\gamma\cos\alpha)z + (v\cos\delta_2)t + b, \\
z' &= (\sin\gamma\cos\alpha)x - (\sin\gamma\cos\beta)y + (\cos\gamma)z + (v\cos\delta_3)t + c, \\
t' &= t + d
\end{aligned}
$$

with $\cos^2\delta_1 + \cos^2\delta_2 + \cos^2\delta_3 = 1.$

Some fundamental properties of curves in 4D Galilean space, is given for the purpose of the requirements in the next section

A curve in $G_4 (I \subset \mathbb{R} \to G_4)$ is given as follows

$$\alpha(t) = (x(t), y(t), z(t), w(t)),$$

where $x(t), y(t), z(t), w(t) \in C^4$ (smooth functions) and $t \in I$. Let α be a curve in G_4, which is parameterized by arc-length $t = s$, and given in the following coordinate form

$$\alpha(s) = (s, y(s), z(s), w(s)).$$

In affine coordinates the Galilean scalar product between two points

$P_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4}), i = 1, 2$ is defined by

$$g(p_1,p_2)=\begin{Bmatrix}\left|x_{21}-x_{11}\right|, if x_{11}\neq x_{21}\\ \sqrt{(x_{22}-x_{11})^2+(x_{23}-x_{13})^2+(x_{24}-x_{14})^2}, if x_{11}\neq x_{21}\end{Bmatrix}$$

We define the Galilean cross product in G_4 for the vectors $a=(a_1,a_2,a_3,a_4)$,

$b=(b_1,b_2,b_3,b_4)$ and $c=(c_1,c_2,c_3,c_4)$ as follows:

$$a\times b\times c=\begin{vmatrix}0 & e_2 & e_3 & e_4\\ a_1 & a_2 & a_3 & a_4\\ b_1 & b_2 & b_3 & b_4\\ c_1 & c_2 & c_3 & c_4\end{vmatrix},$$

where e_i are the standard basic vectors.

In this paper, we shall denote the inner product of two vectors a,b in the sense of Galilean by the notation $\langle a,b\rangle_G$.

Let $\alpha(s)=(s,y(s),z(s),w(s))$ be a curve parameterized by arc-length s in G_4. For a α Frenet curve, the Frenet formulas can be given as following form

$$\begin{bmatrix}t'\\ n'\\ b'\\ e'\end{bmatrix}=\begin{bmatrix}0 & \kappa & 0 & 0\\ 0 & 0 & \tau & 0\\ 0 & -\tau & 0 & \sigma\\ 0 & 0 & -\sigma & 0\end{bmatrix}\begin{bmatrix}t\\ n\\ b\\ e\end{bmatrix}.$$

We can know that t,n,b,e are mutually orthogonal vector fields satisfying equations

$$\langle t,t\rangle_G=\langle n,n\rangle_G=\langle b,b\rangle_G=\langle e,e\rangle_G=1$$
$$\langle t,n\rangle_G=\langle t,b\rangle_G=\langle t,e\rangle_G=\langle n,b\rangle_G=\langle n,e\rangle_G=\langle b,e\rangle_G=0$$

3. Inelastic flows of curves in 4-dimensional Galilean space

We assume that $F:[0,l]\times[0,w]\to G_4$ is a one parameter family of smooth curve in the Galilean space G_4, where l is the arc-length of initial curve. let u be the curve parametrization variable, $0 \le u \le l$. We put $v = \left\| \frac{\partial F}{\partial u} \right\|$, from which the arc-length of F is defined by $s(u) = \int_0^u v du$. Also, the operator $\frac{\partial}{\partial s}$ is given in terms of u by $\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u}$ and the arc-length parameter is given by $\partial s = v\partial u$ [7].

Any flow of F can be given by

$$\frac{\partial F}{\partial t} = f_1 t + f_2 n + f_3 b + f_4 e. \tag{1}$$

We put $s(u,t) = \int_0^u v du$, it is called the arc-length variation of F. From this, the requirement that the curve not be subject to any elongation or compression can be by the condition

$$\frac{\partial}{\partial t} s(u,t) = \int_0^u \frac{\partial v}{\partial t} du = 0, \tag{2}$$

for all $u \in [0,l]$.

Definition 3.1 A curve evolution $F(u,t)$ and its flow $\frac{\partial F}{\partial t}$ in G_4 are said to be inelastic if

$$\frac{\partial}{\partial t}\left\| \frac{\partial F}{\partial u} \right\| = 0.$$

Theorem3.2 *(Necessary and Sufficient Conditions for an Inelastic Flow) Let* $\frac{\partial F}{\partial t} = f_1 t + f_2 n + f_3 b + f_4 e$ *be a flow of* F *in* G_4*. The flow is inelastic if and only if* f_1 *is constant.*

Proof

From the definition of F, we have

$$v^2 = \left\langle \frac{\partial F}{\partial u}, \frac{\partial F}{\partial u} \right\rangle. \tag{3}$$

Since u and t are independent coordinates, $\frac{\partial}{\partial u}$ and $\frac{\partial}{\partial t}$ commute. So, by differentiating of (3) we have

$$2v\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}\left\langle \frac{\partial F}{\partial u}, \frac{\partial F}{\partial u} \right\rangle.$$

On the other hand, changing $\frac{\partial}{\partial u}$ and $\frac{\partial}{\partial t}$, we get

$$v\frac{\partial v}{\partial t} = \left\langle \frac{\partial F}{\partial u}, \frac{\partial}{\partial u}(\frac{\partial F}{\partial t}) \right\rangle.$$

From (1), we obtain

$$v\frac{\partial v}{\partial t} = \left\langle \frac{\partial F}{\partial u}, \frac{\partial}{\partial u}(f_1 t + f_2 n + f_3 b + f_4 e) \right\rangle.$$

By the formula of the Frenet, we have

$$\frac{\partial v}{\partial t} = \left\langle t, (\frac{\partial f_1}{\partial u})t + (\frac{\partial f_2}{\partial u} + f_1 v\kappa - f_3 v\tau)n + (\frac{\partial f_3}{\partial u} + f_2 v\tau - f_4 v\sigma)b + (\frac{\partial f_4}{\partial u} + f_3 v\sigma)e \right\rangle.$$

Making necessary calculations from above equation, we get

$$\frac{\partial v}{\partial t} = \frac{\partial f_1}{\partial u}.$$

So, $\frac{\partial f_1}{\partial s} = 0$, f_1 const.

We now restrict ourselves to arc length parametrized curves. That is, $v = 1$ and the local coordinate u corresponds to the curve arc length s. We require the following lemma.

Lemma3.3 *Le* $\frac{\partial F}{\partial t} = f_1 t + f_2 n + f_3 b + f_4 e$ *be arc- length parametrized curve with the Frenet vectors* $\{t, n, b, e\}$ *in* G_4 *.If the flow of the curve* F *is inextensible, then the derivatives of* $\{t, n, b, e\}$ *with respect to* t *are*

$$\frac{\partial t}{\partial t}=(\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau)n+(\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma)b+(\frac{\partial f_4}{\partial s}+f_3\sigma)e,$$
$$\frac{\partial n}{\partial t}=(\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau)t+\psi_1 b+\psi_2 e,$$
$$\frac{\partial b}{\partial t}=(\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma)t-\psi_1 n+\psi_3 e,$$
$$\frac{\partial e}{\partial t}=(\frac{\partial f_4}{\partial s}+f_3\sigma)t-\psi_2 n-\psi_3 b,$$

where

$$\psi_1=\left\langle\frac{\partial n}{\partial t},b\right\rangle,\psi_2=\left\langle\frac{\partial n}{\partial t},e\right\rangle,\psi_3=\left\langle e,\frac{\partial b}{\partial t}\right\rangle.$$

Proof

Using the Frenet formula and Theorem1, we calculate

$$\frac{\partial t}{\partial t}=\frac{\partial}{\partial t}\frac{\partial F}{\partial s}=\frac{\partial}{\partial s}(f_1 t+f_2 n+f_3 b+f_4 e).$$

Thus, it is seen that

$$\frac{\partial t}{\partial t}=(\frac{\partial f_1}{\partial s})t+(\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau)n+(\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma)b+(\frac{\partial f_4}{\partial s}+f_3\sigma)e$$

Substituting $\frac{\partial f_1}{\partial s}=0$ in above equation, we get

$$\frac{\partial t}{\partial t}=(\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau)n+(\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma)b+(\frac{\partial f_4}{\partial s}+f_3\sigma)e.$$

Differentiating Frenet frame with respect to t, we obtain:

$$0=\frac{\partial}{\partial t}\langle t,n\rangle=\left\langle\frac{\partial t}{\partial t},n\right\rangle+\left\langle t,\frac{\partial n}{\partial t}\right\rangle$$
$$=\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau+\left\langle t,\frac{\partial n}{\partial t}\right\rangle$$
$$0=\frac{\partial}{\partial t}\langle t,b\rangle=\left\langle\frac{\partial t}{\partial t},b\right\rangle+\left\langle t,\frac{\partial b}{\partial t}\right\rangle$$
$$=\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma+\left\langle t,\frac{\partial b}{\partial t}\right\rangle$$

$$0=\frac{\partial}{\partial t}\langle t,e\rangle=\left\langle\frac{\partial t}{\partial t},e\right\rangle+\left\langle t,\frac{\partial e}{\partial t}\right\rangle$$
$$=\frac{\partial f_4}{\partial s}+f_3\sigma+\left\langle t,\frac{\partial b}{\partial t}\right\rangle$$
$$0=\frac{\partial}{\partial t}\langle n,b\rangle=\left\langle\frac{\partial n}{\partial t},b\right\rangle+\left\langle n,\frac{\partial b}{\partial t}\right\rangle$$
$$=\psi_1+\left\langle n,\frac{\partial b}{\partial t}\right\rangle$$
$$0=\frac{\partial}{\partial t}\langle n,e\rangle=\left\langle\frac{\partial n}{\partial t},e\right\rangle+\left\langle n,\frac{\partial e}{\partial t}\right\rangle$$
$$=\psi_2+\left\langle n,\frac{\partial e}{\partial t}\right\rangle$$
$$0=\frac{\partial}{\partial t}\langle b,e\rangle=\left\langle\frac{\partial b}{\partial t},e\right\rangle+\left\langle b,\frac{\partial e}{\partial t}\right\rangle$$
$$=\psi_3+\left\langle b,\frac{\partial e}{\partial t}\right\rangle$$

From the above equations and using

$$\left\langle\frac{\partial n}{\partial t},n\right\rangle=\left\langle\frac{\partial b}{\partial t},b\right\rangle=\left\langle\frac{\partial e}{\partial t},e\right\rangle=0,$$

we obtain

$$\frac{\partial n}{\partial t}=(\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau)t+\psi_1 b+\psi_2 e,$$
$$\frac{\partial b}{\partial t}=(\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma)t-\psi_1 n+\psi_3 e,$$

$$\frac{\partial e}{\partial t}=(\frac{\partial f_4}{\partial s}+f_3\sigma)t-\psi_2 n-\psi_3 b,$$

where

$$\psi_1=\left\langle\frac{\partial n}{\partial t},b\right\rangle,\psi_2=\left\langle\frac{\partial n}{\partial t},e\right\rangle,\psi_3=\left\langle e,\frac{\partial b}{\partial t}\right\rangle.$$

The following theorem states the conditions on the curvature and the torsion for the curve flow $F(u,t)$ to be inelastic

Theorem 3.4 *(Equations for inelastic Evolution) If the curve flow* $\frac{\partial F}{\partial t}= f_1t+ f_2n+ f_3b+ f_4e$ *is inelastic, then the following system of partial differential equations holds:*

$$\frac{\partial \kappa}{\partial t}=\frac{\partial^2 f_2}{\partial s^2}-\frac{\partial}{\partial s}(f_3\tau)+\frac{\partial}{\partial s}(f_1\kappa)-\frac{\partial f_3}{\partial s}\tau- f_2\tau^2+ f_4\tau\sigma .$$

Proof

Noting that $\frac{\partial}{\partial s}\frac{\partial t}{\partial t}=\frac{\partial}{\partial t}\frac{\partial t}{\partial s}$,

$$\begin{aligned}\frac{\partial}{\partial s}\frac{\partial t}{\partial t}&=\frac{\partial}{\partial s}((\frac{\partial f_2}{\partial s}+ f_1\kappa- f_3\tau)n+(\frac{\partial f_3}{\partial s}+ f_2\tau- f_4\sigma)b+(\frac{\partial f_4}{\partial s}+ f_3\sigma)e)\\&=(\frac{\partial^2 f_2}{\partial s^2}-\frac{\partial}{\partial s}(f_3\tau)+\frac{\partial}{\partial s}(f_1\kappa))n+(\frac{\partial f_2}{\partial s}+ f_1\kappa- f_3\tau)(\tau b)+(\frac{\partial^2 f_3}{\partial s^2}+\frac{\partial}{\partial s}(f_2\tau)-\frac{\partial}{\partial s}(f_4\sigma))b\\&+(\frac{\partial f_3}{\partial s}+ f_2\tau- f_4\sigma)(-\tau n+\sigma e)+(\frac{\partial^2 f_4}{\partial s^2}+\frac{\partial}{\partial s}(f_3\sigma))e+(\frac{\partial f_4}{\partial s}+ f_3\sigma)(-\sigma b)\end{aligned}$$

Making necessary arrangements from above equation, we get

$$\begin{aligned}\frac{\partial}{\partial s}\frac{\partial t}{\partial t}&=\left(\frac{\partial^2 f_2}{\partial s^2}-\frac{\partial}{\partial s}(f_3\tau)+\frac{\partial}{\partial s}(f_1\kappa)-\frac{\partial f_3}{\partial s}\tau- f_2\tau^2+ f_4\tau\sigma\right)n\\&+\left(\frac{\partial^2 f_3}{\partial s^2}+\frac{\partial}{\partial s}(f_2\tau)-\frac{\partial}{\partial s}(f_4\sigma)+\frac{\partial f_2}{\partial s}\tau+ f_1\tau\kappa- f_3\tau^2-\frac{\partial f_4}{\partial s}\sigma+ f_3\sigma^2\right)b .\\&+\left(\frac{\partial^2 f_4}{\partial s^2}+\frac{\partial}{\partial s}(f_3\sigma)+\frac{\partial f_3}{\partial s}\sigma+ f_2\tau\sigma- f_4\sigma^2\right)e\end{aligned}$$

By using lemma 2, we have the following equation:

$$\frac{\partial}{\partial t}\frac{\partial t}{\partial s}=\frac{\partial}{\partial t}(\kappa n)$$
$$=\left(\frac{\partial f_2}{\partial s}\kappa+f_1\kappa^2-f_3\kappa\tau\right)t+\frac{\partial\kappa}{\partial t}n+(\psi_1\kappa)b+(\psi_2\kappa)e$$

By combining the above two equations, we have

$$\frac{\partial\kappa}{\partial t}=\frac{\partial^2 f_2}{\partial s^2}-\frac{\partial}{\partial s}(f_3\tau)+\frac{\partial}{\partial s}(f_1\kappa)-\frac{\partial f_3}{\partial s}\tau-f_2\tau^2+f_4\tau\sigma\,.$$

Theorem 3.5 *(Equations for inelastic Evolution) If the curve flow* $\frac{\partial F}{\partial t}=f_1t+f_2n+f_3b+f_4e$ *is inelastic, then the following system of partial differential equations holds:*

$$\frac{\partial\tau}{\partial t}=\frac{\partial\psi_1}{\partial s}-\psi_2\sigma\,,$$
$$\frac{\partial^2 f_3}{\partial s^2}+\frac{\partial f_2}{\partial s}\tau-\frac{\partial f_4}{\partial s}\sigma=-f_1\kappa\tau+f_3\tau^2++f_3\sigma^2+\frac{\partial}{\partial s}(f_4\sigma)-\frac{\partial}{\partial s}(f_2\tau)\,,$$
$$\frac{\partial\sigma}{\partial t}=\psi_2\tau+\frac{\partial\psi_3}{\partial s}\,.$$

Proof

Noting that $\frac{\partial}{\partial s}\frac{\partial n}{\partial t}=\frac{\partial}{\partial t}\frac{\partial n}{\partial s}$,

$$\frac{\partial}{\partial s}\frac{\partial n}{\partial t}=\frac{\partial}{\partial s}\left(\left(\frac{\partial f_2}{\partial s}+f_1\kappa-f_3\tau\right)t+\psi_1b+\psi_2e\right)$$
$$=\left(\frac{\partial^2 f_2}{\partial s^2}-\frac{\partial}{\partial s}(f_3\tau)+\frac{\partial}{\partial s}(f_1\kappa)\right)t+\left(\frac{\partial f_2}{\partial s}\kappa+f_1\kappa^2-f_3\tau\kappa-\psi_1\tau\right)n$$
$$+\left(\frac{\partial\psi_1}{\partial s}-\psi_2\sigma\right)b+\left(\frac{\partial\psi_2}{\partial s}-\psi_1\sigma\right)e$$

Also,

$$\frac{\partial}{\partial t}\frac{\partial n}{\partial s}=\frac{\partial}{\partial t}(\tau b)$$
$$=\left(\frac{\partial f_3}{\partial s}\tau+f_2\tau^2-f_4\sigma\tau\right)t-(\psi_1\tau)n+\frac{\partial\tau}{\partial t}b+(\psi_3\tau)e$$

By combining the above two equations, we have

$$\frac{\partial\tau}{\partial t}=\frac{\partial\psi_1}{\partial s}-\psi_2\sigma\,.$$

Similarly

$$\frac{\partial}{\partial s}\frac{\partial b}{\partial t}=\frac{\partial}{\partial s}\left((\frac{\partial f_3}{\partial s}+f_2\tau-f_4\sigma)t-\psi_1 n+\psi_3 e\right)$$
$$=\left(\frac{\partial^2 f_3}{\partial s^2}+\frac{\partial}{\partial s}(f_2\tau)-\frac{\partial}{\partial s}(f_4\sigma)\right)t+\left(-\frac{\partial\psi_1}{\partial s}+\frac{\partial f_3}{\partial s}\kappa+f_2\tau\kappa-f_4\sigma\kappa\right)n$$
$$+(-\psi_1\tau-\psi_3\sigma)b+\left(\frac{\partial\psi_3}{\partial s}\right)e$$

$$\frac{\partial}{\partial t}\frac{\partial b}{\partial s}=\frac{\partial}{\partial t}(-\tau n+\sigma e)$$
$$=\left(-\frac{\partial f_2}{\partial s}\tau-f_1\kappa\tau+f_3\tau^2+\frac{\partial f_4}{\partial s}\sigma+f_3\sigma^2\right)t+\left(-\frac{\partial\tau}{\partial t}-\psi_2\sigma\right)n$$
$$+(-\psi_1\tau-\psi_3\sigma)b+\left(\frac{\partial\sigma}{\partial t}-\psi_2\tau\right)e$$

By combining the above two equations, we have

$$\frac{\partial^2 f_3}{\partial s^2}+\frac{\partial f_2}{\partial s}\tau-\frac{\partial f_4}{\partial s}\sigma=-f_1\kappa\tau+f_3\tau^2++f_3\sigma^2+\frac{\partial}{\partial s}(f_4\sigma)-\frac{\partial}{\partial s}(f_2\tau)\,.$$

Similarly

$$\frac{\partial}{\partial s}\frac{\partial e}{\partial t}=\frac{\partial}{\partial s}\left((\frac{\partial f_4}{\partial s}+f_3\sigma)t-\psi_2 n-\psi_3 b\right)$$

$$=\left(\frac{\partial^2 f_4}{\partial s^2}+\frac{\partial}{\partial s}(f_3\sigma)\right)t+\left(-\frac{\partial\psi_2}{\partial s}+\frac{\partial f_4}{\partial s}\kappa+f_3\kappa\sigma+\psi_3\tau\right)n$$

$$+(-\psi_2\tau-\frac{\partial\psi_3}{\partial s})b-(\psi_3\sigma)e$$

$$\frac{\partial}{\partial t}\frac{\partial e}{\partial s}=\frac{\partial}{\partial t}(-\sigma b)$$

$$=(-\frac{\partial f_3}{\partial s}\sigma+f_2\tau\sigma-f_4\sigma^2)t+(\psi_1\sigma)n-\frac{\partial\sigma}{\partial t}b-(\psi_3\sigma)e$$

By combining the above two equations, we have

$$\frac{\partial\sigma}{\partial t}=\psi_2\tau+\frac{\partial\psi_3}{\partial s}.$$

Conflict of Interests

The author declares that there is no conflict of interests.

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