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INTEGRAL OPERATORS ACTING BETWEEN SOME SPACES OF ANALYTIC TYPE

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Abstract. The aim of this paper is to define some classes of analytic function spaces in the unit disc. The boundedness of a certain integral-type operator acting between these classes is investigated.

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1. Introduction

Let $\mathbb{D}$ denote the open unit disk in the complex plane $\mathbb{C}$ and $H(\mathbb{D})$ the space of all holomorphic functions on $\mathbb{D}$. Throughout this paper ϕ denotes a nonconstant holomorphic self-map of $\mathbb{D}$ and u a fixed analytic function on $\mathbb{D}$. Associated with $f,\mathbf{g}\in H(\mathbb{D})$, the integral-type operators $J_{\mathbf{g}}$ and $I_{\mathbf{g}}$ are defined as follows:

$$J_{\mathbf{g}}f(z)=\int_0^z f(\zeta)\mathbf{g}^{'}(\zeta)d\zeta \ and \ I_{\mathbf{g}}f(z)=\int_0^z f^{'}(\zeta)\mathbf{g}(\zeta)d\zeta.$$

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The importance of the operators $J_{\mathbf{g}}$ and $I_{\mathbf{g}}$ comes from the fact that

$$J_{\mathbf{g}}f + I_{\mathbf{g}}f = M_{\mathbf{g}}f - f(0)\mathbf{g}(0),$$

where $M_{\mathbf{g}}$ is the multiplication operator defined by

$$M_{\mathbf{g}}f(z) = \mathbf{g}(z)f(z),\ f \in H(\mathbb{D}),\ z \in \mathbb{D}.$$

Boundedness and compactness of the operators $J_{\mathbf{g}}$ and $I_{\mathbf{g}}$ in one-dimensional, as well as their n-dimensional extensions, acting on various function spaces were investigated intensively in [1-4] and [6, 17, 40]. Let ϕ be a positive continuous function on $[0,1)$, then ϕ is called a normal function if there are three constants a, b, t_0, where $0 < a < b$ and $t_0 \in [0,1)$, such that

$$\frac{\phi(t)}{(1-t^2)^a}\ decreases\ for\ t_0 \leq t \leq 1\ and\ \lim_{t \to 1^-} \frac{\phi(t)}{(1-t^2)^a} = 0,$$

$$\frac{\phi(t)}{(1-t^2)^b}\ decreases\ for\ t_0 \leq t \leq 1\ and\ \lim_{t \to 1^-} \frac{\phi(t)}{(1-t^2)^b} = \infty.$$

Now, we give the following definitions;

For a given reasonable function $\omega : (0,1] \to (0,\infty)$ satisfying the condition $\omega(1-|z|) \approx \omega^n(1-|z|)\,;\, n \geq 0$, for $0 < p < \infty$ and a normal function ϕ, let $H(p,p,\omega,\phi)$ denote the space of all analytic functions f on the unit disk $\mathbb{D}$ such that

$$\|f\|_{p,p,\phi} = \left(\int_0^1 M_p^p(f,r) \frac{\omega(1-r)\phi^p(r)}{(1-r)} r dr \right)^{1/p},$$

where the integral means $M_p(f,r)$ are defined by

$$M_p(f,r) = \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta \right)^{1/p},\ 0 \leq r < 1.$$

For $1 \leq p < \infty$, $H(p,p,\omega,\phi)$ equipped with the norm $\|\cdot\|$, is a Banach space. When $0 < p < 1$, $\|\cdot\|_{p,p,\omega,\phi}$ is quipped on $H(p,p,\omega,\phi)$, and $H(p,p,\omega,\phi)$ is a Frechét space but not a Banach space. If $0 < p < \infty$, then $H(p,p,\omega,\phi)$ is the weighted Bergman-type space

$$H(p,p,\omega,\phi) = \left\{ f \in H(\mathbb{D}) : \int_{\mathbb{D}} |f(z)|^p \frac{\omega(1-|z|)\phi^p(|z|)}{(1-|z|)} dA(z) < \infty \right\},$$

where $dA(z)$ denotes the normalized Lebesgue area measure on the unit disk $\mathbb{D}$ with $A(\mathbb{D}) \equiv 1$. Note that if $\phi(r) = (1-r)^{(\alpha+1)/p}$, then $H(p,p,\omega,\phi)$ is the weighted Bergman space $A^p_\alpha(\mathbb{D})$ defined for $0 < p < \infty$ and $\alpha > -1$, as the space of all $f \in H(\mathbb{D})$ such that

$$\|f(z)\|^p_{A^p_\alpha} = \int_{\mathbb{D}} |f(z)|^P \omega(1-|z|)(1-|z|^2)^\alpha dA(z) < \infty.$$

Now, we define the analytic weighted logarithmic Bloch-type space $\mathscr{B}^\alpha_{\omega,log^\beta}(\mathbb{D})$ (where $\alpha > 0$ and $\beta \geq 0$) as follows:

$$\mathscr{B}^\alpha_{\omega,log^\beta}(f) = \sup_{z\in\mathbb{D}} \frac{(1-|z|)^\alpha}{\omega(1-|z|)} \left(\ln \frac{e^{\beta/\alpha}}{1-|z|}\right)^\beta |f'(z)| < \infty.$$

We define the norm on $\mathscr{B}^\alpha_{\omega,log^\beta}$ as follows:

$$\|f\|_{\mathscr{B}^\alpha_{\omega,log^\beta}} = |f(0)| + \mathscr{B}^\alpha_{\omega,log^\beta}(f).$$

The little weighted logarithmic Bloch-type space consists of all $f \in \mathscr{B}^\alpha_{\omega,log^\beta}$ such that

$$\lim_{|z|\to 1^-} \frac{(1-|z|)^\alpha}{\omega(1-|z|)} \left(\ln \frac{e^{\beta/\alpha}}{1-|z|}\right)^\beta |f'(z)| = 0.$$

Remark 1.1 It should be remarked that when $\omega = 1$, then we obtain the space $\mathscr{B}^\alpha_{log^\beta}$ as defined in [38]. When $\omega = 1$ and $\beta = 0$, then $\mathscr{B}^\alpha_{\omega,log^\beta}$ becomes the α-Bloch space $\mathscr{B}^\alpha$, which appeared in characterizing the multipliers of the Bloch space (see [5, 39]).

Remark 1.2 We recall that there are some recent articles used the weight function ω to define and study some function spaces of analytic type (see [14, 15, 22, 23, 24, 25, 36, 37]).

Throughout this article, the letter C denotes a positive constant which may vary at each occurrence but is independent of the essential variables. We use the notation $a \simeq b$ to denote the comparability of the quantities a and b, i.e. the existence of two positive constants C_1 and C_2 satisfying $C_1 a \leq b \leq C_2 a$.
Recall that a linear operator is said to be bounded if the image of a bounded set is a bounded set, while a linear operator is compact if it takes bounded sets to sets with compact closure.

2. Boundedness Of Integral Operator

In this section we characterize the boundedness of the integral-type operator $I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\omega,log^{\beta}} \to H(p,p,\omega,\phi)$. It is interesting to provide a function theoretic characterization of **g**, when **g** induces a bounded or compact integral-type operator on various spaces. For this purpose, we start this section by stating some lemmas that are used in the proofs of main results of this article.

Lemma 2.1 *There exist two functions* $f, \mathbf{g} \in \mathscr{B}^{\alpha}_{\omega,\log^{\beta}}(\mathbb{D})$, *(where* $\alpha > 0$ *and* $\beta \geq 0$*) such that for each* $z \in \mathbb{D}$, *we have*

$$|f^{'}(z)| + |\mathbf{g}^{'}(z)| \geq \frac{C\omega(1-|z|)}{(1-|z|)^{\alpha} \ln^{\beta} \frac{e^{\beta/\alpha}}{1-|z|}},$$

for some positive constant C.

Proof. The proof is similar to the corresponding result in ([30, 37] with simple modifications so it will be omitted.

Lemma 2.2 *Let* $f \in \mathscr{B}^{\alpha}_{\omega,\log^{\beta}}$, *(where* $\alpha > 0$ *and* $\beta \geq 0$*), then*

$$\|f_t\| \leq C\|f\|, \text{ where } f_t(z) = f(tz), \; 0 < t < 1.$$

Proof. The proof is much akin to the corresponding result in ([38]).

Lemma 2.3 *Let* $0 < p < \infty$, $\alpha > 0$, $\beta \geq 0$. *If* $f \in H(\mathbb{D})$, *then*

$$\|f\|^{p}_{p,p,\phi} \simeq |f(0)|^{p} + \int_{\mathbb{D}} |f^{'}(z)|^{p} \omega(1-|z|)(1-|z|^{2})^{p} \frac{\phi^{p}(|z|)}{(1-|z|)} dA(z).$$

Proof. The proof is similar to the corresponding result in ([29]) with simple modifications so it will be omitted.

Theorem 2.1 *Let* $\mathbf{g} \in H(\mathbb{D})$, $0 < p < \infty$, $\alpha > 0$ *and* $\beta \geq 0$. *For a given reasonable function* $\omega : (0,1] \to (0,\infty)$ *assume that* $\omega(1-|z|) \approx \omega^{n}(1-|z|)$; $n \geq 0$. *Then the following statements are equivalent:*

$(a)\, I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\omega,log^{\beta}} \to H(p,p,\omega,\phi)$ *is bounded,*

$(b)\, I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\omega,log^{\beta},0} \to H(p,p,\omega,\phi)$ *is bounded,*

(c)

$$\int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \omega(1-|z|)\phi^p(|z|)(1-|z|)^{p(1-\alpha)}}{(1-|z|)\big(\ln(e^{\beta/\alpha}/(1-|z|^2))\big)^{\beta p}} dA(z) < \infty. \tag{2.1}$$

Proof. $(a) \Rightarrow (b)$. This implication is clear.

$(b) \Rightarrow (c)$. Assume that $I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\omega,log^{\beta},0} \to H(p,p,\omega,\phi)$ is bounded. In view of Lemma 2.1 there are $h_1, h_2 \in \mathscr{B}^{\alpha}_{\omega,log^{\beta}}$ such that

$$\frac{C\omega(1-|z|)}{(1-|z|^2)^{\alpha} \ln^{\beta} \frac{e^{\beta/\alpha}}{1-|z|^2}} \leq \frac{C\omega(1-|z|)}{(1-|z|)^{\alpha} \ln^{\beta} \frac{e^{\beta/\alpha}}{1-|z|}} \leq C(|h_1^{'}| + |h_2^{'}|).$$

Let $\{t_n\} \subset (0,1)$ be a sequence converging to 1, $(h_j)_n = h_j(t_n z)$ for $j = 1,2$, then $(h_j)_n \in \mathscr{B}^{\alpha}_{\omega,\log^{\beta},0}$, and $I_{\mathbf{g}}(h_1)_n, I_{\mathbf{g}}(h_2)_n \in H(p,p,\omega,\phi)$, hence

$$\begin{aligned}
\frac{\omega(1-|t_n z|)|\mathbf{g}(z)t_n|^p}{(1-|t_n z|^2)^{\alpha p} \left(ln^{\beta} \frac{e^{\beta/\alpha}}{1-|t_n z|^2} \right)^p} &\leq C|\mathbf{g}(z)|^p (|t_n h_1^{'}(t_n z)|^p + |t_n h_2^{'}(t_n z)|^p) \\
&= C|\mathbf{g}(z)|^p \left(|((h_1)_n)^{'}(z)|^p + |((h_2)_n)^{'}(z)|^p \right) \\
&\leq C\left(|(I_{\mathbf{g}}(h_1)_n)^{'}(z)|^p + |(I_{\mathbf{g}}(h_2)_n)^{'}(z)|^p \right).
\end{aligned}$$

From Lemmas 2.2 and 2.3, we have that

$$\begin{aligned}
&\int_{\mathbb{D}} \frac{\omega(1-|t_n z|)|\mathbf{g}(z)t_n|^p}{(1-|t_n z|^2)^{\alpha p} \left(ln^{\beta} \frac{e^{\beta/\alpha}}{1-|t_n z|^2} \right)^p} (1-|z|^2)^p \frac{\phi^p(|z|)}{1-|z|} dA(z) \\
\leq\ & C \int_{\mathbb{D}} |(I_{\mathbf{g}}(h_1)_n)^{'}(z)|^p (1-|z|^2)^p \frac{\phi^p(|z|)}{1-|z|} dA(z) \\
+\ & C \int_{\mathbb{D}} |(I_{\mathbf{g}}(h_1)_n)^{'}(z)|^p (1-|z|^2)^p \frac{\phi^p(|z|)}{1-|z|} dA(z) \\
\leq\ & C(\|I_{\mathbf{g}}(h_1)_n\|^p_{p,p,\phi} + \|I_{\mathbf{g}}(h_2)_n\|^p_{p,p,\phi}) \\
\leq\ & C\|I_{\mathbf{g}}\|^p < \infty.
\end{aligned}$$

Thus by Fatou's lemma, we obtain

$$\int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \omega(1-|z|)}{\left(\ln(e^{\beta/\alpha}/(1-|z|^2)) \right)^{\beta p}} \frac{\phi^p(|z|)}{1-|z|} dA(z) \leq C,$$

which proves condition (2.1). To prove $(c) \Rightarrow (a)$, we assume that

$$L = \int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \omega(1-|z|)(1-|z|)^{p(1-\alpha)}}{\left(\ln(e^{\beta/\alpha}/(1-|z|^2))\right)^{\beta p}} \frac{\phi^p(|z|)}{1-|z|} dA(z) < \infty.$$

For each $f \in \mathscr{B}^{\alpha}_{\omega,\log^{\beta}}$, we have

$$\begin{aligned}
&\int_{\mathbb{D}} |(I_{\mathbf{g}}f)'(z)|^p \omega(1-|z|)(1-|z|^2)^p \frac{\phi^p(|z|)}{1-|z|} dA(z) \\
&= \int_{\mathbb{D}} |f'(z)\mathbf{g}(z)|^p \omega(1-|z|)(1-|z|^2)^p \frac{\phi^p(|z|)}{1-|z|} dA(z) \\
&\leq C\|f\|^p_{\mathscr{B}^{\alpha}_{\omega,\log^{\beta}}} \int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \omega^2(1-|z|)(1-|z|)^{p(1-\alpha)}}{\left(\ln(e^{\beta/\alpha}/(1-|z|))\right)^{\beta p}} \frac{\phi^p(|z|)}{1-|z|} dA(z) \\
&\leq C\|f\|^p_{\mathscr{B}^{\alpha}_{\omega,\log^{\beta}}} \int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \omega(1-|z|)(1-|z|)^{p(1-\alpha)}}{\left(\ln(e^{\beta/\alpha}/(1-|z|))\right)^{\beta p}} \frac{\phi^p(|z|)}{1-|z|} dA(z) \leq CL\|f\|^p_{\mathscr{B}^{\alpha}_{\log^{\beta}}},
\end{aligned}$$

then $I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\log^{\beta}} \to H(p,p,\omega,\phi)$ is bounded.

Corollary 2.1 *Let* $\mathbf{g} \in H(\mathbb{D})$, $0 < p < \infty$, $\alpha > 0$ *and* $\beta \geq 0$. *For a given reasonable function* $\omega : (0,1] \to (0,\infty)$ *assume that* $\omega(1-|z|) \approx \omega^n(1-|z|)$; $n \geq 0$. *Then the following statements are equivalent:*

$(a)\, I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\omega} \to H(p,p,\omega,\phi)$ *is bounded,*

$(b)\, I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{\omega,0} \to H(p,p,\omega,\phi)$ *is bounded,*

(c)

$$\int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \omega(1-|z|)\phi^p(|z|)(1-|z|)^{p(1-\alpha)}}{(1-|z|)} dA(z) < \infty. \tag{2.2}$$

Proof. The proof follows by letting $\ln(e^{\beta/\alpha}/(1-|z|^2)) = 1$ in Theorem 2.1.

When $\omega = 1$, we can obtain the following result.

Corollary 2.2 *Let* $\mathbf{g} \in H(\mathbb{D})$, $0 < p < \infty$, $\alpha > 0$ *and* $\beta \geq 0$. *Then the following statements are equivalent:*

$(a)\, I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{log^{\beta}} \to H(p,p,\phi)$ *is bounded,*

$(b)\, I_{\mathbf{g}} : \mathscr{B}^{\alpha}_{log^{\beta},0} \to H(p,p,\phi)$ *is bounded,*

(c)

$$\int_{\mathbb{D}} \frac{|\mathbf{g}(z)|^p \phi^p(|z|)(1-|z|)^{p(1-\alpha)}}{(1-|z|)\big(\ln(e^{\beta/\alpha}/(1-|z|^2))\big)^{\beta p}} dA(z) < \infty. \tag{2.3}$$

Remark 2.1 It is still an open problem to study integral operators on some hyperbolic classes. For more information on such classes, we refer to [13, 15, 20, 35].

Remark 2.2 It is still an open problem to study integral operators on some analytic, harmonic and meromorphic classes which defined and studied in [8, 18, 19, 26].

Remark 2.3 It is still an open problem to study integral operators in quaternion function spaces. For more details on some classes of quaternion function spaces, we refer to [7, 9, 10, 11, 12, 16, 21, 27, 28, 31, 32, 33] and others.

Remark 2.4 How one can investigate the order and type of weighted logarithmic Bloch functions that defined in this paper? For some studies on the order and type in several function spaces, we refer to [9, 34] and others.

Conflict of Interests

The author declares that there is no conflict of interests.

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