Available online at http://scik.org

J. Math. Comput. Sci. 3 (2013), No. 6, 1586-1600

ISSN: 1927-5307

STOCHASTIC ANALYSIS OF A TWO UNIT WARM STANDBY SYSTEM WITH TWO PHASE REPAIR AND GEOMETRIC DISTRIBUTIONS OF THE EVENTS

RAKESH GUPTA*, AND ARTI TYAGI

Department of Statistics, Ch. Charan Singh University, Meerut-250004, India

Abstract: The paper deals with stochastic model of a two identical unit warm standby system with two modes- normal (N) and total failure (F) of a unit. The repair of a failed unit is completed in two phases. Two repairmen are always available with the system- Skilled and ordinary for the purpose of phase-1 and phase-2 repairs respectively. The failure and repair times of a unit are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Keywords: Regenerative point, reliability, MTSF, availability of system, busy period of repairmen, net expected profit.

2000 AMS Subject Classification: 90B25

1. Introduction

Two-unit redundant system models have been analyzed widely in the literature of reliability by several authors due to their vital existence in modern business and industries. To increase the system effectiveness parallel and standby redundancies are used by various researchers including [3,4,8,9]. The reliability of the system may further be enhanced by introducing the concept of repair and preventive maintenance. The authors as mentioned in references above have analyzed the reparable system models by assuming that a failed unit immediately goes into repair and after completing the repair it becomes as good as new. In real

*Corresponding author

Received September 20, 2013

existing situations we observed many times that the repair of a failed unit is completed in a number of phases. Keeping this fact in view, Khaled et al [2] analyzed a two identical unit cold standby system model assuming that a single repairman completes the repair of a failed unit in two phases. They have assumed the general distributions of time to failure and repair in both phases, but the equations developed in the paper for various characteristics are true only for exponential distributions i.e. for other then exponential distributions of failure and repair times the results obtain by Khaled et al are incorrect. Gupta [5] have studied a two identical unit cold standby system with two phase repair and preventive maintenance (P.M.). The PM of an operating unit starts after a fixed amount of time only when the other unit is in standby. In analyzing the system models by all the above authors, the continuous distributions of all the random variables have been considered. Only Gupta and Varshney [6,7] analyzed very elementary two-unit system models with discrete parametric Markov-chain. Sometimes we may come across the situations when the repair process of a failed unit is completed in various phases and for each phase a separate repairman is needed. For example - Upon failure of any electronic device such as computer, air conditioner, refrigerator or automobile etc, it may be attended first by skilled person to inspect the failed unit for identifying the faults and then the ordinary repairman performs its repair as per the directions of skilled repairman. More so, it is worth to mentioned that the study of warm standby is the generalization over the study of parallel and standby redundancies as the results of parallel and standby redundancies can easily be drown as the particular cases from the results of warm standby redundant system.

Keeping the above facts in view, the purpose of the present paper is to analyze a two identical unit warm standby system model assuming that the repair of a failed unit is completed in two phases-phase-I and phase-II by two different persons. In phase-I the failed unit is first attended by the skilled repairman to inspect and identify the faults and then in phase-II, the process of repair is performed by ordinary repairman. Both types of repairman are always available with the system. After completion of phase-I and phase-II repairs, unit becomes as good as new.The random variables denoting time to failures and repairs are taken of discrete nature having geometric distributions with different parameters. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.

iii) Point-wise and steady-state availability of the system as well as expected up time of the system during time (0, t-1).

iv) Expected busy period of skilled and ordinary repairman during time (0, t-1).

v) Net expected profit incurred by the system during a finite and steady-state are obtained.

2. JUSTIFICATION FOR CONSIDERATION OF GEOMETRIC DISTRIBUTIONS

The phenomena of discrete failure and repair time distributions may be observed in the following situation.

Let the continuous time period $(0, \infty)$ is divided as 0, 1, 2,…, n,… of equal distance on real line and the probability of failure of a unit during time (i, i+1); i = 0, 1, 2,….. is p, then the probability that the unit will fail during (t, t+1) i.e. after passing successfully t intervals of time is given by $p(1-p)^t$; t = 0, 1, 2,…

This is the p.m.f of geometric distribution. Similarly, if r denotes the probability that a failed unit is repaired during (i, i+1); i = 0, 1, 2,... then the probability that the unit will be repaired during (t, t+1) is given by $r(1-r)^t$; t = 0, 1, 2,….. In view of the discrete distributions, the stochastic model under study leads to the discrete parametric Markov-Chain with state space S_0 to S_5 as shown in section-4(b).

3. MODEL DESCRIPTION AND ASSUMPTIONS

i) The system comprises of two identical units. Initially, one unit is operative and other is kept into warm standby.

ii) A unit of the system has two modes- normal (N) and total failure (F).

iii) The repair of a failed unit is completed in two phases (phase-I and phase-II) i.e. a failed unit first enters in phase-I for its repair and after the completion of phase-I repair it enters into phase-II for final repair.

iv) Two repairmen (skilled and ordinary) are always available with the system. Skilled repairman is to performed phase-I repair while the ordinary repairman is available to carry out the phase-II repair.

v) The random variables denoting the failure times and repair times of phase-I and phase-II repairs are independent of discrete nature and follow geometric distributions with different parameters.

vi) After the repair a unit works as good as new.

4. NOTATIONS AND STATES OF THE SYSTEM

a) Notations :

pq^t : p.m.f. of failure time of operative unit; $p+q=1$.

$p_s q_s^t$: p.m.f. of failure time of standby unit; $p_s+q_s=1$.

$r_i s_i^t$: p.m.f. of repair time of unit for phase-I and phase-II respectively for i=1, 2 $\left(r_i+s_i=1\right)$.

$q_{ij}(\bullet), Q_{ij}(\bullet)$: p.m.f. and c.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : Steady state transition probability from state S_i to S_j.

$$p_{ij}=Q_{ij}(\infty)$$

$Z_i(t)$: Probability that the system sojourn in the state S_i up to epochs 0, 1, 2, . . . , (t-1)..

ψ_i : Mean sojourn time in state S_i.

$*, h$: Symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right]=q_{ij}^*(h)=\sum_{t=0}^{\infty} h^t q_{ij}(t)$$

b) Symbols for the states of the system:

N_0 / N_S : Unit is in N-mode and operative/standby.

F_{R1} / F_{W1} : Unit is in Total failure (F) mode and under phase-I repair/waits for phase-I repair.

F_{R2} / F_{W2} : Unit is in Total failure (F) mode and under phase-II repair/waits for phase-II repair.

With the help of above symbols the possible states of the system are:

$S_0 \equiv (N_O, N_S)$, $S_1 \equiv (F_{R1}, N_O)$ $S_2 \equiv (F_{R2}, N_O)$

$S_3 \equiv (F_{R1}, F_{W1})$, $S_4 \equiv (F_{R2}, F_{R1})$, $S_5 \equiv (F_{R2}, F_{W2})$

The transition diagram of the system model is shown in fig. 1.

5. TRANSITION PROBABILITIES

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments we have

$$Q_{01}(t) = \frac{pq_s + p_s q}{1 - qq_s}\left[1 - (qq_s)^{t+1}\right], \qquad Q_{03}(t) = \frac{pp_s}{1 - qq_s}\left[1 - (qq_s)^{t+1}\right]$$

$$Q_{12}(t) = \frac{r_1 q}{1 - s_1 q}\left[1 - (s_1 q)^{t+1}\right], \qquad Q_{13}(t) = \frac{ps_1}{1 - s_1 q}\left[1 - (s_1 q)^{t+1}\right]$$

$$Q_{14}(t) = \frac{r_1 p}{1 - s_1 q}\left[1 - (s_1 q)^{t+1}\right], \qquad Q_{20}(t) = \frac{r_2 q}{1 - s_2 q}\left[1 - (s_2 q)^{t+1}\right]$$

$$Q_{21}(t) = \frac{r_2 p}{1 - s_2 q}\left[1 - (s_2 q)^{t+1}\right], \qquad Q_{24}(t) = \frac{s_2 p}{1 - s_2 q}\left[1 - (s_2 q)^{t+1}\right]$$

$$Q_{34}(t) = 1 - s_1^{t+1}, \qquad Q_{41}(t) = \frac{s_1 r_2}{1 - s_1 s_2}\left[1 - (s_1 s_2)^{t+1}\right]$$

$$Q_{42}(t) = \frac{r_1 r_2}{1 - s_1 s_2}\left[1 - (s_1 s_2)^{t+1}\right], \qquad Q_{45}(t) = \frac{r_1 s_2}{1 - s_1 s_2}\left[1 - (s_1 s_2)^{t+1}\right]$$

$$Q_{52}(t) = 1 - s_2^{t+1} \tag{1-13}$$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-13) by taking $t \to \infty$, as follows:

$$p_{01} = \frac{pq_s + qp_s}{1 - qq_s}, \quad p_{03} = \frac{pp_s}{1 - qq_s}, \quad p_{12} = \frac{r_1 q}{1 - s_1 q}, \quad p_{13} = \frac{s_1 p}{1 - s_1 q}$$

$$p_{14} = \frac{r_1 p}{1 - s_1 q}, \quad p_{20} = \frac{r_2 q}{1 - s_2 q}, \quad p_{21} = \frac{r_2 p}{1 - s_2 q}, \quad p_{24} = \frac{s_2 p}{1 - s_2 q}$$

$p_{34}=1$, $p_{41}=\dfrac{s_1r_2}{1-s_1s_2}$, $p_{42}=\dfrac{r_1r_2}{1-s_1s_2}$, $p_{45}=\dfrac{r_1s_2}{1-s_1s_2}$

$p_{52}=1$

We observe that the following relations hold-

$$p_{01}+p_{03}=1,\quad p_{12}+p_{13}+p_{14}=1,\quad p_{20}+p_{21}+p_{24}=1$$

$$p_{34}=p_{52}=1,\quad p_{41}+p_{42}+p_{45}=1 \qquad (14\text{-}18)$$

6. MEAN SOJOURN TIMES

Let T_i be the sojourn time in state S_i (i=0, 1, 2, 3, 4, 5) then ψ_i mean sojourn time in state S_i is given by

$$\psi_i=\sum_{t=1}^{\infty}P\left[T\geq t\right]$$

In particular,

$$\psi_0=\frac{qq_s}{1-qq_s},\quad \psi_1=\frac{qs_1}{1-qs_1},\quad \psi_2=\frac{qs_2}{1-qs_2}$$

$$\psi_3=\frac{s_1}{r_1},\quad \psi_4=\frac{s_1s_2}{1-s_1s_2},\quad \psi_5=\frac{s_2}{r_2} \qquad (19\text{-}24)$$

7. METHODOLOGY FOR DEVELOPING EQUATIONS

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system-

Here we define $R_i\left(t\right)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i. To determine it, we regard the failed state S_3, S_4 and S_5 as absorbing state. Now, the expression for $R_i\left(t\right)$; i=0, 1, 2 : we have the following set of convolution equations.

$$R_0\left(t\right)=q^tq_S^t+\sum_{u=0}^{t-1}q_{01}\left(u\right)duR_1\left(t-1-u\right)$$

$$= Z_0(t) + q_{01}(t-1) © R_1(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{12}(t-1) © R_2(t-1)$$

$$R_2(t) = Z_2(t) + q_{20}(t-1) © R_0(t-1) + q_{21}(t-1) © R_1(t-1) \quad (25\text{-}27)$$

Where,

$$Z_1(t) = p^t r_1^t, \qquad Z_2(t) = p^t r_2^t$$

b) Availability of the system-

Let $A_i(t)$ be the respective probabilities that the system is up at epoch (t-1) when it initially started from state S_i . Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t)$; i=0 to 5.

$$A_0(t) = Z_0(t) + q_{01}(t-1) © A_1(t-1) + q_{03}(t-1) © A_3(t-1)$$

$$A_1(t) = Z_1(t) + q_{12}(t-1) © A_2(t-1) + q_{13}(t-1) © A_3(t-1) + q_{14}(t-1) © A_4(t-1)$$

$$A_2(t) = Z_2(t) + q_{20}(t-1) © A_0(t-1) + q_{21}(t-1) © A_1(t-1) + q_{24}(t-1) © A_4(t-1)$$

$$A_3(t) = q_{34}(t-1) © A_4(t-1)$$

$$A_4(t) = q_{41}(t-1) © A_1(t-1) + q_{42}(t-1) © A_2(t-1) + q_{45}(t-1) © A_5(t-1)$$

$$A_5(t) = q_{52}(t-1) © A_2(t-1) \quad (28\text{-}33)$$

Where,

The values of $Z_i(t)$; i=0 to 2 are same as given in section 6(a).

c) Busy period of repairman-

Let $B_i^S(t)$ and $B_i^O(t)$ be the respective probabilities that the skilled repairman and ordinary repairman is busy at epoch (t-1) in the phase-I repair and phase-II repair of failed unit when system initially starts from state S_i . Using simple probabilistic arguments in case of reliability analysis, the relations for $B_i^k(t)$; i=0 to 5 and k=S, O can be easily developed as below.

$$B_0^k(t)=q_{01}(t-1)©B_1^k(t-1)+q_{03}(t-1)©B_3^k(t-1)$$
$$B_1^k(t)=(1-\delta)Z_1(t)+q_{12}(t-1)©B_2^k(t-1)+q_{13}(t-1)©B_3^k(t-1)+q_{14}(t-1)©B_4^k(t-1)$$
$$B_2^k(t)=\delta Z_2+q_{20}(t-1)©B_0^k(t-1)+q_{21}(t-1)©B_1^k(t-1)+q_{24}(t-1)©B_4^k(t-1)$$
$$B_3^k(t)=(1-\delta)Z_3(t)+q_{34}(t-1)©B_4^k(t-1)$$
$$B_4^k(t)=Z_4(t)+q_{41}(t-1)©B_1^k(t-1)+q_{42}(t-1)©B_2^k(t-1)+q_{45}(t-1)©B_5^k(t-1)$$
$$B_5^k(t)=\delta Z_5(t)+q_{52}(t-1)©B_2^k(t-1) \qquad (34\text{-}39)$$

Where,

$\delta=0$ and 1 respectively for k=O and S. The values of $Z_i(t)$; i=1, 2 are same as given in section 6(a) and $Z_3(t)=s_1^t$, $Z_4(t)=s_1^t s_2^t$, $Z_5(t)=s_2^t$

8. ANALYSIS OF RELIABILITY AND MTSF

Taking geometric transform of (25-27) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h)=\frac{N_1(h)}{D_1(h)} \qquad (40)$$

Where,

$$N_1(h)=\left[1-h^2q_{12}^*q_{21}^*\right]Z_0^*+hq_{01}^*Z_1^*+h^2q_{01}^*q_{12}^*Z_2^*$$
$$D_1(h)=1-h^2q_{12}^*q_{21}^*-h^3q_{01}^*q_{12}^*q_{20}^*$$

Collecting the coefficient of h^t from expression (41), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to 1}R_0^*(h)=\frac{N_1(1)}{D_1(1)}\text{-}1 \qquad (41)$$

Where,

$$N_1(1)=(1-p_{21}p_{12})\psi_0+p_{01}\psi_1+p_{01}p_{12}\psi_2$$

$$D_1(1) = 1 - p_{12}(p_{21} + p_{20}p_{01})$$

9. AVAILABILITY ANALYSIS

On taking geometric transform of (28-33) and simplifying the resulting equations for we get

$$A_0(h) = N_2(h)/D_2(h) \tag{42}$$

Where,

$$N_2(h) = \Big[1 - h^2 q_{24}^* q_{42}^* - h^3 q_{24}^* q_{45}^* q_{52}^* - hq_{21}^* \left\{hq_{12}^* + \left(h^2 q_{13}^* q_{34}^* + hq_{14}^*\right)\left(hq_{42}^* + h^2 q_{45}^* q_{52}^*\right)\right\}$$
$$- hq_{41}^* \left\{h^2 q_{12}^* q_{24}^* + h^2 q_{13}^* q_{34}^* + hq_{14}^*\right\}\Big] Z_0^* + \Big[hq_{01}^* \left\{1 - hq_{24}^* \left(hq_{42}^* + h^2 q_{45}^* q_{52}^*\right)\right\}$$
$$+ h^2 q_{03}^* q_{34}^* \left\{hq_{21}^* \left(hq_{42}^* + h^2 q_{45}^* q_{52}^*\right) + hq_{41}^*\right\}\Big] Z_1^* + \Big[hq_{01}^* \Big\{hq_{12}^* + \left(h^2 q_{13}^* q_{34}^* + hq_{14}^*\right)$$
$$\left(hq_{42}^* + h^2 q_{45}^* q_{52}^*\right)\Big\} + h^2 q_{03}^* q_{34}^* \left\{hq_{42}^* + h^2 q_{45}^* q_{52}^* + h^2 q_{41}^* q_{12}^*\right\}\Big] Z_2^*$$

and

$$D_2(h) = 1 - h^2 q_{24}^* q_{42}^* - h^3 q_{24}^* q_{45}^* q_{52}^* - hq_{21}^* \left\{hq_{12}^* + \left(h^2 q_{13}^* q_{34}^* + hq_{14}^*\right)\left(hq_{42}^* + h^2 q_{45}^* q_{52}^*\right)\right\}$$
$$- hq_{41}^* \left\{h^2 q_{12}^* q_{24}^* + h^2 q_{13}^* q_{34}^* + hq_{14}^*\right\}\Big] - hq_{20}^* \Big[hq_{01}^* \Big\{hq_{12}^* + \left(h^2 q_{13}^* q_{34}^* + hq_{14}^*\right)$$
$$\left(hq_{42}^* + h^2 q_{45}^* q_{52}^*\right)\Big\} + h^2 q_{03}^* q_{34}^* \left\{hq_{42}^* + h^2 q_{45}^* q_{52}^* + h^2 q_{41}^* q_{12}^*\right\}\Big]$$

The steady state availabilities of the system due to operation of the operative unit of the system is given by-

$$A_0 = \lim_{t \to \infty} A_0(t) = \lim_{h \to 1} (1 - h) \frac{N_2(h)}{D_2(h)}$$

Now since $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule we get

$$A_0 = -N_2(1)/D_2'(1) \tag{43}$$

Where,

$$N_2(1) = \left[1 - p_{41}(1 - p_{12})\right](p_{20}\psi_0 + \psi_2) + \left[(p_{21} + p_{20}p_{01})(1 - p_{41}) + p_{41}\right]\psi_1$$

and

$$D_2'(1) = -\Big[\left\{1 - p_{41}(1 - p_{12})\right\}(p_{20}\psi_0 + \psi_2) + \left\{(p_{21} + p_{20}p_{01})(1 - p_{41}) + p_{41}\right\}\psi_1$$

$$+\left\{p_{13}\left(\left(p_{21}+p_{20}p_{01}\right)\left(1-p_{41}\right)+p_{41}\right)+p_{20}p_{03}\left(1-p_{41}\left(1-p_{12}\right)\right)\right\}\psi_3$$
$$+\left\{1-p_{12}\left(p_{21}+p_{20}p_{01}\right)\right\}\left(\psi_4+p_{45}\psi_5\right)\Big]$$

Now the expected uptime of the system due to operative unit upto epoch (t-1) is given by

$$\mu_{up}\left(t\right)=\sum_{x=0}^{t-1}A_0\left(x\right)$$

so that

$$\mu_{up}^{*}\left(h\right)=A_0^{*}\left(h\right)/\left(1-h\right) \tag{44}$$

10. BUSY PERIOD ANALYSIS

On taking geometric transform of (35-40) and simplifying the resulting equations for k=S and O we get

$$B_0^{S*}\left(h\right)=\frac{N_3\left(h\right)}{D_2\left(h\right)} \quad \text{and} \quad B_0^{O*}\left(h\right)=\frac{N_4\left(h\right)}{D_2\left(h\right)} \tag{45-46}$$

Where,

$$N_3\left(h\right)=\Big[hq_{01}^{*}\left\{1-hq_{24}^{*}\left(hq_{42}^{*}+h^2q_{45}^{*}q_{52}^{*}\right)\right\}-h^2q_{03}^{*}q_{34}^{*}\left\{hq_{21}^{*}\left(hq_{42}^{*}+h^2q_{45}^{*}q_{52}^{*}\right)+hq_{41}^{*}\right\}\Big]Z_1^{*}$$
$$+\Big[hq_{01}^{*}q_{13}^{*}\left\{1-hq_{24}^{*}\left(hq_{42}^{*}+h^2q_{45}^{*}q_{52}^{*}\right)\right\}+hq_{03}^{*}\left\{1-hq_{24}^{*}\left(hq_{42}^{*}+h^2q_{45}^{*}q_{52}^{*}\right)\right.$$
$$\left.-hq_{12}^{*}\left(hq_{21}^{*}+h^2q_{24}^{*}q_{41}^{*}\right)-h^2q_{21}^{*}q_{14}^{*}\left(hq_{42}^{*}+h^2q_{45}^{*}q_{54}^{*}\right)+h^2q_{14}^{*}q_{41}^{*}\right\}\Big]Z_3^{*}$$
$$+\Big[hq_{01}^{*}\left\{h^2q_{12}^{*}q_{24}^{*}+h^2q_{13}^{*}q_{34}^{*}+hq_{14}^{*}\right\}+hq_{03}^{*}\left\{hq_{34}^{*}\left(1-h^2q_{12}^{*}q_{21}^{*}\right)\right\}\Big]Z_4^{*}$$

$$N_4\left(h\right)=\Big[hq_{01}^{*}\left\{hq_{12}^{*}+\left(h^2q_{13}^{*}q_{34}^{*}+hq_{14}^{*}\right)\left(hq_{42}^{*}+h^2q_{45}^{*}q_{52}^{*}\right)\right\}+h^2q_{03}^{*}q_{34}^{*}\left\{hq_{42}^{*}+h^2q_{45}^{*}q_{52}^{*}\right.$$
$$\left.+h^2q_{41}^{*}q_{12}^{*}\right\}\Big]Z_2^{*}+\Big[hq_{01}^{*}\left\{h^2q_{12}^{*}q_{24}^{*}+h^2q_{13}^{*}q_{34}^{*}+hq_{14}^{*}\right\}$$
$$+hq_{03}^{*}\left\{hq_{34}^{*}\left(1-h^2q_{12}^{*}q_{21}^{*}\right)\right\}\Big]\left(Z_4^{*}+q_{45}^{*}Z_5^{*}\right)$$

and $D_2\left(h\right)$ is same as in availability analysis.

In the long run the respective probabilities that the skilled repairman and ordinary repairman is busy in the repair of failed unit are given by-

$$B_0^S = \lim_{t\to\infty} B_o^S(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

$$B_0^O = \lim_{t\to\infty} B_o^O(t) = \lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^S = -\frac{N_3(1)}{D_2'(1)} \quad \text{and} \quad B_0^O = -\frac{N_4(1)}{D_2'(1)} \tag{47-48}$$

Where,

$$N_3(1) = \left[(p_{21}+p_{20}p_{01})(1-p_{41})+p_{41}\right]\psi_1 + \left[p_{13}\left((p_{21}+p_{20}p_{01})(1-p_{41})+p_{41}\right)\right.$$
$$\left.+p_{20}p_{03}\left(1-p_{41}(1-p_{12})\right)\right]\psi_3 + \psi_4\left[1-p_{12}(p_{21}+p_{20}p_{01})\right]$$

$$N_4(1) = \left[1-p_{41}(1-p_{12})\right]\psi_2 + \left[1-p_{12}(p_{21}+p_{20}p_{01})\right](\psi_4+p_{45}\psi_5)$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of the skilled repairman and ordinary repairman in repair of failed unit up to epoch (t-1) are respectively given by-

$$\mu_b^S(t) = \sum_{x=0}^{t-1} B_0^S(x), \qquad \mu_b^O(t) = \sum_{x=0}^{t-1} B_0^O(x)$$

So that,

$$\mu_b^{S*}(h) = \frac{B_0^{S*}(h)}{(1-h)}, \qquad \mu_b^{O*}(h) = \frac{B_0^{O*}(h)}{(1-h)} \tag{49-50}$$

11. PROFIT FUNCTION ANALYSIS

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section.

Let us consider,

K_0 =revenue per-unit time by the system when main unit is operative unit.

K_1 =cost per-unit time when skilled repairman is busy in the repairing phase-I repair of failed unit.

K_2 =cost per-unit time when ordinary repairman is busy in the repairing phase-II repair of failed unit.

Then, the net expected profit incurred up to epoch (t-1) given by

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^S(t) - K_2\mu_b^O(t) \quad (51)$$

The expected profit per unit time in steady state is given by-

$$P = \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{h\to 1}(1-h)^2 P^*(h)$$

$$= K_0 \lim_{h\to 1}(1-h)^2 \frac{A_0^*(h)}{(1-h)} - K_1 \lim_{h\to 1}(1-h)^2 \frac{B_0^{S*}(h)}{(1-h)} - K_2 \lim_{h\to 1}(1-h)^2 \frac{B_0^{O*}(h)}{(1-h)}$$

$$= K_0A_0 - K_1B_0^S - K_2B_0^O \quad (52)$$

Remark:

All the results obtained under study approach to the results of two-identical unit cold standby system with two-phase repair and discrete distribution of random variables if we consider distribution of random variables if we consider $p_s = 0$. Further, if we assume $p_s = p$ then we get the results of a discrete parametric Markov-chain model of a two identical unit parallel system with two phase repair. In view of this our work is generalization of these two research studies.

12. GRAPHICAL REPRESENTATION

The curves for MTSF and profit function have been drawn for different values of parameters. Fig.2 depicts the variations in MTSF with respect to failure rate (p) of operative unit for different values of repair rate (r_1) of first phase repair and failure rate p_s of standby unit when repair rate of second phase repair is kept fixed as $r_2 = 0.07$.The smooth curves shows the trends for three different values 0.04, 0.10 and 0.20 of r_1 when p_s is taken as 0.005 where as dotted curves shows the trends for same three values of r_1 as above when p_s is taken as 0.010. From the curves we observed that MTSF decreases uniformly as the values of p and p_s increase and increases with the increase in r_1.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to p for varying values of r_1 and p_s as taken in case of MTSF, when the values of other parameters are kept fix as $r_2 = 0.07$, $K_0 = 180$, $K_1 = 320$ and $K_2 = 200$. From this figure same trends in respect of p, r_1 and p_s have been observed as in MTSF. Further it is also revealed by smooth curves that system is profitable only if p is less than 0.0143, 0.0290 and 0.0410 respectively for $r_1 = 0.04$, 0.10 and 0.20 for fixed $p_s = 0.005$. From dotted curves that system is profitable only if 'p' is less than 0.0115, 0.0260 and 0.0380 respectively for $r_1 = 0.04$, 0.10 and 0.20 for fixed $p_s = 0.010$.

Conflict of Interests

The author declares that there is no conflict of interests.

REFERENCES

1. M. El-Said Khaled and M. S. EL-Sherbeny, Stochastic Analysis of a Two-unit Cold Standby System with Two-Stage Repair and Waiting Time. Sankhya-B 72(1):1-10 (2010).
2. R. Gupta and Shivakar, Cost Benefit Analysis of a Two-Unit Parallel System with Correlated Failure and Repair Times. IAPQR Transaction 35 (2):1-10 (2010).
3. R. Gupta, Probabilistic analysis of a two-unit cold standby system with two-phase repair and preventive maintenance, Microelectron Reliab., Vol.-26, No. 1, pp. 13-18(1986).
4. R. Gupta and G. Varshney, A Two Non-Identical Unit Parallel System with Geometric Failure and Repair Time Distributions, IAPQR Trans., Vol.31, No. 2 (Science), pp. 127-139(2006).
5. R. Gupta and G. Varshney, A Two Identical Unit Parallel System with Geometric Failure and Repair Time Distributions, J. of Comb. Info. And Sciences, Vol. 32, No. 1-4, pp. 127-136 (2007).
6. R. Gupta, S.C. Sharma and Vikas Saxena, A Two Non-Identical Unit Cold Standby System with Random Change, Imperfect Switching Device and Correlated Failure and Repair of Units, Int. Transactions in Mathematical Sciences and Computers, Vol. 13, No. 2, pp. 287-296 (2010).
7. G. S. Mokaddis, M. S. El-Sherbeny, and Entesar Al-Esayeh, A dissimilar two-unit parallel system with common-cause failure and preventive maintenance, The Journal of Mathematics & Computer Science, vol. 19, no. 2, pp. 153–167(2008).
8. V. Sridharan and T. V. Kalyani, Stochastic analysis of a non-identical two-unit parallel system with common-cause failure using GERT technique, International Journal of Information and Management Sciences, vol. 13, no. 1, pp. 49–57(2002).

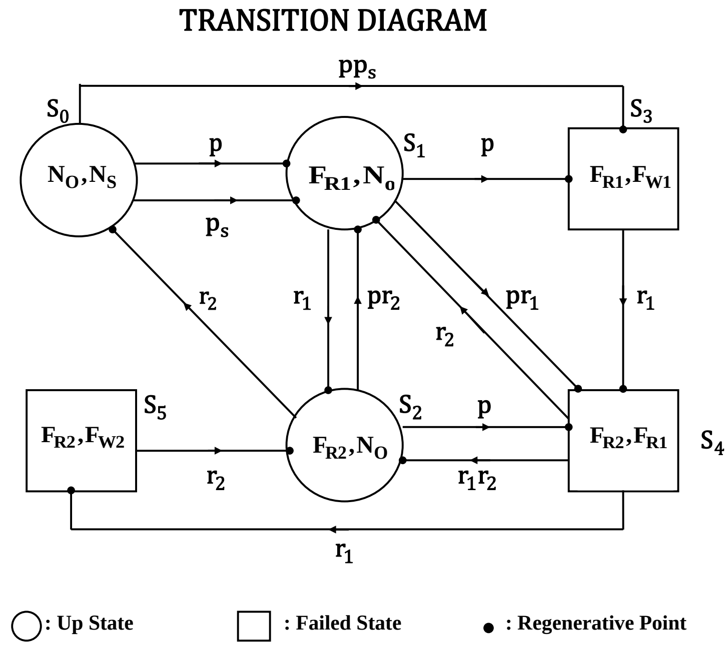
TRANSITION DIAGRAM
pp_s
S_0
N_O, N_S
p
p_s
S_1
F_{R1}, N_o
p
S_3
F_{R1}, F_{W1}
r_2
r_1
pr_2
pr_1
r_2
r_1
S_5
F_{R2}, F_{W2}
r_2
S_2
F_{R2}, N_O
p
$r_1 r_2$
F_{R2}, F_{R1}
S_4
r_1
: Up State
: Failed State
: Regenerative Point

Fig.1

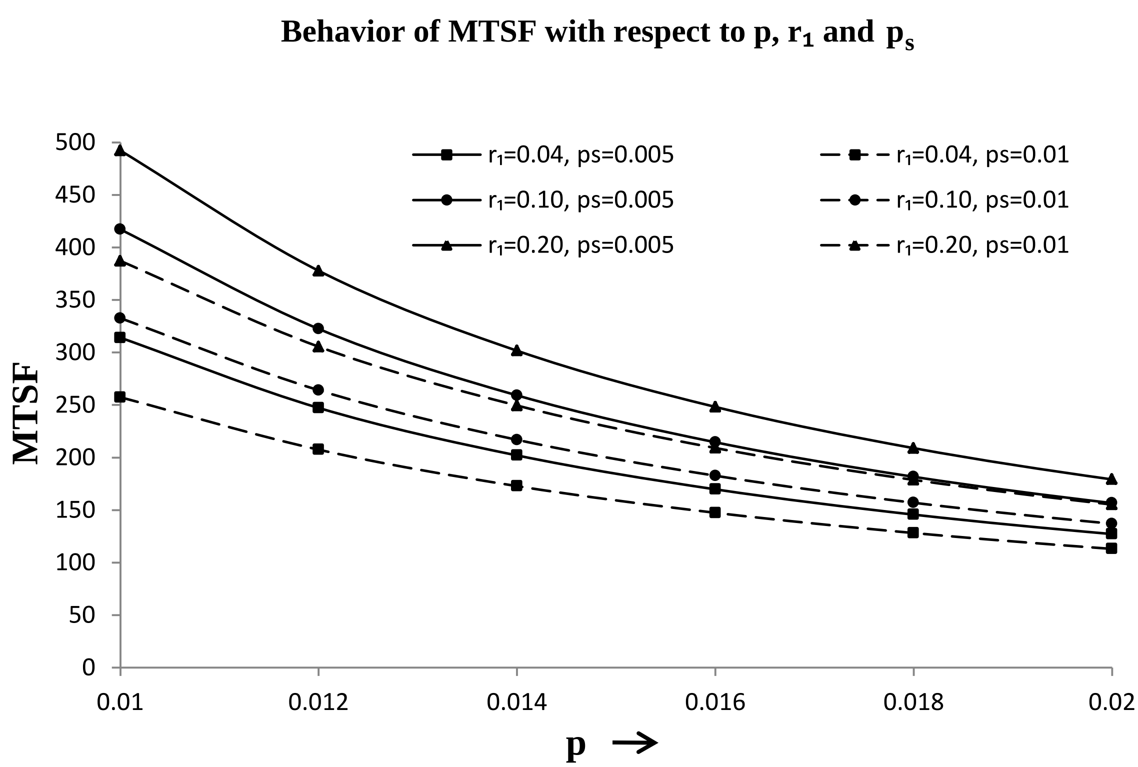
Behavior of MTSF with respect to p, r_1 and p_s
r_1=0.04, ps=0.005
r_1=0.04, ps=0.01
r_1=0.10, ps=0.005
r_1=0.10, ps=0.01
r_1=0.20, ps=0.005
r_1=0.20, ps=0.01
MTSF
500
450
400
350
300
250
200
150
100
50
0
0.01
0.012
0.014
0.016
0.018
0.02
p →

Fig. 2

Behavior of Profit (P) with respect to p, r_1 and p_s

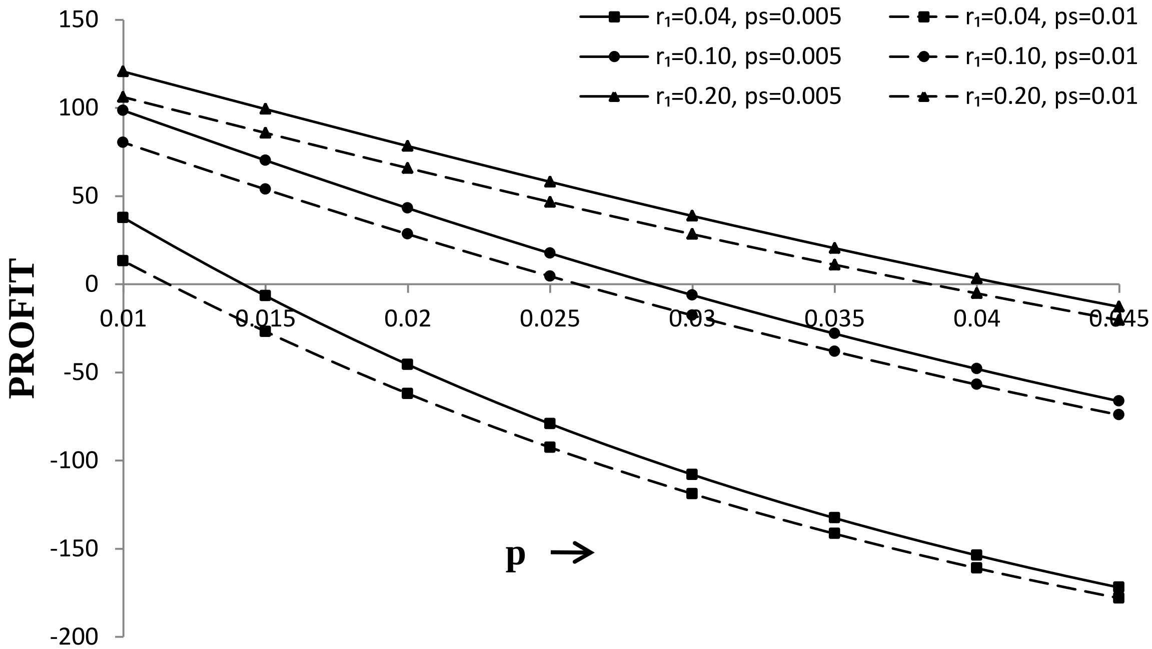

Fig. 3