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UNBIASED ESTIMATION IN BURR DISTRIBUTION

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Abstract. The aim of the present study is to discuss the uniformly minimum variance unbiased estimate (UMVUE) of the probability density function (pdf) of the Burr distribution. The UMVUE of the cumulative distribution function (cdf), pth quantile and rth moment of the Burr distribution are also obtained as the corresponding functional of this density estimator.

Keywords: Burr distribution, density function, UMVUE, moments, cumulative distribution function, complete sufficient statistic.

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1. Introduction

Let $X_1, \ldots, X_n$ be a random sample from a distribution with pdf $f(x,\theta)$, $\theta \in \Omega$ (parameter space), $x \in S$ (sample space). An important objective of the statistical theory of parametric estimation is to estimate $f(x,\theta)$. To estimate $f(x,\theta)$,we estimate θ (parameter) by a suitable estimator U and then replace θ by U in $f(x,\theta)$. Thus $f(x,U)$ is parametric estimate of the true density $f(x,\theta)$. However in many situations, we can obtain the density estimator better than $f(x,U)$. For example, if unbiasedness is desired, then $f(x,U)$ is not an unbiased estimator of $f(x,\theta)$ i.e. E[$f(x,U)$] $\neq$ $f(x,\theta)$. Further if we want to get a good estimator for a class of parametric function, then it is better to obtain a good estimator of the underlying density function

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and then obtain the estimators for these parametric function as the corresponding functional of the density estimator.

To obtain good estimator of the density function we can consider an unbiased estimator having minimum variance. That is, we can obtain the uniformly minimum variance unbiased estimator of the density function $f(x,\theta)$. In this paper we obtain the UMVUE of the pdf of Burr distribution. A random variable X is said to have a Burr distribution if its pdf is given by

$$f(x,\alpha,\beta)=\begin{cases}\dfrac{\alpha\beta x^{\alpha-1}}{(1+x^{\alpha})^{\beta+1}} & ,x\geq 0,\alpha>0,\beta>0\\ 0, & \text{otherwise}\end{cases} \tag{1}$$

The Burr distribution is an important distribution in statistical theory and this distribution finds many important applications in engineering, survival analysis, industry and actuarial science.

The UMVUE of the pdf of statistical distributions have been the subject of investigation by many authors. Asrabasdi [1] obtained the UMUVUE of the pdf of the Pareto distribution. Dixit and Jabbari Nooghabi [2] obtained the UMVUE of the pdf of Pareto distribution with the presence of outliers. Singh [3] has discussed the UMVUE of the pdf of several univariate probability distributions.

The organization of this paper is as follows. In section 2, we discuss the method of obtaining the UMVUE of the pdf of a statistical distribution. In section 3, we obtained the UMVUE of the pdf of Burr distribution. In section 4, we derived the UMVUE of cdf, pth quantile and rth moment of the Burr distribution as the corresponding functional of the true density estimator. Finally at the end of this paper a conclusion is given.

2. Preliminaries

Let $X_1,\ldots,X_n$ be a random sample from a distribution with pdf $f(x,\theta)$, $\theta\in\Omega$, $x\in S$ and T be complete sufficient statistic for the family { $f(x,\theta)$, $\theta\in\Omega$ }. The conditional density of X_1 given T=t, denoted by g(x|t), is unbiased for $f(x,\theta)$ since

$$E[g(x|t)] = \int_t g(x\,|\,t)h(t,\theta)dt\ ,$$

$$= \int_t k(x,t,\theta)dt$$

$$= f(x,\theta)$$

where $k(x,t,\theta)$ and $h(t,\theta)$ denote respectively the joint pdf of X_1 and T and marginal pdf of T. Since g(x|t) is a function of complete sufficient statistic which is unbiased for $f(x,\theta)$ hence g(x|t) is the UMVUE of $f(x,\theta)$.

3. Main Results

3.1 UMVUE

Let $X_1,\ldots,X_n$ be a random sample from $f(x,\theta)$ in (1). We shall obtain the UMVUE of $f(x,\theta)$ when α is known. If α is known, then T = $\sum \log(1+x^{\alpha})$ is complete sufficient statistic for β or for the family { $f(x,\theta), \theta \in \Omega$ }. To obtain g(x|t), the UMVUE of $f(x,\theta)$, we proceed as follows. Note that by definition,

$$g(x\,|\,t) = \frac{k(x,t,\theta)}{h(t,\theta)} \tag{2}$$

$$= \frac{p(t\,|\,x)\,f(x,\theta)}{h(t,\theta)} \tag{3}$$

where p(t|x) is the conditional pdf of T given X_1 at x. The statistic T has gamma distribution with parameter n and β. To obtain p(t|x), we note that

$$T = \sum_{i=1}^{n} \log(1+x_i^{\alpha})$$

$$= \log(1+x_1^{\alpha}) + \sum_{i=2}^{n} \log(1+x_i^{\alpha})$$

$$T = \log(1+x_1^{\alpha}) + W \tag{4}$$

where

$$W = \sum_{i=2}^{n} \log(1+x_i^{\alpha})$$

The conditional density of T given X_1=x is the unconditional density of W evaluated at t – log(1+x^{α}). The statistic W has gamma distribution with parameter (n-1) and β. Thus from (3) g(x|t) comes out as below

$$g(x|t) \quad = \frac{\frac{\beta^{n-1}}{\lceil n-1} e^{[t-\log(1+x^{\alpha})]\beta} (t-\log(1+x^{\alpha}))^{n-2} \times \frac{\alpha\beta x^{\alpha-1}}{(1+x^{\alpha})^{\beta+1}}}{\frac{\beta^{n}}{\lceil n} e^{-t\beta} t^{n-1}}$$

$$= \begin{cases} \left(\frac{n-1}{t}\right) \frac{\alpha x^{\alpha-1}}{(1+x^{\alpha})} \left(1-\frac{\log(1+x^{\alpha})}{t}\right)^{n-2} & if \quad t > \log(1+x^{\alpha}) \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

3.2 UMVUE of cdf, pth quantile and rth moment

In this section we obtained the UMVUE of cdf, pth quantile and rth moment of the Burr distribution as the corresponding functional of density estimator g(x|t) given in (5). The cdf of the Burr distribution is given by:

$$F(x) = \begin{cases} 1-\left[1+x^{\alpha}\right]^{-\beta} & if \;\; 0 \le x < \infty \\ 0, & \text{otherwise} \end{cases}$$

The UMVUE of F(x) is given by

$$\hat{F}(u) = \int_0^u g(x|t)dx$$

$$\hat{F}(u) = \frac{(n-1)}{t} \int_0^u \frac{\alpha x^{\alpha-1}}{(1+x^{\alpha})} \left[1-\frac{\log(1+x^{\alpha})}{t}\right]^{n-2} dx$$

$$\hat{F}(u) = \begin{cases} 0 & if \;\; u < 0 \\ 1 - \left[1 - \dfrac{\log(1+x^{\alpha})}{t}\right]^{n-1} & if \;\; 0 < u < (e^t - 1)^{1/\alpha} \\ 1 & if \;\; u > (e^t - 1)^{1/\alpha} \end{cases} \tag{6}$$

The UMVUE $\hat{\theta}_p$ of the pth quantile θ_p of the Burr distribution is given by

$$\hat{F}(\hat{\theta}_p) = p$$

$$= \int_0^{\hat{\theta}_p} g(x \mid t) dx = p$$

$$(\hat{\theta}_p) = e^{t[(1-p)^{\frac{1}{n-1}} - 1]^{1/\alpha}} \tag{7}$$

Further, the UMVUE $\hat{\mu}'_r$ of the rth moment μ'_r of the Burr distribution is given by

$$\hat{\mu}'_r = E[X^r \mid T(X) = t]$$

$$= \int_0^{(e^t-1)^{1/\alpha}} x^r g(x \mid t) dx$$

$$= \int_0^{(e^t-1)^{1/\alpha}} x^r \frac{(n-1)}{t} \frac{\alpha x^{\alpha-1}}{(1+x^{\alpha})} \left[1 - \frac{\log(1+x^{\alpha})}{t}\right]^{n-2} dx$$

Let $I_r^{n-1} = \dfrac{(n-1)}{t} \displaystyle\int_0^{(e^t-1)^{1/\alpha}} x^r \frac{\alpha x^{\alpha-1}}{\log(1+x^{\alpha})} \left[1 - \frac{\log(1+x^{\alpha})}{t}\right]^{n-2} dx$

On integration by parts we get the following recurrence relation

$$I_r^{n-1} = \frac{rt}{\alpha n} I_{r-\alpha}^{n} + \frac{rt}{\alpha n} I_r^{n}, \quad n > 2$$

Thus

$$\hat{\mu}'_r = \frac{rt}{\alpha n} I^n_{r-\alpha} + \frac{rt}{\alpha n} I^n_r, \quad n > 2$$

Conclusion

In this paper we obtained the UMVUE of the pdf of Burr distribution. This UMVUE can be used to characterize the Burr distribution and to obtain a goodness of fit test for Burr distribution.

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