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(1, 2) – DOMINATION IN GRAPHS

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Abstract. In this paper we discuss (1,2) – domination in different graphs and results obtained are compared with the domination in graphs.

Keywords: Dominating set, Domination number, (1,2) – dominating set, (1,2) – domination number.

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1. Introduction

Domination in a graph along with its many variations provide an extremely rich area of study. Berge [2] and Ore[6] were the first to define dominating sets. A new type of dominating set, (1,2) – dominating set is introduced by Steve Hedetniemi and Sandee Hedetniemi [5]. In this paper we present some basic theorems on these sets and the relation between the usual domination and (1,2)- domination.

2. Preliminaries

Let G = (V,E) be a simple graph. A subset D of V is a *dominating set* of G if every vertex of V- D is adjacent to a vertex of D. The *domination number* of G, denoted by γ (G),is the minimum cardinality of a dominating set of G.

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A *(1,2) – dominating set* in a graph G = (V,E) is a set S having the property that for every vertex v in V – S there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v. The order of the smallest (1,2)- dominating set of G is called the *(1,2) – domination number* of G and we denote it by $\gamma_{(1,2)}$

From the definition of (1,2) – dominating sets, we see that a (1,2) – dominating set contains atleast 2 vertices, (1,2) – domination number of a graph will be always ≥ 2 and (1,2) – dominating sets occur in graphs of order atleast 3.

A graph is said to be complete if each of its vertices is adjacent to every other vertex. A graph is said to be regular if each of its vertices has the same degree. A graph is said to be cubic graph if each of its vertices is of degree three. A bipartite graph is a graph in which vertices can be divided into two disjoint sets A and B such that every edge connects a vertex in A to one in B.

For each vertex x in a graph G, we introduce a new vertex x' and join x and x' by an edge. The resulting graph is called the *corona* of G.

3. Main results

Theorem 3.1

All (1,2) – dominating sets are dominating sets.

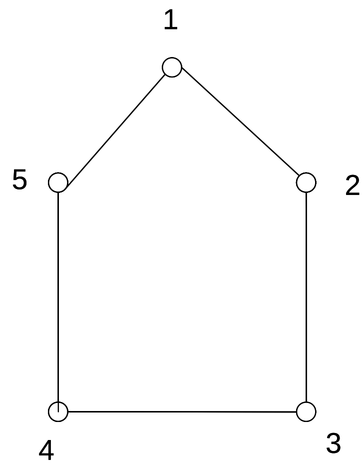
Proof:

The result is trivial from the definition of (1,2)- dominating sets.

But the converse need not be true.

Example:

Consider the graph



For this, { 1,4} is a dominating set.

But it is not a (1,2) – dominating set.

{2,3,4} is a (1,2) – dominating set. And it is a dominating set also.

3.1. 2. (1,2) – domination in Complete Graphs

Theorem 3.2

(1,2) – domination is not possible in complete graphs.

Proof: In a complete graph, each vertex is adjacent to every other vertices. So we cannot find a

(1,2) – dominating set. No vertex can be found at a distance atmost 2 from any other vertex.

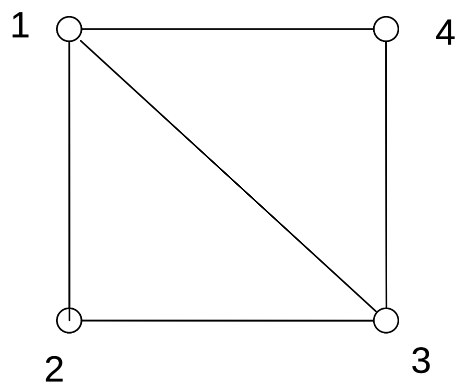
Let G be a complete graph with n vertices. Then it will have nC_2 edges and each vertex is of degree n – 1.The minimum number of edges to be deleted so as to become the resulting graph (1,2) – dominating is n - 2 . If we delete n – 2 edges from a complete graph, then in the resulting graph , we can find a (1,2) – dominating set.

Lemma 3.3

If a graph G with n vertices, has a vertex of degree n – 1 ,we cannot find a

(1,2) – dominating set.

Example:

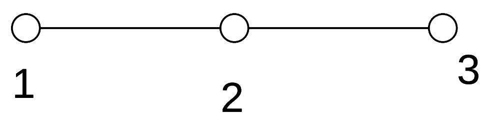


In this graph, we cannot find a (1,2) – dominating set since each vertex is adjacent to all other vertices.

3.1.3. Relation between domination number and (1,2) – domination number

In this section we consider different types of graphs and find out their domination number and (1,2)- domination number and check the relation between them.

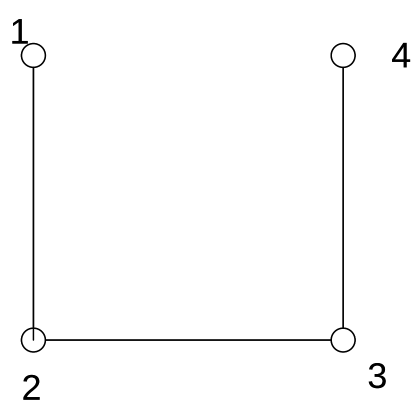
Consider the following graphs



Here {2} is a dominating set. γ (G) = 1.

{2,3} is a (1,2) – dominating set. $\gamma_{(1,2)} = 2$.

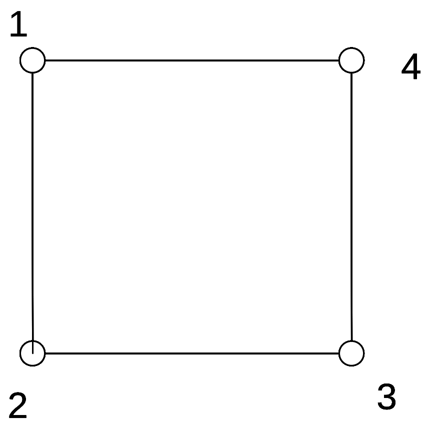
That is, $\gamma < \gamma_{(1,2)}$



Here {1,3}, {1,4}, {2,4}, {2,3} are all dominating sets. γ (G) = 2.

{1,4} is a (1,2) – dominating set. . $\gamma_{(1,2)} = 2$.

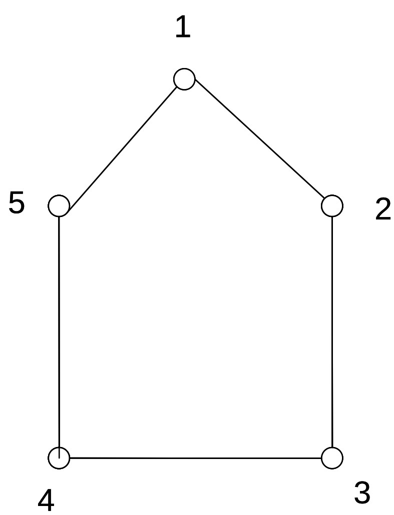
$\gamma = \gamma_{(1,2)}$



Here {1,4} , {2,3} ,{1,3}, {2,4} are dominating sets. γ (G) = 2.

{2,3} is a (1,2) – dominating set. $\gamma_{(1,2)} = 2$.

$\gamma = \gamma_{(1,2)}$

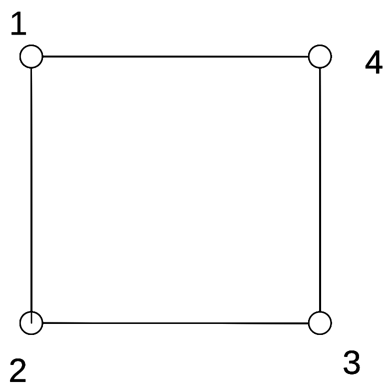


Here {1,4}, {1,3},{2,4},{3,5} are dominating sets. γ (G) = 2.

{2,3} is a (1,2) – dominating set. . $\gamma_{(1,2)} = 3$.

$\gamma < \gamma_{(1,2)}$

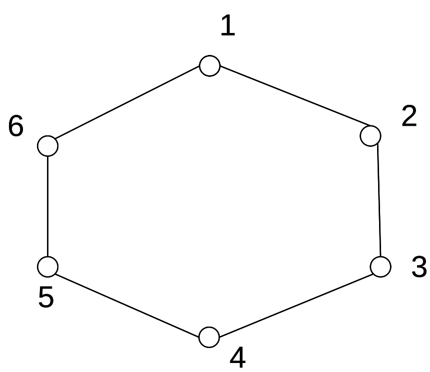
Consider some regular graphs with even number of vertices



Here {1,4}, {2,3},{1,3},{2,4} are dominating sets. γ (G) = 2.

{2,3} is a (1,2) – dominating set. $\gamma_{(1,2)} = 2$.

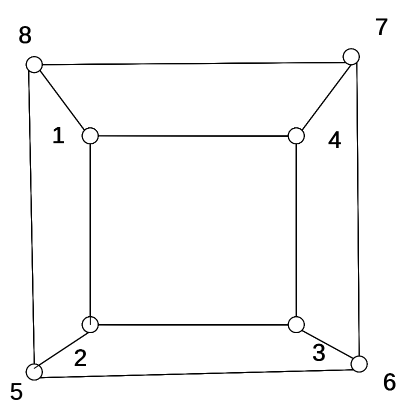
$\gamma = \gamma_{(1,2)}$



Here {1,3,5}, {2,4,6} are dominating sets. γ (G) = 3.

{1,4,6} is a (1,2) – dominating set. $\gamma_{(1,2)} = 3$.

$\gamma = \gamma_{(1,2)}$

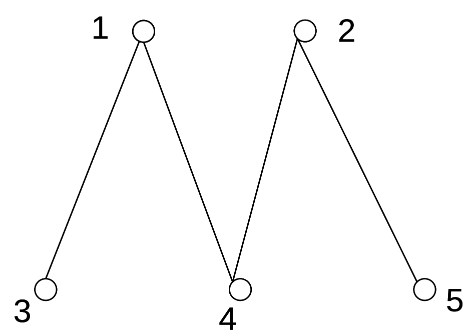


Here {1,2,3,4} , {5,6,7,8} are dominating. γ (G) = 4.

{1,2,3,4} is a (1,2) – dominating set. $\gamma_{(1,2)} = 4$.

$\gamma = \gamma_{(1,2)}$

Consider the bipartite graph given below



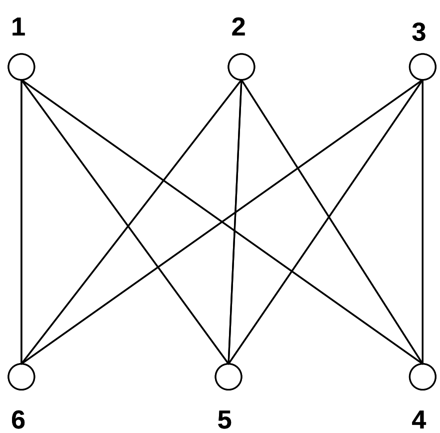
{1,2} is a dominating set. $\gamma(G) = 2$.

{1,4,5} is a (1,2) – dominating set. $\gamma_{(1,2)} = 3$.

$\gamma < \gamma_{(1,2)}$

Consider the cubic bipartite graphs

For n = 6,

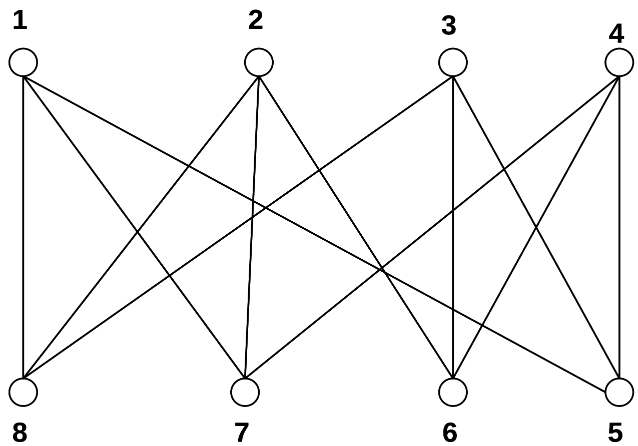


{1,5}, {2,6} are dominating sets. $\gamma(G) = 2$.

{1,5} is a (1,2) – dominating set. $\gamma_{(1,2)} = 2$.

$\gamma = \gamma_{(1,2)}$

For n = 8,

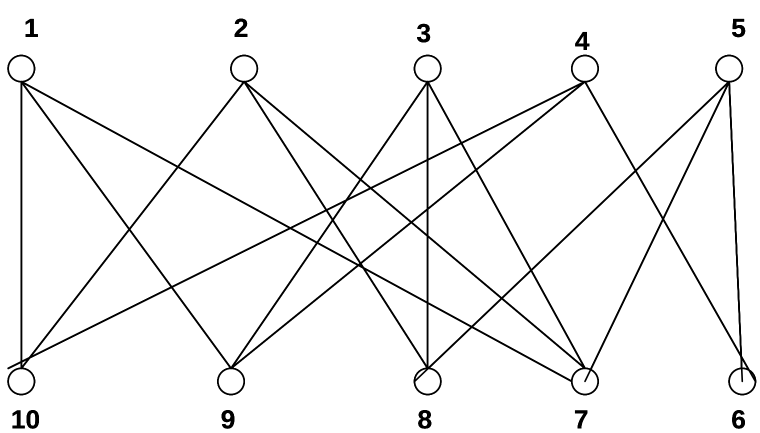


{1,6} is a dominating set. γ (G) = 2.

{1,6} is a (1,2) – dominating set. $\gamma_{(1,2)} = 2$.

$\gamma = \gamma_{(1,2)}$

For n = 10,



{2,5,9} is a dominating set. γ (G) = 3.

{2,4,6,8,10} is a (1,2) – dominating set. $\gamma_{(1,2)} = 5$.

$\gamma < \gamma_{(1,2)}$

In all the above cases, domination number is less than or equal to (1,2)- domination number.
From the above examples we have the following theorem.

Theorem 3.4
In a graph G, domination number is less than or equal to (1,2) – domination number.
Proof:
Let G be a graph and D be its dominating set. Then every vertex in V – D is adjacent to a vertex in D. That is, in D , for every vertex u, there is a vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance atmost 2 from u. So if we find a (1,2) – dominatinating set, it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than or equal to (1,2) – domination number.
Theorem 3.5
If G is a 2-regular graph, then the (1,2) – domination number of the corona of G is equal to the number of vertices of G.
Proof: Let G be a 2- regular graph. Then each of its vertices will be of degree 2.
In the corona of G, for each vertex x, we introduce a new vertex and join them.
Consequently, an edge is added to each of its vertices. By the definition of (1,2) – dominating set each vertex v in V – S has atleast one vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v. Hence (1,2) – dominating set of the corona of G will consist of all the vertices of G.

Theorem 3.6
If in a graph G, an edge e is added, $\gamma_{(1,2)}(G+ e) \geq \gamma_{(1,2)}(G)$
Proof: Let G be a graph. Let S be the (1,2) – dominating set of G.
If we add an edge to a vertex in S, that will not affect the cardinality of S.

If we add an edge to a vertex in V- S , the cardinality of (1,2) – dominating set will increase.

Therefore, $\gamma_{(1,2)}(G+e) \geq \gamma_{(1,2)}(G)$.

Theorem 3.7

If G is a complete bipartite graph, then the (1,2) – domination number $\gamma_{(1,2)}$ is 2.

Proof: Let G be a complete bipartite graph. Then V(G) can be partitioned in to 2 disjoint sets X and Y and each edge has one end in X and other end in Y. Since G is complete bipartite, each vertex of X is joined to every vertex in Y. A set of 2 vertices, one from X and another from Y will constitute a (1,2) – dominating set.

Therefore, $\gamma_{(1,2)} = 2$.

4. Conclusion

We found (1,2) – domination number of some graphs and compared them with the domination number. Also some preliminary theorems on (1,2)- dominating sets are proved.

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