Available online at http://scik.org
J. Math. Comput. Sci. 3 (2013), No. 6, 1475-1480
ISSN: 1927-5307

SOLITON SOLUTIONS OF (2+1)-ZOOMERON EQUATION AND DUFFING EQUATION AND SRLW EQUATION

AMINAH QAWASMEH

Department of Mathematics and Statistics,

Jordan University of Science and Technology, Irbid 22110, Jordan

Abstract. In this paper, we use the sine-cosine function method to construct the traveling wave solutions for three models; namely the (2+1)-dimensional Zoomeron equation, the Duffing equation and the Symmetric Regularized Long Wave equation (SRLW). These equations play a very important role in mathematical physics and engineering sciences.

Keywords: Sine-cosine function method, (2+1)-Zoomeron equation, Duffing equation and SRLW equation, Traveling wave solution.

2000 AMS Subject Classification: 74J35; 35C08

1. Introduction

Exact solutions to nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. In the literature, many significant methods have been proposed for obtaining exact solutions of nonlinear partial differential equations (PDEs) such as the tanh method, trigonometric and hyperbolic function methods, the rational sine-cosine method, the extended tanh-function method, the Exp-function method, the Hirota's method, Hirota bilinear forms, the tanh-sech method and

E-mail address: ammon_q@hotmail.com

Received October 30, 2013

so on [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The main aim of this paper is to apply the sine-cosine function method with the help of symbolic computation to obtain soliton solutions of the (2+1)-dimensional Zoomeron equation, the Duffing equation and the Symmetric Regularized Long Wave equation (SRLW) given, respectively, by:

$$(\frac{u_{xy}}{u})_{tt} - (\frac{u_{xy}}{u})_{xx} + 2(u^2)_{xt} = 0, \tag{1}$$

$$u_{tt} + au + bu^3 = 0, \tag{2}$$

$$u_{tt} + u_{xx} + u_{xxtt} + (uu_x)_t = 0. \tag{3}$$

Next, we survey in brief the construction of the sine-cosine function method.

2. The sine-cosine method

Restricting our attention to traveling waves, we use the transformation $u(x,t) = u(\zeta)$, where the wave variable $\zeta = (x - ct)$ transforms the PDE to an equivalent ODE. The sine-cosine algorithm [12, 13, 14, 15, 16] admits the use of the ansatz

$$u(x,t) = \lambda \cos^{\beta}(\mu\zeta), \quad |\zeta| \leq \frac{\pi}{2\mu} \tag{4}$$

and the ansatz

$$u(x,t) = \lambda \sin^{\beta}(\mu\zeta), \quad |\zeta| \leq \frac{\pi}{\mu}, \tag{5}$$

where $\lambda,\ \mu,\ c$ and β are parameters that will be determined. Substituting (4) or (5) into the reduced ODE gives a polynomial equation of cosine or sine terms. Balancing the exponents of the trigonometric functions cosine or sine, collecting all terms with same power in $\cos^k(\mu\zeta)$ or $\sin^k(\mu\zeta)$ and set to zero their coefficients to get a system of algebraic equation among the unknowns $\lambda,\ \mu,\ c$ and β. The problem is now completely reduced to an algebraic one. Having determined $\lambda,\ \mu,\ c$ and β by algebraic calculations or by using symbolic computerized calculations, the solutions proposed in (4) and (5) follow immediately.

3. The (2+1)-dimensional Zoomeron equation

In this section we construct explicit traveling wave solutions of an evolution equation that called Zoomeron equation given by:

$$(\frac{u_{xy}}{u})_{tt} - (\frac{u_{xy}}{u})_{xx} + 2(u^2)_{xt} = 0, \tag{6}$$

where $u(x,y,t)$ is the amplitude of the relevant wave mode. In the literature, there are few articles about this equation. We only know that this equation was introduced by Calogero and Degasperis. Recently, Reza [17] obtained periodic and soliton solutions to Zoomeron equation by means of G'/G expansion method method. Alquran and Al-Khaled [18] investigated this model by means of Exp-function method, extended tanh method and the sech-tanh function method.

Now, using the wave variable $\zeta = x + by - ct$ transforms Equation (6) into the ODE:

$$b(1-c^2)u'' - 2cu^3 + Ru = 0 \tag{7}$$

Where R is the integration constant. Substituting ansatz (4) in equation (7) yields

$$\lambda R\cos(\zeta\mu)^{\beta} - 2\lambda^3 c\cos(\zeta\mu)^{3\beta} + \lambda b(1-c^2)(-\beta\mu^2\cos(\zeta\mu)^{\beta} + (-1+\beta)\beta\mu^2\cos(\zeta\mu)^{-2+\beta}\sin(\zeta\mu)^2) = 0. \tag{8}$$

Using the identity $\sin(\zeta\mu)^2 + \cos(\zeta\mu)^2 = 1$, then equation (8) can be rewritten as

$$(-\lambda\beta b\mu^2 + \lambda\beta^2 b\mu^2 + \lambda b\beta c^2\mu^2 - \lambda b\beta^2 c^2\mu^2)\cos(\zeta\mu)^{-2+\beta} + (\lambda R - \lambda b\beta^2\mu^2 + \lambda b\beta^2 c^2\mu^2)\cos(\zeta\mu)^{\beta} - 2\lambda^3 c\cos(\zeta\mu)^{3\beta} = 0. \tag{9}$$

Balancing the exponents of the cosine function in equation (9) we get $\beta = -1$ and the following system:

$$\begin{aligned} 0 &= R + b(-1+c^2)\mu^2, \\ 0 &= \lambda^2 c + b(-1+c^2)\mu^2. \end{aligned} \tag{10}$$

Solving for λ and μ we obtain

$$\lambda = -\frac{\sqrt{R}}{\sqrt{c}}, \qquad \mu = -\frac{\sqrt{R}}{\sqrt{b-bc^2}}. \tag{11}$$

Substituting (11) in (4), the first solution of the (2+1)-dimensional Zoomeron equation is

$$u_1(x,t) = \pm\frac{\sqrt{R}}{\sqrt{c}}\ \sec\left(\frac{\sqrt{R}}{\sqrt{b-bc^2}}(-ct+x+by)\right). \tag{12}$$

If we use ansatz (5) the following second solution of Zoomeron is obtained

$$u_2(x,t) = \pm\frac{\sqrt{R}}{\sqrt{c}}\ \csc\left(\frac{\sqrt{R}}{\sqrt{b-bc^2}}(-ct+x+by)\right). \tag{13}$$

4. The Duffing equation

The Duffing equation [19] reads

$$u_{tt} + au + bu^3 = 0, \tag{14}$$

where a and b are real constants. The Duffing equation describes the motion of a classical particle in a double well potential. This equation can display chaotic behavior. For $b > 0$, the equation represents a hard spring, and for

$b<0$ it represents a soft spring, Using the wave variable $\zeta = x - ct$ transforms (14) into the following ODE

$$c^2u'' + au + bu^3 = 0. \tag{15}$$

Now, substituting ansatz (4) in equation (15) we get the following equation of cosine terms

(16)

$$-\lambda\beta c^2\mu^2\cos(\zeta\mu)^{-2+\beta} + \lambda\beta^2c^2\mu^2\cos(\zeta\mu)^{-2+\beta} + a\lambda\cos(\zeta\mu)^{\beta} - \lambda\beta^2c^2\mu^2\cos(\zeta\mu)^{\beta} + \lambda^3 b\cos(\zeta\mu)^{3\beta} = 0.$$

Balancing the exponents of the cosine function in equation (16) we get $\beta = -1$ and the following system:

$$\begin{aligned} 0 &= a - c^2\mu^2, \\ 0 &= \lambda^2 b + 2c^2\mu^2. \end{aligned} \tag{17}$$

Solving the above system we obtain

$$\lambda = \pm\sqrt{2a}, \qquad \mu = \frac{\sqrt{a}}{c}, \tag{18}$$

provided that b is prescribed to be one. Substituting (18) in (4) the first solution of the Duffing equation is

$$u_1(x,t) = \pm\sqrt{2a}\ \sec(\frac{\sqrt{a}\ (x-ct)}{c}). \tag{19}$$

Moreover, if we use ansatz (5), the second solution is

$$u_2(x,t) = \pm\sqrt{2a}\ \csc(\frac{\sqrt{a}\ (x-ct)}{c}). \tag{20}$$

5. The SRLW equation

The Symmetric Regularized Long Wave equation (SRLW) [20, 21] is given by

$$u_{tt} + u_{xx} + u_{xxtt} + (uu_x)_t = 0, \quad x\varepsilon R\ , t > 0. \tag{21}$$

This equation was shown to describe weekly nonlinear ion acoustic and space-charge waves, and the real-valued $u(x,t)$ corresponds to the dimensionless fluid velocity with a decay condition. Using the wave variable $\zeta = x - ct$ transforms (21) into the following ODE

$$c^2u'' + (c^2+1)u - \frac{c}{2}u^2 = 0. \tag{22}$$

Now, substituting ansatz (4) in equation (22) we get the following equation of cosine terms

$$-\lambda\beta c^2\mu^2\cos(\zeta\mu)^{-2+\beta} + \lambda\beta^2c^2\mu^2\cos(\zeta\mu)^{-2+\beta} + \lambda\cos(\zeta\mu)^{\beta} + \lambda c^2\cos(\zeta\mu)^{\beta} - \lambda\beta^2c^2\mu^2\cos(\zeta\mu)^{\beta} \tag{23}$$

$$-\frac{1}{2}\lambda^2 c\cos(\zeta\mu)^{2\beta} = 0.$$

Balancing the exponents of the cosine function in equation (23) we get $\beta = -2$ and the following system:

$$\begin{aligned} 0 &= -2 + c^2(-2+8\mu^2), \\ 0 &= \lambda - 12c\mu^2. \end{aligned} \tag{24}$$

Solving for λ and μ we obtain

$$\lambda = \frac{3(1+c^2)}{c}, \qquad \mu = -\frac{\sqrt{1+c^2}}{2c}. \tag{25}$$

Substituting (25) in (4), the first solution of the SRLW equation is

$$u_1(x,t) = \frac{3(1+c^2)\sec(\frac{\sqrt{1+c^2}(-ct+x)}{2c})^2}{c}, \tag{26}$$

and by ansatz (5) the second solution is

$$u_2(x,t) = \frac{3(1+c^2)\csc(\frac{\sqrt{1+c^2}(-ct+x)}{2c})^2}{c}. \tag{27}$$

6. Conclusion

The sine-cosine method has been successfully implemented to establish solitary wave solutions for various type of nonlinear PDEs. The method can be used for many other nonlinear equations or coupled ones.

Conflict of Interests

The author declares that there is no conflict of interests.

REFERENCES

[1] H. Triki, A.M. Wazwaz, Bright and dark soliton solution for a K(m,n) equation with t−dependent coefficients, Phys. Lett. A 373 (2009) 2162-2165.

[2] H. Triki, A.M. Wazwaz, Sub-ODE method and soliton solutions for the variable-coefficient mKdV equation, Appl. Math. Comput. 214 (2009) 370-373.

[3] A. Biswas, Solitary wave solution for the generalized Kawahara equation, Appl. Math. Lett. 22 (2009) 208-210.

[4] A. Biswas, 1−soliton solution of the B(m,n) equation with generalized evolution, Nonlinear Sci. Numer. Simul. 14 (2009) 3226-3229.

[5] S.L. Palacios, Two simple ansatze for obtaining exact solutions of high dispersive nonlinear schrodinger equations, Chaos, Solitons and Fractals 19 (2004) 203-207.

[6] S. Shukri, K. Al-khaled, The extended tanh method for solving systems of nonlinear wave equations, Appl. Math. Comput. 217 (2010) 1997-2006.

[7] M. Alquran, Bright and dark soliton solutions to the Ostrovsky-Benjamin-Bona-Mahony (OS-BBM) equation, J. Math. Comput. Sci. 2 (2012) 15-22.

[8] H.M. Jaradat, S. Al-Shara, F. Awawdeh, M. Alquran, Variable coefficient equations of the Kadomtsev-Petviashvili hierarchy: multiple soliton solutions and singular multiple soliton solutions, Phys. Scr. 85 (2012) 035001.

[9] M. Alquran, M. Ali, K. Al-Khaled, Solitary wave solutions to shallow water waves arising in fluid dynamics, Nonlinear Stud. 19(4) (2012) 555-562.

[10] M. Alquran, R. Al-Omary, Q. Katatbeh, New explicit solutions for homogeneous KdV equations of third order by trigonometric and hyperbolic function methods, Appl. Appl. Math. 7 (2012) 211-225.

[11] Marwan Alquran, Kamel Al-Khaled, Hassan Ananbeh, New Soliton Solutions for Systems of Nonlinear Evolution Equations by the Rational Sine-Cosine Method. *Studies in Mathematical Sciences.*, Volume 3(1) (2011) pp. 1-9.

[12] M. Alquran, Solitons and periodic solutions to nonlinear partial differential equations by the Sine-Cosine method, Appl. Math. Inf. Sci. 6(1) (2012) 85-88

[13] A.H.A. Ali, A.A. Soliman, K.R. Raslan, Soliton solution for nonlinear partial differential equations by cosine-function method, Phys. Lett. A 368 (2007) 299-304.

[14] M. Alquran, K. Al-Khaled, Sinc and solitary wave solutions to the generalized Benjamin-Bona-Mahony-Burgers equations, Phys. Scr. 83 (2011) 065010.

[15] M. Alquran, K. Al-Khaled, The tanh and sine-cosine methods for higher order equations of Korteweg-de Vries type, Phys. Scr. 84 (2011) 025010.

[16] M. Alquran, A. Qawasmeh, Classifications of solutions to some generalized nonlinear evolution equations and systems by the sine-cosine method, Nonlinear Stud. 20(2) (2013) 261-270.

[17] R. Abazari, The solitary wave solutions of Zoomeron equation, Appl. Math. Sci. 5 (2012) 2943-2949.

[18] M. Alquran, K. Al-khaled, Mathematical methods for a reliable treatment of the (2+1)-dimensional Zoomeron equation, Math. Sci. 6(11) (2012).

[19] A. Bekir, O. Unsal, Exact solutions for a class of nonlinear wave equations by using first integral method, Int. J. Nonlinear Sci. 15 (2013) 99-110.

[20] C. Yong, L. Biao, Travelling wave solutions for generalized symmetric regularized long-wave equations with high-order nonlinear terms, Chinese Phys. doi:10.1088/1009-1963/13/3/007.

[21] B. Zheng, O. Unsal, Traveling Wave Solution For The SRLW Equation, Int. J. Nonlinear Sci. 15 (2013) 99-110.