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$\mu-$ NECKS OF FUZZY AUTOMATA

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Abstract. We introduce $\mu-$ necks of fuzzy automata, that is we find a word that brings each state of a fuzzy automata to a single state with minimal weight μ $[0 < \mu \leq 1]$ and also we introduce local $\mu-$ necks of fuzzy automata that is, it is a $\mu-$ neck of some subautomata of a fuzzy automata. Further, we study the structural properties of fuzzy automata using the notions of their $\mu-$necks and local $\mu-$ necks.

Keywords: $\mu-$ Necks & Local $\mu-$ necks of fuzzy automata, $\mu-$directable fuzzy automata, $\mu-$ Reversible fuzzy automata, Monogenically & Uniformly monogenically $\mu-$ directable fuzzy automata.

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1. Introduction

Directable automata is also known as Synchronizable which are significant type of automata with very interesting algebraic properties and important applications in various branches of computer science [2]. Various specializations and generalizations of directable automata have appeared recently. T. Petkovic *et al*. [5] introduced and studied monogenically, locally and generalized directable automata. These automata are also refered by Z. Popovic *et al*. [6] and [7]. Milena Bogdanovic *et al*. [1] studied directable automata

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using their necks. The theory of fuzzy set was introduced by L.A. Zadeh in 1965 [11]. The mathematical formulation of a fuzzy automaton was first proposed by W.G. Wee in 1967 [10]. E.S. Santos 1968 [8] proposed fuzzy automata as a model of pattern recognition. John N. Mordeson and D.S. Malik gave a detailed account of fuzzy automata and languages in their book 2002 [4].

We introduce $\mu-$ necks of a fuzzy automata, that is we find a word that brings each state of a fuzzy automata to a single state with minimal weight μ $[0 < \mu \leq 1]$. We introduce local $\mu-$ necks of fuzzy automata. It is a $\mu-$ neck of some subautomata of a fuzzy automata. We shown that set of $\mu-$necks in a fuzzy automata is a subautomata and it is a least subautomata of a fuzzy automata. We obtain a necessary and sufficient condition for a fuzzy automata to be strongly $\mu-$ directable. Also we obtain a condition for fuzzy automata under which it is not $\mu-$ directable. Also we establish some equivalent conditions for uniformly monogenically strongly $\mu-$ directable fuzzy automata.

2. Basic concepts

2.1. **Fuzzy automata** [3]. A finite fuzzy automata is a system of 5 tuples, $M = (Q,\ \Sigma,\ \pi,\ \eta,\ f_M)$ where Q-set of states $\{q_1,\ q_2, ...,\ q_n\}$
Σ-alphabets (or) input symbols
π-$Q \rightarrow [0, 1]$ initial state designator
η-$Q \rightarrow [0, 1]$ final state designator
f_M-function from $Q \times \Sigma \times Q \rightarrow [0, 1]$
$f_M(q_i,\ \sigma,\ q_j) = \mu$ $[0 < \mu \leq 1]$ means when M is in state q_i and reads the input σ will move to the state q_j with weight function μ. For each $\sigma \in \Sigma$ we can form a $n \times n$ matrix $F(\sigma)$ whose (i, j) the element is $f_M(q_i,\ \sigma,\ q_j)$. For $x \in \Sigma^*$ and if $x = \sigma_1\ \sigma_2\ ....\ \sigma_m$
$F(x) = F(\sigma_1) \circ F(\sigma_2) \circ ..... \circ F(\sigma_m)$
In otherwords $F(x)$ is the fuzzy sum of fuzzy products of weights taken over the paths in the automata.

Note

$f_M(i,\ x,\ j)$ is the (i, j) the element of $F(x)$

$f_M(s,\ x,\ t)$ =Max$\{$Min$\{f_M(s,\ \sigma_1,\ q_1), f_M(q_1,\ \sigma_2,\ q_2), \ldots.., f_M(q_{m-1},\ \sigma_m,\ t)\}\}$ where Max is taken over all the paths from s to t.

Note

$F_{pq}(w)$ denotes p^{th} row and q^{th} column of a matrix $F(w)$.

2.2. **Sub automata** [4]. Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. An automaton $N = (Q_1,\ \Sigma,\ f_N)$ is called subautomata of M if for any $u \in \Sigma^*$ and $q \in Q_1$, then there exists $q' \in Q_1$ such that $f_N(q,\ u,\ q') > 0$ where f_N is the restriction of f_M into N.

2.3. **Strongly connected fuzzy automata.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. M is said to be strongly connected if for every $p,\ q \in Q$ there exists $u \in \Sigma^*$such that $f_M(p,\ u,\ q) > 0$. Equivalently, M is strongly connected if it has no proper subautomata.

2.4. **Subautomata generated by** q**.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata and let $q \in Q$. The subautomata of M generated by q is denoted by $< q >$. It is given by $< q >= \{q_1\ /\ f_M(q,\ u,\ q_1) > 0,\ u \in \Sigma^*\}$. It is called least subautomata of M containing q and it is also called monogenic subautomata of M.

2.5. **Subautomata generated by** H**.** For any non-empty $H \subseteq Q$, the subautomata of M generated by H is denoted by $< H >$ and is given by
$< H >= \{q_1\ /\ f_M(q,\ w,\ q_1) > 0,\ q \in H,\ w \in \Sigma^*\}$. It is called least subautomata of M containing H. The least subautomata of a fuzzy automata M, if it exists is called the kernel of M.

2.6. **Necks of fuzzy automata.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. A state $q \in Q$ is called a neck of M if there exists $u \in \Sigma^*$ such that $f_M(p,\ u,\ q) > 0$ for every $p \in Q$.

In that case q is also said to be a u-neck of M and the word u is called a directing word of M. If M has a directing word, then M is called directable fuzzy automata.

2.7. $\mu-$ **Necks of fuzzy automata.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. A state $q \in Q$ is called a $\mu-$ neck of M if there exists $u \in \Sigma^*$ and minimal weight μ in M $[0 < \mu \leq 1]$ such that $f_M(p,\ u,\ q) = \mu$ for every $p \in Q$.

In that case q is also said to be a u-$\mu-$neck of M and the word u is called a $\mu-$directing word of M. If M has a $\mu-$ directing word then M is called $\mu-$ directable fuzzy automata.

Note

1) The set of all $\mu-$ necks of a fuzzy automata M is denoted by $\mu N(M)$.
2) The set of all $\mu-$directing words of a fuzzy automata M is denoted by $\mu DW(M)$.
3) If a fuzzy automata M is strongly $\mu-$ directable then $M = \mu N(M)$

Example

Fuzzy automata with µ-necks

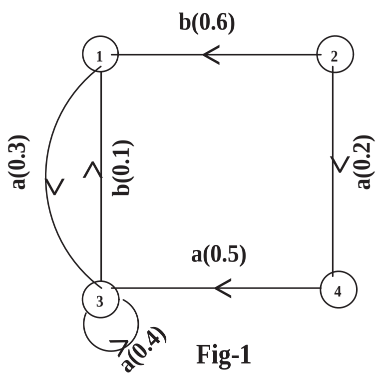

Fig-1

In Fig-1, $f_M(p,\ aab,\ 1) = 0.1\ \forall\ p \in Q$and $f_M(p,\ aaba,\ 3) = 0.1\ \forall\ p \in Q$. Hence the states 1 and 3 are $\mu-$ necks of M.

Fuzzy automata with no µ-necks

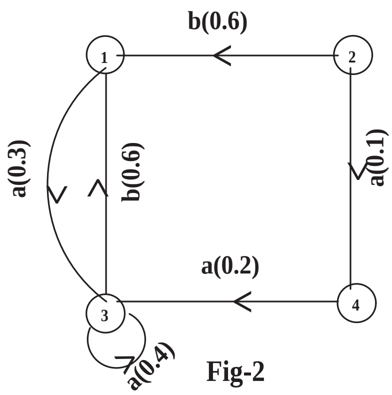

Fig-2

In Fig-2, the states 1 and 3 are necks of M but not $\mu-$ necks and directing words are aab and $aaba$ but not $\mu-$ directing words.

2.8. $\mu-$ **Reversible fuzzy automata.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. A state $q \in Q$ is called $\mu-$ reversible. If for everyword $v \in \Sigma^*$ there exists a word $u \in \Sigma^*$ such that $f_M(q,\ vu,\ q) = \mu$ and the set of all $\mu-$ reversible states of M called the $\mu-$reversible part of M is denoted by $\mu R(M)$.

If it is non empty $\mu R(M)$ is a subautomata of M.

Note

(i) If all states of a fuzzy automata M are $\mu-$ reversible, then the fuzzy automata $M = (Q,\ \Sigma,\ f_M)$ is called $\mu-$ reversible fuzzy automata.

(ii) If M is a $\mu-$ directable fuzzy automata implies that it is a directable fuzzy automata. Then the converse need not be true. i.e If M is directable fuzzy automata then it need not be a $\mu-$ directable fuzzy automata.

2.9. **Local** $\mu-$ **necks of fuzzy automata.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. We say that a state $q \in Q$ is called local $\mu-$ neck of M, if it is $\mu-$neck of some $\mu-$ directable fuzzy subautomata of M. The set of all local $\mu-$ necks of M is denoted by $L\mu N(M)$.

Example

Fuzzy automata M with local μ-necks

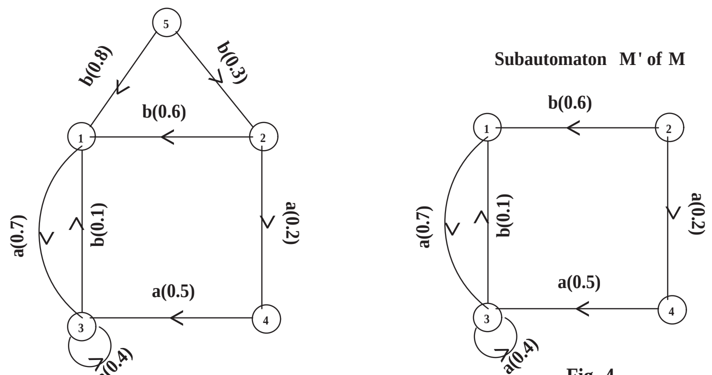

Fig- 3 **Fig- 4**

In Fig-3, 1 and 3 are local $\mu-$ necks as it is a $\mu-$ neck of subautomata M'(Fig-4) of M with $\mu-$ directing words aab and $aaba$ respectively.

Fuzzy automata M with no local μ-necks

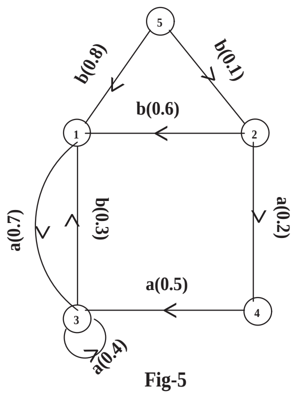

Fig-5

In Fig-5, 1 and 3 are local necks but not local $\mu-$ necks with directing words aab, $aaba$ and not $\mu-$ directing words.

2.10. **Monogenically $\mu-$ directable fuzzy automata.** A fuzzy automata M is called monogenically $\mu-$ directable, if every monogenic subautomata of M is $\mu-$ directable fuzzy automata.

Example

Monogenically fuzzy μ-directable automata M

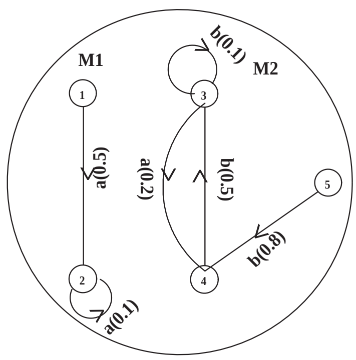

Fig- 6

In Fig-6, M_1 and M_2 are monogenic subautomata with $\mu-$ directing words aa and bbb respectively.

2.11. **Monogenically strongly $\mu-$ directable fuzzy automata.** A fuzzy automata M is called monogenically strongly $\mu-$ directable, if every monogenic subautomata of M is strongly $\mu-$ directable.

Example

Monogenically strongly fuzzy μ- directable automata M

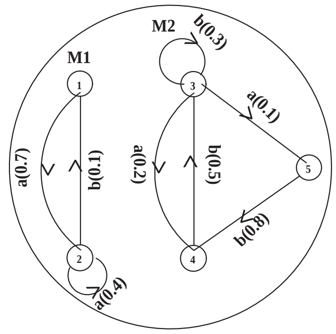

Fig - 7

In Fig-7, M_1 and M_2 are strongly monogenic subautomata with $\mu-$ directing words aab and bba respectively.

2.12. **Common $\mu-$ directing word.** Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. We define a word $u \in \Sigma^*$ to be a common $\mu-$directing word of M, if u is a $\mu-$ directing word of every monogenic subautomata of M i.e., if $u \in \mu DW(< p >)$, for every $p \in Q$.

2.13. **Uniformly monogenically $\mu-$ directable fuzzy automata.** Let M be a fuzzy automata. M is called uniformly monogenically $\mu-$directable fuzzy automata, if every monogenic subautomata of M is $\mu-$ directable and have atleast one common $\mu-$ directing word.

2.14. **Uniformly monogenically strongly $\mu-$ directable fuzzy automata.** Let M be a fuzzy automata. M is called uniformly monogenically strongly $\mu-$ directable fuzzy automata, if every monogenic subautomata of M is strongly $\mu-$ directable and have atleast one common $\mu-$ directing word.

Note

(i) If M is monogenically strongly $\mu-$ directable fuzzy automata implies that M is monogenically strongly directable fuzzy automata. The converse is need not be true.

(ii) If M is uniformly monogenically strongly $\mu-$ directable fuzzy automata implies that M is uniformly monogenically strongly directable fuzzy automata. The converse is need not be true.

3. $\mu-$ NECKS OF FUZZY AUTOMTA

The following lemma is easily proved from [1]

Lemma3.1 Let M be a fuzzy automata. If $\mu N(M) \neq \phi$ then $\mu N(M)$ is a subautomata of M.

Lemma3.2 Let M be a $\mu-$ directable fuzzy automata. Then $\mu N(M)$ is the kernel of M and $\mu N(M) = \mu R(M)$.

Theorem3.3 A fuzzy automata M is strongly directable if and only if it is strongly $\mu-$ directable.

Proof. Let M be a strongly directable fuzzy automata. Let $q \in N(M)$ and there exists $u \in \Sigma^*$ such that $f_M(p,\ u,\ q) > 0\ \forall\ p\ \in Q$. In M, there exists two states q_i, q_j such that $f_M(q_i,\ a,\ q_j) = \mu$ [where μ is minimal weight in M] for some $a \in \Sigma$.

Since $N(M) = M$ choose the suitable word v, that reaches the state q_i from the state q i.e., $f_M(p,\ uv,\ q_i) > 0\ \forall\ p \in Q$.

Now,$f_M(p,\ uva,\ q_j) = Max\{Min_{q_i \in Q}\{f_M(p,\ uv,\ q_i), f_M(q_i,\ a,\ q_j)\}\} = \mu\ \forall\ p \in Q$.

Hence M is strongly $\mu-$ directable fuzzy automata.

Conversly, let M be a stronly $\mu-$ directable fuzzy automata. Then $\mu N(M) \neq \phi$ and by lemma 3.1 $\mu N(M)$ is a subautomata of M. But, since M is strongly $\mu-$ directable, it follows that $M = \mu N(M)$ i.e., for any $q \in Q$ there exists $u \in \Sigma^*$ such that $f_M(p,\ u,\ q) = \mu\ \forall\ p\ \in\ Q$

$$\Longrightarrow f_M(p,\ u,\ q) > 0\ \forall\ p\ \in\ Q.$$

Hence M is strongly directable fuzzy automata.

Theorem3.4 Let M be a directable fuzzy automata with minimal weight $\mu \in M$. If there exists $p,\ q \notin N(M)$ and $a \in \Sigma$ such that $f_M(p,\ a,\ q) = \mu$. Then M is not $\mu-$ directable fuzzy automata.

Proof. Assume that M be a $\mu-$ directable fuzzy automata. Then for every $p \in Q$ and $u \in \Sigma^*$ there exist $q \in Q$ such that $f_M(p,\ u,\ q) = \mu$. Let $p_1 \in \mu N(M)$,

$f_M(p_1,\ u,\ q) = f_M(p_1,\ u_1 a,\ q)$ where $u = u_1 a,\ u_1 \in \Sigma^*,\ a \in \Sigma$

$\Longrightarrow Max\{Min_{r \in Q}\{f_M(p_1,\ u_1,\ r), f_M(r,\ a,\ q)\}\} = \mu_1 > \mu$. [Since, there is no

p_1 & $p_2 \in N(M)$ such that $f_M(p_1,\ a,\ p_2) = \mu$.] Which is a contradiction. Therefore M is not $\mu-$ direcable fuzzy automata.

Theorem3.5 Let $M = (Q,\ \Sigma,\ f_M)$ be a $\mu-$ directable fuzzy automata. Let $p \in Q$. Then the following conditions are equivalent.

(i) p is a $\mu-$ neck.

(ii)$< p >$ is a strongly $\mu-$ directable fuzzy automata.

(iii) for every $v \in \Sigma^*$, there exists $u \in \Sigma^*$ such that $f_M(p,\ vu,\ p) = \mu$.

Proof. $(i) \Rightarrow (ii)$

Let p is a $\mu-$ neck of M. For every $q \in Q$ there exist a $\mu-$ directing word $u \in \Sigma^*$ such that $f_M(q,\ u,\ p) = \mu$. For any $q_1 \in < p >$ and $v \in \Sigma^*$ such that $f_M(q_1,\ uv,\ q_2)=$ $Max\{Min_{r\in<p>}\{f_M(q_1,\ u,\ r), f_M(r,\ v,\ q_2)\}\} = \mu$ for some $q_2 \in < q >$. Hence $< p >$ is strongly connected. Let $p_1 \in < p >$ and u is a $\mu-$ directing word of M. Then $f_M(p_1,\ u,\ p) = \mu$. Hence $< p >$ is a $\mu-$ directable.

$(ii) \Rightarrow (iii)$

Let $< p >$ be a strongly $\mu-$ directable fuzzy automata. Then p is a $u - \mu-$ neck of $< p >$ for some $u \in \Sigma^*$. Since $< p >$ is strongly $\mu-$ fuzzy directable, there exists some $p_1 \in < p >$ and $v \in \Sigma^*$ such that $f_M(p,\ v,\ p_1) > 0$.

Now, $f_M(p,\ vu,\ p) = Max\{Min_{r\in<p>}\{f_M(p,\ v,\ r), f_M(r,\ u,\ p)\}\} = \mu$.

$(iii) \Rightarrow (i)$

Since M is $\mu-$ fuzzy directable, there exists $u-$ directing word and $p_1 \in Q$ such that $f_M(q,\ u,\ p_1) = \mu \,\forall\, q \in Q$. For any $u \in \Sigma^*$ there exists $v \in \Sigma^*$ such that $f_M(p,\ uv,\ p) =$ μ. Let $q_1 \in Q$, $f_M(q_1,\ uv,\ p)= Max\{Min_{p_1 \in Q}\{f_M(q_1,\ u,\ p_1), f_M(p_1,\ v,\ p)\}\} = \mu$. Hence p is a $\mu-$neck.

4. Local $\mu-$ necks of fuzzy automta

Theorem 4.1.. *Let M be a monogenically strongly directable fuzzy automata with minimal weight $\mu \in M$. If there exists $p,\ q \in LN(M)$ and $\forall u \in \Sigma^*$ such that $f_M(p,\ u,\ q) \neq \mu$. Then M is not a monogenically strongly $\mu-$ directable fuzzy automata.*

Proof. Assume that M be a monogenically strongly $\mu-$ directable fuzzy automata. Then for every $p \in Q$ and $u \in \Sigma^*$ there exist $q \in Q$ such that $f_M(p,\ u,\ q) = \mu$. Let

$p_1 \in L\mu N(M)$,

$f_M(p_1,\ u,\ q) = f_M(p_1,\ u_1u_2,\ q)$ where $u = u_1u_2$where $u_1, u_2 \in \Sigma^*$

$\implies Max\{Min_{r\in Q}\{f_M(p_1,\ u_1,\ r), f_M(r,\ u_2,\ q)\}\} = \mu_1 > \mu$. [Since, there is no p_1 & $p_2 \in LN(M)$ and for any $u \in \Sigma^*$ such that $f_M(p_1,\ u,\ p_2) \neq \mu$.] Which is a contradiction. Therefore M is not a monogenically strongly $\mu-$ direcable fuzzy automata.

Theorem 4.2.. *Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automata. Then the following conditions are equivalent.*

(i) Every state of M is a local $\mu-$ neck, and $u \in \Sigma^*$ is a common $\mu-$ directing word of M.

(ii)M is uniformly monogenically strongly $\mu-$ directable fuzzy automata.

(iii)M is uniformly monogenically $\mu-$ directable and $\mu-$ reversible fuzzy automata.

(iv)M is direct sum of strongly $\mu-$directable fuzzy automata.

Proof.$(i) \Rightarrow (ii)$

If every state $p \in Q$ is a local $\mu-$ neck of M, then by lemma 3.1 we have for every $p \in Q$ the monogenic subautomata $< p >$ of M is strongly $\mu-$directable and $u \in \Sigma^*$ is a common $\mu-$ directing word of M, then every monogenic subautomaton of M have u as $\mu-$ directing word. Therefore, M is uniformly monogenically strongly $\mu-$ directable fuzzy automata.

$(ii) \Rightarrow (iii)$

If M is uniformly monogenically strongly $\mu-$ directable fuzzy automata, then it is clear that it is uniformly monogenically $\mu-$ directable fuzzy automata. On the otherhand, every monogenic subautomata of M is strongly connected, hence it follows that M is $\mu-$ reversible.

$(iii) \Rightarrow (iv)$

In [9] If M is reversible then it is a direct sum of strongly connected fuzzy automata M_α, $\alpha \in Y$. Let $\alpha \in Y$ and $p \in Q_\alpha$. Then $< p >= M_\alpha$. Since M_α is strongly connected and by the monogenic $\mu-$ directability of a fuzzy automata M we have that $M_\alpha =< p >$ is $\mu-$ directable. Therefore M_α is strongly $\mu-$ directable for any $\alpha \in Y$.

$(iv) \Rightarrow (i)$

Let M be a direct sum of strongly $\mu-$ directable fuzzy automata M_α, $\alpha \in Y$. Then for each state $p \in Q$ there exists $\alpha \in Y$ such that $p \in M_\alpha$, that is $p \in M_\alpha = \mu N(M_\alpha)$. So p is local $\mu-$ neck of M.

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