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RELATIONS ON FUZZY SOFT SET

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Abstract. The aim of this paper is to introduce the concept of relation on fuzzy soft sets. We have studied some related properties and also put forward some propositions on fuzzy soft relations with proofs and examples. Finally the notions of symmetric, transitive, reflexive, irreflexive and equivalence fuzzy soft relations have been established in our work.

Keywords: Fuzzy Set, Soft Set, Fuzzy Soft Set, Fuzzy Soft Relation.

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1. Introduction

In real world, the complexity generally arises from uncertainty in the form of ambiguity. Uncertainty may arise due to partial information about the problem, or due to information which is not fully reliable, or due to receipt of information from more than one source. Fuzzy set theory, Rough set theory, Vague set theory, theory of probability are mathematical tools to deal with uncertainty arising due to vagueness. In 1965, Zadeh [5] introduced the notion of fuzzy set theory. Later in 1999, Moldstov [3] initiated the novel concept of soft set theory which can be considered as a

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generalization of fuzzy sets initiated by Zadeh [5]. Maji, Biswas and Roy [2] studied the theory of soft sets in 2003 and put forward definitions of equality of two soft sets, subset and super set of a soft set and complement of a soft set.

In this paper we give the proof of some propositions of fuzzy soft relation and support them with examples. We have also discussed the notions of symmetric, transitive, reflexive, irreflexive and equivalence fuzzy soft relations and some algebraic properties of fuzzy soft relation.

The content of this paper is as follows:

In section 2, some basic definitions with examples and preliminary results are given which would be used in the rest of the paper. In section 3, intersection and union of fuzzy soft sets have been redefined.

2. Preliminaries

Definition 2.1 [3]

A pair (*F*, *E*) is called a soft set over *U* if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε-approximate elements of the soft set.

Example 2.1

Let $U=\{S_1,S_2,S_3,S_4\}$ be the set of students under consideration and

$E = \{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)} be the set of parameters and $A=\{e_1,e_3,e_4\} \subseteq E$, Then

$(F, A) = F\{F(e_1) = \{S_1,S_3\}, F(e_3) = \{S_1,S_2,S_4\}, F(e_4) = \{S_4\}\}$ is the soft set representing over all expertness of the students.

Definition 2.2 [2]

A pair (F, A) is called a fuzzy soft set over U where $F: A \to \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$

Example 2.2

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

$E = \{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_3, e_4\} \subseteq E$,

Then

$(F, A) = \{F(e_1) = \{S_1/0.3, S_2/0.5, S_3/0.1, S_4/0.7\},$

$F(e_3) = \{S_1/0.4, S_2/0.2, S_3/0.6, S_4/0.1\},$

$F(e_4) = \{S_1/0.2, S_2/0.7, S_3/0.2, S_4/0.3\}\}$ is the fuzzy soft set representing overall expertness of the students.

Definition 2.3 [4]

A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is continuous t - norm if $*$ satisfies the following conditions.

(i) $*$ is commutative and associative

(ii) $*$ is continuous

(iii) $a * 1 = a \;\; \forall a \in [0,1]$

(iv) $a * b \leq c * d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$

An example of continuous t - norm is $a * b = ab$.

Definition 2.4 [4]

A binary operation $\Diamond: [0,1] \times [0,1] \to [0,1]$ is continuous t - conorm if $\Diamond$ satisfies the following conditions:

(i) $\lozenge$ is commutative and associative

(ii) $\lozenge$ is continuous

(iii) $a \lozenge 0 = a \quad \forall a \in [0,1]$

(iv) $a \lozenge b \le c \lozenge d$ whenever $a \le c, b \le d$ and $a,b,c,d \in [0,1]$

An example of continuous t - conorm is $a * b = a + b - ab$.

Definition 2.5 [2]

For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E), we say that (F, A) is a fuzzy soft subset of (G, B) if

(i) $A \subseteq B$

(ii) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \tilde{\subseteq} (G, B)$

Example 2.5

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

E= $\{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)} be the set of parameters and A=$\{e_1, e_2\} \subseteq E$ and B=$\{e_1, e_2, e_4\} \subseteq E$. We consider the fuzzy soft sets

(F, A)= $\{F(e_1) = \{S_1/0.7, S_2/0.1, S_3/0.2, S_4/0.6\}$,

$F(e_2) = \{S_1/0.8, S_2/0.6, S_3/0.1, S_4/0.5\}\}$

and

$(G, B) = \{G(e_1) = \{S_1/0.7, S_2/0.2, S_3/0.3, S_4/0.7\}$,

$G(e_2) = \{S_1/0.9, S_2/0.7, S_3/0.3, S_4/1\}$,

$G(e_4) = \{S_1/0.1, S_2/0.2, S_3/0.7, S_4/0.6\}\}$

Here $A \subseteq B$ and for all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$

Thus $(F, A) \tilde{\subseteq} (G, B)$.

Definition 2.6 [1]

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \varphi$. Then intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write $(F, A) \tilde{\cap} (G, B) = (H, C)$

Example 2.6

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

E= $\{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)$\}$ be the set of parameters and $A=\{e_1, e_2, e_4\} \subseteq E$ and $B=\{e_1, e_5\} \subseteq E$. Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$

$F(e_2) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\},$

$F(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$

$(G, B) = \{G(e_1) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\},$

$G(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Then $(F, A) \tilde{\cap} (G, B) = (H, C)$, where $C = A \cap B = \{e_1\}$

Thus $(H, C) = \{H(e_1) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\}\}$

Definition 2.7 [2]

Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a fuzzy soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Example 2.7

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

E = $\{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)$\}$ be the set of parameters and

$A = \{e_1, e_3, e_4\} \subseteq E$ and $B = \{e_1, e_5\} \subseteq E$. Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\}$,

$F(e_3) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}$,

$F(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$

$(G, B) = \{G(e_1) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\}$,

$G(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Then $(F, A)\tilde{\cup}(G, B) = (H, C)$, where $C = A \cup B = \{e_1, e_3, e_4, e_5\}$

$(H, C) = \{H(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\}$,

$H(e_3) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}$,

$H(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}$,

$H(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Definition 2.8 [2]

The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \to \tilde{P}(U)$ is a mapping given by $F^c(\sigma) = (F(\neg\sigma))^c \; \forall \sigma \in \neg A$.

Example 2.8

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and E = $\{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)} be the set of parameters and $A = \{e_1, e_2\} \subseteq E$

Then

$(F, A) = \{F(e_1) = \{S_1/0.7, S_2/0.1, S_3/0.2, S_4/0.6\},$

$F(e_2) = \{S_1/0.8, S_2/0.6, S_3/0.1, S_4/0.5\} \}$

$(F, A)^c = \{F^c(\neg e_1) = \{S_1/0.3, S_2/0.9, S_3/0.8, S_4/0.4\},$

$F^C(\neg e_2) = \{S_1/0.2, S_2/0.4, S_3/0.9, S_4/0.5 \}$

3. Fuzzy Soft Relation

Definition 3.1

Let (F, A) and (G, B) be two fuzzy soft sets in the fuzzy soft class (U, E). Their intersection is defined as the fuzzy soft set (H, C) where $C = A \cap B$ and $H(\alpha) = F(\alpha) * G(\alpha) \; \forall \alpha \in C$, where * is the operation t - norm between the fuzzy sets $F(\alpha)$ and $G(\alpha)$.

Example 3.1

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

E= $\{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)} be the set of parameters and A=$\{e_1, e_2, e_3, e_4\} \subseteq E$ and B=$\{e_1, e_5\} \subseteq E$ Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$

$$F(e_2)=\{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\},$$

$$F(e_3)=\{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\},$$

$$F(e_4)=\{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$$

$$(G,B)=\{G(e_1)=\{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\},$$

$$G(e_5)=\{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$$

$$(F\,\tilde{\cap}\,G)(e_1) = \{S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64\}$$

Definition 3.2

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in the fuzzy soft class (*U*, *E*). Their union is defined as the fuzzy soft set (*H*, *C*) = (*F*, *A*) Ũ (*G*, *B*) where $C = A \cup B$ and

$$H(\alpha) = \begin{cases} F(\alpha) & \text{if } \alpha \in A - B \\ G(\alpha) & \text{if } \alpha \in B - A \\ F(\alpha) \Diamond G(\alpha) & \text{if } \alpha \in B \cap A \end{cases}$$

Where $\Diamond$ is the operation $t-$conorm between the fuzzy sets $F(\alpha)$ and $G(\alpha)$.

Example 3.2

From **Example 3.1**

$(F\,\tilde{\cup}\,G)(e_1) = \{S_1/0.98, S_2/0.36, S_3/0.80, S_4/0.96\}$,

$(F\,\tilde{\cup}\,G)(e_2) = F(e_2)$,

$(F\,\tilde{\cup}\,G)(e_3) = F(e_3)$,

$(F\,\tilde{\cup}\,G)(e_4) = F(e_4)$,

$(F\,\tilde{\cup}\,G)(e_5) = G(e_5)$.

Definition 3.3

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in the fuzzy soft class (*U*, *E*). Their certesian product is defined as (*F*, *A*) × (*G*, *B*) = (*H*, *C*) Where $C = A \times B$ and

$H(\alpha,\beta) = F(\alpha) * G(\beta)\ \forall(\alpha,\beta) \in C$

Here * is the operation t - norm between the fuzzy sets $F(\alpha)$ and $G(\alpha)$.

Example 3.3

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

E= $\{e_1$ (expertise in english), e_2 (expertise in mathematics), e_3 (expertise in chemistry), e_4 (expertise in computer science)} be the set of parameters and A=$\{e_1, e_2, e_3, e_4\} \subseteq E$ and B=$\{e_1, e_5\} \subseteq E$. Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\}$,

$F(e_2) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\}$,

$F(e_3) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}$,

$F(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$

$(G, B) = \{G(e_1) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\}$,

$G(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Then $(F, A) \times (G, B) = (H, C)$ where $C = A \times B$ and

$(H, C) = \{H(e_1, e_1) = \{S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64\}$,

$H(e_1, e_5) = \{S_1/0.54, S_2/0.06, S_3/0.54, S_4/0.56\}$,

$H(e_2, e_1) = \{S_1/0.24, S_2/0.18, S_3/0.20, S_4/0.24\}$,

$H(e_2, e_5) = \{S_1/0.18, S_2/0.27, S_3/0.36, S_4/0.21\}$,

$H(e_3, e_1) = \{S_1/0.48, S_2/0.06, S_3/0.45, S_4/0.64\}$,

$H(e_3, e_5) = \{S_1/0.36, S_2/0.09, S_3/0.81, S_4/0.56\}$,

$H(e_4, e_1) = \{S_1/0.08, S_2/0.14, S_3/0.15, S_4/0.16\}$,

$H(e_4, e_5) = \{S_1/0.06, S_2/0.21, S_3/0.27, S_4/0.14\}\}$

Definition 3.4

Let (F, A) and (G, B) be two fuzzy soft sets in the fuzzy soft class (U, E). Then fuzzy soft relation (R, C) from (F, A) to (G, B) is a fuzzy soft subset of $(F, A) \times (G, B)$ where $C \subseteq A \times B$ and $(R, C) \cong (F, A) \times (G, B)$. If (R, C) is a fuzzy soft subset of (F, A) to (F, A), then it is called a fuzzy soft relation on (F, A).

Example 3.4

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

A= { expertise in physics (p), expertise in chemistry (c), expertise in biology (b)},

B = { expertise in physics (p), expertise in history (h)}

$$(F, A) = \{F(p) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$$

$$F(c) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\},$$

$$F(b) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}\}$$

$$(G, B) = \{G(p) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\},$$

$$G(h) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$$

We take $C = \{(p, p), (c, p), (b, p)\} \subseteq A \times B$

Let

$$(R, C) = \{R(p, p) = \{S_1/0.71, S_2/0.02, S_3/0.30, S_4/0.54\},$$

$$R(c, p) = \{S_1/0.14, S_2/0.18, S_3/0.10, S_4/0.14\},$$

$$R(b, p) = \{S_1/0.28, S_2/0.06, S_3/0.05, S_4/0.64\}$$

Clearly $(R, C) \cong (F, A) \times (G, B)$ and hence a fuzzy soft relation from (F, A) to (G, B).

Definition 3.5

A fuzzy soft relation R on (F, A) is said to be fuzzy soft symmetric relation if

$R(p, q) = R(q, p), \ \forall p, q \in A.$

Definition 3.6

Let R be fuzzy soft relation from (F, A) to (G, B) then inverse relation R^{-1} is defined as

$R^{-1}(p, q) = R(q, p), \ \forall p, q \in D \subseteq B \times A.$

Example 3.5

Define a fuzzy soft inverse relation R^{-1} from (G, B) to (F, A). From **Example 3.4**
$(p, p), (p, c), (p, b) \in D \subseteq B \times A.$

Then

$$(R^{-1}, D) = \{R^{-1}(p,p) = \{S_1/0.71, S_2/0.02, S_3/0.30, S_4/0.54\},$$
$$R^{-1}(p,c) = \{S_1/0.14, S_2/0.18, S_3/0.10, S_4/0.14\},$$
$$R^{-1}(p,b) = \{S_1/0.28, S_2/0.06, S_3/0.05, S_4/0.64\}$$

Proposition 3.1

If R is fuzzy soft relation from (F, A) to (G, B) then R^{-1} is a fuzzy soft relation from (G, B) to (F, A).

Proof

$R^{-1}(p, q) = R(q, p) \subseteq G(q) \cap F(p), \ \forall (p, q) \in C \subseteq B \times A.$

Hence R^{-1} is fuzzy soft relation from (G, B) to (F, A).

Proposition 3.2

If R and S be two fuzzy soft relations from (F, A) to (G, B) then

(i) $(R^{-1})^{-1} = R$

(ii) $R \,\tilde{\subseteq}\, S \Rightarrow R^{-1} \,\tilde{\subseteq}\, S^{-1}$

Proof

(i) $(R^{-1})^{-1}(p, q) = R^{-1}(q, p) = R(p, q)$

It follows that $(R^{-1})^{-1} = R$

(ii) $R(p, q) \subseteq S(p, q) \Rightarrow R^{-1}(q, p) \subseteq S^{-1}(q, p)$

$$\Rightarrow R^{-1} \tilde{\subseteq} S^{-1}$$

Definition 3.7

Let R and S be two fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C) respectively. Then the composition $\circ$ of R and S is defined by

$(R \circ S)(p, q) = R(p, r) \cap S(r, q)$

Example 3.6

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

A = { expertise in physics (p), expertise in chemistry (c), expertise in biology (b)}

B = { expertise in physics (p), expertise in history (h)}

C = { expertise in chemistry (c), expertise in education (e)}

Then

$(F, A) = \{F(p) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$

$F(c) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\},$

$F(b) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}\}$

$(G, B) = \{G(p) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\},$

$G(h) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}$

$(H,C) \;=\{H(c)=\{S_1/0.3,S_2/0.5,S_3/0.2,S_4/0.1\},$

$H(e)=\{S_1/0.4,S_2/0.2,S_3/0.7,S_4/0.1\}$

Thus we have

$(R, E) \;=\; \{R(p,p)=\{S_1/0.72,S_2/0.04,S_3/0.30,S_4/0.64\}$

$R(c,p)=\{S_1/0.24,S_2/0.18,S_3/0.20,S_4/0.24\},$

$R(b,p)=\{S_1/0.48,S_2/0.06,S_3/0.45,S_4/0.64\}\}$ where $E \subseteq A\times B$ and

$(S, F)=\{\;S(p,c)=\{S_1/0.24,S_2/0.10,S_3/0.10,S_4/0.08\}\}$ where $F \subseteq B\times C$

$\therefore (R \circ S)(p, c)$

$= R(p, p) \cap S(p, c)$

$= \{S_1/0.1728,S_2/0.004,S_3/0.03,S_4/0.0512\}$

Proposition 3.3

If R and S be two fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C) respectively, satisfying t - norm condition. Then $R \circ S$ is fuzzy soft relation from (F, A) to (H, C).

Proof

By definition

$R(p,q) \;=\{x/F(p)*G(q): x\in U\}, \forall(p,q)\in A\times B$

$S(q,r) \;=\{x/G(q)*H(r): x\in U\}, \forall(q,r)\in B\times C$

Therefore

$(R\circ S)(p,r) \;=\{x/(F(p)*G(q))*(G(q)*H(r)): x\in U\}, \forall(p,q,r)\in A\times B\times C$

Now

$(F(p) * G(q)) *(G(q) *H(r))$

$= (F(p) * G(q)) *H(r) \subseteq F(p)*1*H(r)$

$= F(p) * H(r)$

Hence

$R(p,q) \cap S(q,r) \subseteq F(p) \cap H(r)$

Thus $R \circ S$ is fuzzy soft relation from (F, A) to (H, C).

Proposition 3.4

The composition operation of soft mappings is associative.

Proof

Let R, S, T are three fuzzy soft relation from (F, A) to (G, B), (G, B) to (H, C) and (H, C) to (L, D) .Then

$$\begin{aligned}
((R \circ S) \circ T)(a, d) &= (R \circ S)(a, c) \cap T(c, d) \\
&= (R(a, b) \cap S(b, c)) \cap T(c, d) \\
&= R(a, b) \cap (S(b, c)) \cap T(c, d) \\
&= R(a, b) \cap (S \circ T)(b, d) \\
&= (R \circ (S \circ T))(a, d)
\end{aligned}$$

Hence $(R \circ S) \circ T = R \circ (S \circ T)$

Definition 3.8

A fuzzy soft relation R on (F, A) is said to be fuzzy soft reflexive relation if

$R(p, q) \subseteq R(p, p)$ and $R(q, p) \subseteq R(p, p)$, $\forall p, q \in A$

Definition 3.9

A fuzzy soft relation R on (F, A) is said to be fuzzy soft irreflexive relation if

$R(p,q) \not\subset R(p,p)$ and $R(q,p) \not\subset R(p,p)$, $\forall p, q \in A$

Example 3.7

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

A = { expertise in physics (p), expertise in chemistry (c), expertise in biology (b)}

Then

$$(F, A) = \{F(p) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$$

$$F(c) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\},$$

$$F(b) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}\}$$

Define a fuzzy soft relation (R, C) from (F, A) to (F, A) as (p, p), (c, p), (p, c) $\in C \subseteq A \times A$

Then

$$(R, C) = \{R(p, p) = \{S_1/0.81, S_2/0.04, S_3/0.36, S_4/0.64\},$$

$$R(p, c) = \{S_1/0.27, S_2/0.18, S_3/0.24, S_4/0.24\},$$

$$R(c, p) = \{S_1/0.26, S_2/0.11, S_3/0.20, S_4/0.14\}\}$$

$\therefore$ $R(p, c) \not\subset R(p, p)$ and $R(c, p) \not\subset R(p, p)$ and hence R is irreflexive relation.

Definition 3.10

A fuzzy soft relation R on (F, A) is said to be fuzzy soft transitive relation if

$R \circ R \,\tilde{\subseteq}\, R$

Definition 3.11

If a fuzzy soft relation R on (F, A) is simultaneously reflexive, symmetric and transitive then it is known as a fuzzy soft equivalence relation.

Proposition 3.5

For a fuzzy soft relation S on (F, A), $S = S^{-1}$ if and only if S is symmetric fuzzy soft relation on (F, A).

Proof

Let $S = S^{-1}$

$$S(p, q) = S^{-1}(q, p) = S(q, p)$$

$\therefore$ S is symmetric.

Conversely, Let S be a symmetric fuzzy soft relation.

$S^{-1}(p, q) = S(q, p) = S(p, q)$

$\therefore$ $S = S^{-1}$

Proposition 3.6

If S be a fuzzy soft equivalence relation on (F, A) then $S \circ S$ is also a fuzzy soft equivalence relation.

Proof

Since S is a fuzzy soft equivalence relation therefore S is reflexive, symmetric and transitive.

Now $S \circ S(p, q) \subseteq S(p, p)$

$\therefore$ $S \circ S$ is reflexive.

Again

$S \circ S(p, q)$

$= S(p, r) \cap S(r, q)$

$= S(r, q) \cap S(p, r)$

$= S(q, r) \cap S(r, p)$, since S is symmetric

$= S \circ S(q, p)$

$\therefore$ $S \circ S$ is symmetric.

$S \circ S(p, q)$

$= S(p, r) \cap S(r, q)$

$\supseteq S \circ S(p, r) \cap S \circ S(r, q)$ since S is symmetric.

$= (S \circ S \circ S \circ S)\ (p, q)$

$\therefore\ S \circ S \circ S \circ S \ \tilde{\subseteq}\ S \circ S$

$\therefore\ S \circ S$ is transitive.

Hence $S \circ S$ is equivalence relation.

Proposition 3.7

If S be a fuzzy soft equivalence relation on (F, A) then S^{-1} is also a fuzzy soft equivalence relation.

Proof

$S^{-1}\ (p, q) = S\ (q, p) \subseteq S\ (p, p) = S^{-1}\ (p, p)$, since S is reflexive.

$S^{-1}\ (q, p) = S\ (p, q) \subseteq S\ (p, p) = S^{-1}\ (p, p)$

S^{-1} is reflexive

$S^{-1}\ (p, q) = S\ (q, p) = S\ (p, q) = S^{-1}\ (q, p)$, since S is symmetric

S^{-1} is symmetric.

$S^{-1}\ (p, q) = S\ (q, p) \supseteq S \circ S\ (q, p)$

$= S(q, r) \cap S(r, p)$

$= S(r, q) \cap S(p, r)$

$= S^{-1}\ (q, r) \cap S^{-1}\ (r, p)$

$= (S^{-1} \circ S^{-1})(p, q)$

$S^{-1} \circ S^{-1} \tilde{\subseteq} S^{-1}$

$\therefore\ S^{-1}$ is transitive

Thus S^{-1} is a fuzzy soft equivalence relation.

Definition 3.12

Let R and S be two fuzzy soft sets from (F, A) to (G, B) .Then

$(R \cap S)\,(p, q) = \min\,\{R\,(p, q), S\,(p, q)\}$

$(R \cup S)\,(p, q) = \max\,\{R\,(p, q), S\,(p, q)\}. \quad \forall (p, q) \in A \times B$

Proposition 3.8

If R and S, T are fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C), then

(*i*) $R \circ (S \tilde{\cap} T) = (R \circ S) \tilde{\cap} (R \circ T)$

(*ii*) $R \circ (S \tilde{\cup} T) = (R \circ S) \tilde{\cup} (R \circ T)$

Proof

Suppose $p \in A, q \in B, r \in C$

(*i*) $R(p, q) \circ S(q, r) \cap T\,(q, r)$

$= R(p, q) \cap \min\,\{ S(q, r), \; T(q, r) \}$

$= \min\,\{ R(p, q) \cap S(q, r), R(p, q)) \cap T(q, r)\}$

$= \min\,\{ (R \circ S)(p, r), (R \circ T)(p, r) \}$

$= (R \circ S)\,(p, r) \cap (R \circ T)\,(p, r)$

Hence $R \circ (S \tilde{\cap} T) = (R \circ S) \tilde{\cap} (R \circ T)$.

(*ii*) $R(p, q) \circ S\,(q, r) \cup T\,(q, r)$

$= R(p, q) \cap \max\,\{ S(q, r), T(q, r) \}$

$= \max\,\{ R(p, q)) \cap S(q, r), R(p, q)) \cap T(q, r)\}$

$= \max\,\{ (R \circ S)(p, r), (R \circ T)(p, r) \}$

$= (R \circ S)(p, r) \cup (R \circ T)(p, r)$

Hence $R \circ (S \tilde{\cup} T) = (R \circ S) \tilde{\cup} (R \circ T)$.

Proposition 3.9

If R and S are fuzzy soft relations from (F, A) to (G, B) then

(*i*) $\left(R \tilde{\cap} S\right)^{-1} = R^{-1} \tilde{\cap} S^{-1}$

(ii) $(R \tilde{\cup} S)^{-1} = R^{-1} \tilde{\cup} S^{-1}$

Proof

Suppose $p \in A, q \in B$

(i) $(R \tilde{\cap} S)^{-1}(p,q) = (R \tilde{\cap} S)(q,p)$

$= \min\{R(q,p), S(q,p)\}$

$= \min\{R^{-1}(p,q), S^{-1}(p,q)\}$

$= (R^{-1} \tilde{\cap} S^{-1})(p,q)$

Hence $(R \tilde{\cap} S)^{-1} = R^{-1} \tilde{\cap} S^{-1}$

(ii) $(R \tilde{\cup} S)^{-1}(p,q) = (R \tilde{\cup} S)(q,p)$

$= \max\{R(q,p), S(q,p)\}$

$= \max\{R^{-1}(p,q), S^{-1}(p,q)\}$

$= (R^{-1} \tilde{\cup} S^{-1})(p,q)$

Hence $(R \tilde{\cup} S)^{-1} = R^{-1} \tilde{\cup} S^{-1}$

4. CONCLUSION

In this paper, we have discussed the concept of relation in fuzzy soft set theory. We have established some properties of fuzzy soft relation such as symmetricity, transitivity, reflexivity, irreflexivity, equivalence etc. In our work, we have also put forward the definitions of union and intersection between two fuzzy soft sets with a new approach. Future works in this regard would be necessary whether the notions put forward in our work bring about a fruitful result.

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