Available online at http://scik.org
J. Math. Comput. Sci. 4 (2014), No. 2, 425-445
ISSN: 1927-5307

ON COMPACT AND SEMICOMPACT FUZZY SOFT TOPOLOGICAL SPACES

PRADIP KUMAR GAIN[1,*], RAMKRISHNA PRASAD CHAKRABORTY[2] AND MADHUMANGAL PAL[3]

[1]Department of Mathematics, Kharagpur College,Inda, Kharagpur, Paschim medinipur-721305, West Bengal, India

[2]Department of Mathematics, Hijli College, Hijli, Kharagpur, Paschim Medinipur-721301, West Bengal, India

[3]Department of Applied Mathematics with Oceanology and Computer Programming,Vidyasagar University, Midnapore-721102, West Bengal, India

Abstract: The concept of compactness is one of the central and important concepts of paramount interest to topologists and it seems to be the most celebrated type among all the covering properties. In this paper the concept of compactness of fuzzy soft topological spaces is introduced and characterized in terms of finite intersection property (FIP) and in terms of fuzzy soft mappings. This concept is also generalized by introducing the concept of fuzzy soft semi-compact topological spaces. Invariance of the property under suitable maps is also taken into consideration.

Keywords: fuzzy soft set, fuzzy soft topological space, fuzzy soft open set, fuzzy soft closed set, fuzzy soft mapping.

2010 AMS Subject Classification: 54A40, 06D72, 03E72, 54D30

---

[*]Corresponding author
Received January 18, 2014

1. Introduction

There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets etc. which can be considered as mathematical tools for dealing with uncertain data, obtained in various fields of engineering, physics, computer science, economics, social science, medical science, and of many other diverse fields. But all these theories have their own difficulties. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets, introduced by L.A. Zadeh [51] in 1965. This theory brought a paradigmatic change in mathematics. But there exists difficulty, how to set the membership function in each particular case. The theory of intuitionistic fuzzy sets is a more generalized concept than the theory of fuzzy sets, but this theory has the same difficulties. All the above mentioned theories are successful to some extent in dealing with problems arising due to vagueness present in the real world. But there are also cases where these theories failed to give satisfactory results, possibly due to inadequacy of the parameterization tool in them. As a necessary supplement to the existing mathematical tools for handling uncertainty, in 1999, Molodtsov [34] initiated the concept of soft set via a set-valued mapping. The theory of soft sets is free from the difficulties mentioned above. Since its introduction, the concept of soft set has gained considerable attention and this concept has been studied in a series of works [1,12,17,18,19,20,21,32,35,36,48,49,50,52] including some successful applications in information processing [15,16,39,54], decision-making [9,13,33,38,42], demand analysis [11], forecasting [47], relations [10], algebraic structures of the set theory [1,2,4,12,21,22,23,24,25,26,27,28,29,41,45], topology [7,12,44,46,53], theory of BCK/BCI-algebras [22], operation research [6,14,30] etc. In recent times, researchers have contributed a lot towards fuzzification of soft theory. Maji et al. [37], introduced the concept of fuzzy soft set and some properties regarding fuzzy soft union, fuzzy soft intersection, complement of fuzzy soft set, De Morgan Law etc.

2. Preliminaries

Definition 2.1. [34**]** Let $A \subseteq E$. A pair (F, A) is called a soft set over U if and only if F is a mapping given by $F : A \to \mathrm{P}(U)$ such that $F(e) = \varphi$ if $e \notin A$ and $F(e) \neq \varphi$ if $e \in A$, where φ stands for the empty set, U is an initial universe set, E is the set of parameters and $\mathrm{P}(U)$ is the set of all subsets of U. Here F is called approximate function of the soft set (F, A) and the value $F(e)$ is a set called e–element of the soft set. In other words, the soft set is a parameterized family of subsets of the set U.

Definition 2.2. [51] A fuzzy set A in any arbitrary set U is defined by the mapping $\mu_A : U \rightarrow [0,1] = I$ [or by $A : U \rightarrow [0,1] = I$], where $\mu_A(u)$ or $A(u)$ states the grade of belongingness (membership) of u in A. That is, a fuzzy set A in U can be represented by the set of ordered pairs $A = \{ (u, \mu_A(u)) : u \in U \}$. The family of all fuzzy sets in U is denoted by I^U.

Definition 2.3. [51] Let A and B be two fuzzy sets in U. That is, $A, B \in I^U$.

1. A is contained in B, denoted by $A \leq B$, *if and only if* $A(u) \leq B(u)$ for all $u \in U$.
2. The union of A and B, denoted by $A \vee B$, is a fuzzy set C defined as $C(u) = (A \vee B)(u) = Max\{ A(u), B(u) \}$ for each $u \in U$.
3. The intersection of A and B, denoted by $A \wedge B$, is a fuzzy set D defined as $D(u) = (A \wedge B)(u) = Min\{ A(u), B(u) \}$ for each $u \in U$.
4. The complement of a fuzzy set A is a fuzzy set, denoted by A', defined as $A'(u) = 1 - A(u)$ for every $u \in U$.

Definition 2.4. [37,43] Let U be an initial universe set, let E be a set of parameters, let $A \subseteq E$.

A pair (F, A) is called a fuzzy soft set over U if and only if F is a mapping given by $F : A \rightarrow I^U$ such that $F(e) = 0_U$ if $e \notin A$ and $F(e) \neq 0_U$ if $e \in A$, where $0_U(u) = 0$ for all $u \in U$. Here F is called approximate function of the fuzzy soft set (F, A) and the value $F(e)$ is a fuzzy set called e–element of the fuzzy soft set (F, A). Thus a fuzzy soft set (F, A) over U can be represented by the set of ordered pairs $(F, A) = \{ (e, F(e)) : e \in A, F(e) \in I^U \}$. In other words, the fuzzy soft set is a parameterized family of fuzzy subsets of the set U.

Definition 2.5. [8,9] A fuzzy soft set (F, A) over U is called a *null* fuzzy soft set, denoted by $\tilde{0}_E$, if $F(e) = 0_U$ for all $e \in A \subseteq E$.

Remark 1. According to the definition of fuzzy soft set, i.e., $F(e) \neq 0_U$ if $e \in A \subseteq E$, 0_U does not belong to the co-domain of F. Therefore, the concept of null fuzzy soft set can be defined as follows.

Definition 2.6. A fuzzy soft set (F, A) over U is called a *null* fuzzy soft set or an *empty* fuzzy soft set, whenever $A = \varphi$.

Definition 2.7. A fuzzy soft set (F, A) over U is said to be an A–*universal* fuzzy soft set if $F(e) = 1_U$ if $e \in A$, where $1_U(u) = 1$ for all $u \in U$.

An A–*universal* fuzzy soft set is denoted by $\tilde{1}_A$.

Definition 2.8. [43] A fuzzy soft set (F, A) over U is said to be an *absolute* fuzzy soft set or a *universal* fuzzy soft set if $A = E$ and $F(e) = 1_U$ for all $e \in E$.

An *absolute* fuzzy soft set is denoted by $\tilde{1}_E$.

Definition 2.9. [37] A fuzzy soft set (F, A) is said to be a fuzzy soft subset of a fuzzy soft set (G, B) over a common universe U if $A \subseteq B$ and $F(e) \leq G(e)$ for all $e \in A$.

We redefine fuzzy soft subset as follows.

Definition 2.10. A fuzzy soft set (F, A) is said to be a fuzzy soft subset of a fuzzy soft set (G, B) over a common universe U if either $F(e) = 0_U$ for all $e \in A$ or $A \subseteq B$ and $F(e) \leq G(e)$ for all $e \in A$.

If a fuzzy soft set (F, A) is a fuzzy soft subset of a fuzzy soft set (G, B) we write $(F, A) \tilde{\subseteq} (G, B)$.

(F, A) is said to be a fuzzy soft superset of a fuzzy soft set (G, B) if (G, B) is a fuzzy soft subset of (F, A) and we write $(F, A) \tilde{\supseteq} (G, B)$.

Definition 2.11. [43] Two fuzzy soft sets (F, A) and (G, B) over a common universe are said to be equal, denoted by $(F, A) = (G, B)$, if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$. That is, if $F(e) \leq G(e)$ and $G(e) \leq F(e)$ for all $e \in E$.

Definition 2.12. [5,43] The intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cap B$ and $H(e) = F(e) \wedge G(e)$ for all $e \in C$ and we write $(H, C) = (F, A) \tilde{\cap} (G, B)$.

In particular, if $A \cap B = \varphi$ or $F(e) \wedge G(e) = 0_U$ for every $e \in A \cap B$, then $H(e) = 0_U$.

Definition 2.13. [37] The union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A - B$, $H(e) = G(e)$ if $e \in B - A$, $H(e) = F(e) \vee G(e)$ if $e \in A \cap B$. In this case we write (H, C)

$= (F, A)\tilde{\cup}(G, B)$.

Definition 2.14. [37] The complement of a fuzzy soft set (F, A), denoted by $(F, A)^C$, is defined as $(F, A)^C = (F^C, \neg A)$, where $F^C : \neg A \to I^U$ is a mapping given by $F^C(e) = (F(\neg e))^C$ for all $e \in \neg A$.

Alternatively, the complement of a fuzzy soft set can be defined as follows.

Definition 2.15. [46] The fuzzy soft complement of a fuzzy soft set (F, A), denoted by $(F, A)^C$, is defined as $(F, A)^C = (F^C, A)$, where $F^C(e) = 1 - F(e)$ for every $e \in A$. Clearly, $((F, A)^C)^C = (F, A)$ and $(\tilde{1}_E)^C = \tilde{0}_E$ and $(\tilde{0}_E)^C = \tilde{1}_E$.

Proposition 2.1. *Let (F, A) be a fuzzy soft set over (U, E). Then*

1. $(F, A)\tilde{\cup}(F, A) = (F, A), (F, A)\tilde{\cap}(F, A) = (F, A)$
2. $(F, A)\tilde{\cup}\tilde{0}_E = (F, A), (F, A)\tilde{\cap}\tilde{0}_E = \tilde{0}_E$
3. $(F, A)\tilde{\cup}\tilde{1}_E = \tilde{1}_E, (F, A)\tilde{\cap}\tilde{1}_E = (F, A)$
4. $(F, A)\tilde{\cup}(F, A)^C = \tilde{1}_E, (F, A)\tilde{\cap}(F, A)^C = \tilde{0}_E$

Proposition 2.2. *Let (F, A), (G B), (H, C) be fuzzy soft sets over (U, E). Then*

1. $(F, A)\tilde{\cup}(G\ B) = (G, B)\tilde{\cup}(F, A), (F, A)\tilde{\cap}(G, B) = (G, B)\tilde{\cap}(F, A)$
2. $((F, A)\tilde{\cup}(G, B))^C = (G, B)^C\tilde{\cap}(F, A)^C, ((F, A)\tilde{\cap}(G, B))^C = (G, B)^C\tilde{\cup}(F, A)^C$
3. $((F, A)\tilde{\cup}(G, B))\tilde{\cup}(H, C) = (F, A)\tilde{\cup}((G, B)\tilde{\cup}(H, C)), ((F, A)\tilde{\cap}(G, B))\tilde{\cap}(H, C) = (F, A)\tilde{\cap}((G, B)\tilde{\cap}(H, C))$
4. $(F, A)\tilde{\cup}((G, B)\tilde{\cap}(H, C)) = ((F, A)\tilde{\cup}(G\ B))\tilde{\cap}((F, A)\tilde{\cup}(H, C)), (F, A)\tilde{\cap}((G, B))\tilde{\cup}(H, C)) = ((F, A)\tilde{\cap}(G, B))\tilde{\cup}((F, A)\tilde{\cap}(H, C))$

Definition 2.16. [43,46] A fuzzy soft topology τ on (U, E) is a family of fuzzy soft sets over (U, E), satisfying the following properties:

1. $\tilde{0}_E, \tilde{1}_E \in \tau$
2. If $(F, A), (G\ B) \in \tau$ then $(F, A)\tilde{\cap}(G, B) \in \tau$.

3. If $(F, A)_\alpha \in \tau$, $\forall \alpha \in \Lambda$ then $\tilde{\bigcup}_{\alpha \in \Lambda} (F, A)_\alpha \in \tau$.

Definition 2.17. [43] If τ is a fuzzy soft topology on (U, E), the triple (U, E, τ) is said to be a fuzzy soft topological space. Each member of τ is called a fuzzy soft open set in (U, E, τ).

Definition 2.18. [40] Let (U, E, τ) be a fuzzy soft topological space. A fuzzy soft set is called fuzzy soft closed if its complement is a member of τ.

Proposition 2.3. [40] *Let (U, E, τ) be a fuzzy soft topological space and let τ' be the collection of all fuzzy soft closed sets. Then*

1. $\tilde{0}_E$, $\tilde{1}_E \in \tau'$
2. *If $(F, A), (G\ B) \in \tau'$ then $(F, A) \tilde{\cup} (G, B) \in \tau'$.*
3. *If $(F, A)_\alpha \in \tau'$, $\forall \alpha \in \Lambda$ then $\tilde{\bigcap}_{\alpha \in \Lambda} (F, A)_\alpha \in \tau'$.*

Definition 2.19. [40,46] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft closure of (F, A), denoted by $\overline{(F, A)}$, is defined as the intersection of all fuzzy soft closed sets which contain (F, A). That is, $\overline{(F, A)} = \tilde{\cap}\{ (G, B) : (G, B)$ is fuzzy soft closed and $(F, A) \tilde{\subseteq} (G, B)\}$. Clearly, $\overline{(F, A)}$ is the smallest fuzzy soft closed set over (U, E) which contains (F, A). It is also clear that $\overline{(F, A)}$ is fuzzy soft closed and $(F, A) \tilde{\subseteq} \overline{(F, A)}$.

Definition 2.20. [40,46] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft interior of (F, A), denoted by $(F, A)^o$, is defined as the union of all fuzzy soft open sets which are contained in (F, A). That is, $(F, A)^o = \tilde{\cup}\{ (G, B) : (G, B)$ is fuzzy soft open and $(G, B) \tilde{\subseteq} (F, A) \}$. Clearly, $(F, A)^o$ is the largest fuzzy soft open set over (U, E) which is contained in (F, A). It is also clear that $(F, A)^o$ is fuzzy soft open and $(F, A)^o \tilde{\subseteq} (F, A)$.

3. Compact Fuzzy Soft Topological Spaces

In this section we define fuzzy soft cover of a fuzzy soft set over (U, E). We also define finite intersection property (*FIP*) of a family of fuzzy soft sets over (U, E). Then we define compact fuzzy soft topological spaces and we characterize compactness in terms of *FIP* and fuzzy soft mappings.

Definition 3.1. A family ζ of fuzzy soft sets is said to be a cover of a fuzzy soft set (F, A) if $(F, A) \tilde{\subseteq} \tilde{\bigcup}_{\alpha \in \Lambda} \{ (F, A)_\alpha : (F, A)_\alpha \in \zeta, \alpha \in \Lambda \}$. A subcover of ζ is a subfamily of ζ which is also a cover.

Definition 3.2. A family ζ of fuzzy soft sets is said to have finite intersection property (*FIP*) if the intersection of the members of each finite subfamily of ζ is not null fuzzy soft set.

Definition 3.3. A fuzzy soft topological space (U, E, τ) is said to be fuzzy soft compact if each cover of $\tilde{1}_E$ by fuzzy soft open sets over (U, E) has a finite subcover.

Theorem 3.1. *A fuzzy soft topological space (U, E, τ) is compact if and only if each family of fuzzy soft closed sets with finite intersection property has a non-null intersection.*

Proof.

Let ψ be a family of fuzzy soft sets in a fuzzy soft topological space(U, E, τ). Then ψ is a cover of $\tilde{1}_E$ if and only if one of the following conditions holds.

1. $\tilde{\bigcup}_{\alpha \in \Lambda} \{ (F, A)_\alpha : (F, A)_\alpha \in \psi \} = \tilde{1}_E$.
2. $(\tilde{\bigcup}_{\alpha \in \Lambda} \{ (F, A)_\alpha : (F, A)_\alpha \in \psi \})^C = (\tilde{1}_E)^C = \tilde{0}_E$.
3. $\tilde{\bigcap}_{\alpha \in \Lambda} \{ ((F, A)_\alpha)^C : (F, A)_\alpha \in \psi \} = \tilde{0}_E$.

Hence the fuzzy soft topological space (U, E, τ) is compact if and only if each family of fuzzy soft open sets over (U, E) such that no finite subfamily covers $\tilde{1}_E$, fails to be a cover and this is true if and only if each family of fuzzy soft closed sets which has the finite intersection property (*FIP*) has a non-null intersection.

Definition 3.4. [31] Let $FSS(U, E_1)$ and $FSS(V, E_2)$ be the families of all fuzzy soft sets over (U, E_1) and (V, E_2) respectively. Let $u : U \to V$ and $p : E_1 \to E_2$ be two functions. Then f_{pu} is called a fuzzy soft mapping from $FSS(U, E_1)$ to $FSS(V, E_2)$, denoted by $f_{pu} : FSS(U, E_1) \to FSS(V, E_2)$ and defined as follows:

(1) Let (F, A) be a fuzzy soft set in $FSS(U, E_1)$. Then the image of (F, A) under the fuzzy soft

mapping f_{pu} is the fuzzy soft set over (V, E_2) defined by $f_{pu}((F, A))$, where

$$f_{pu}((F, A))(e_2)(y) = \bigvee_{x\in u^{-1}(y)} \Big(\bigvee_{e_1\in p^{-1}(e_2)\cap A} F(e_1))(x) \text{ if } u^{-1}(y) \neq \varphi,\ p^{-1}(e_2)\cap A \neq \varphi$$

$$= 0_V \text{ otherwise.}$$

(2) Let (G, B) be a fuzzy soft set in $FSS(V, E_2)$. Then the preimage (inverse image) of (G, B) under the fuzzy soft mapping f_{pu} is the fuzzy soft set over (U, E_1) defined by $f^{-1}_{pu}((G, B))$, where

$$f^{-1}_{pu}((G, B))(e_1)(x) = G(p(e_1))(u(x)) \text{ for } p(e_1)\in B$$

$$= 0_U \text{ otherwise.}$$

Definition 3.5. If p and u are injective in definition 3.4, then the fuzzy soft mapping f_{pu} is said to be injective. If p and u are surjective then the fuzzy soft mapping f_{pu} is said to be surjective. If p and u are constant then f_{pu} is called constant.

Theorem 3.2. [31] *Let $(F, A)\in FSS(U, E_1)$, $\{ (F, A)_\alpha : \forall \alpha \in \Lambda\}\subset FSS(U, E_1)$ and $(G, B)\in FSS(V, E_2)$, $\{ (G, B)_\alpha : \forall\alpha \in \Lambda\}\subset FSS(V, E_2)$, where Λ is an index set. Then*

1. *If $(F, A)_1 \tilde{\subseteq} (F, A)_2$, then $f_{pu}((F, A)_1) \tilde{\subseteq} f_{pu}((F, A)_2)$.*
2. *If $(G, B)_1 \tilde{\subseteq} (G, B)_2$, then $f^{-1}_{pu}((G, B)_1) \tilde{\subseteq} f^{-1}_{pu}((G, B)_2)$.*
3. $f_{pu}(\tilde{\bigcup}_{\alpha\in\Lambda} (F, A)_\alpha) = \tilde{\bigcup}_{\alpha\in\Lambda} f_{pu}((F, A)_\alpha)$.
4. $f_{pu}(\tilde{\bigcap}_{\alpha\in\Lambda} (F, A)_\alpha) \tilde{\subseteq} \tilde{\bigcap}_{\alpha\in\Lambda} f_{pu}((F, A)_\alpha)$.
5. $f^{-1}_{pu}(\tilde{\bigcup}_{\alpha\in\Lambda} (G, B)_\alpha) = \tilde{\bigcup}_{\alpha\in\Lambda} f^{-1}_{pu}((G, B)_\alpha)$.
6. $f^{-1}_{pu}(\tilde{\bigcap}_{\alpha\in\Lambda} (G, B)_\alpha) = \tilde{\bigcap}_{\alpha\in\Lambda} f^{-1}_{pu}((G, B)_\alpha)$.
7. $f^{-1}_{pu}(\tilde{1}_{E_2}) = \tilde{1}_{E_1}$, $f^{-1}_{pu}(\tilde{0}_{E_2}) = \tilde{0}_{E_1}$.
8. $f_{pu}(\tilde{0}_{E_1}) = \tilde{0}_{E_2}$, $f_{pu}(\tilde{1}_{E_1}) \tilde{\subseteq} \tilde{1}_{E_2}$.

Theorem 3.3. *Let $(F, A)\in FSS(U, E_1)$, $\{ (F, A)_\alpha : \forall\alpha \in \Lambda\}\subset FSS(U, E_1)$ and $(G, B)\in FSS(V, E_2)$, $\{ (G, B)_\alpha : \forall\alpha \in \Lambda\}\subset FSS(V, E_2)$, where Λ is an index set. Then*

1. $f_{pu}(\tilde{\bigcap}_{\alpha\in\Lambda} (F, A)_\alpha) = \tilde{\bigcap}_{\alpha\in\Lambda} f_{pu}((F, A)_\alpha)$, *if f_{pu} is injective.*
2. $f_{pu}(\tilde{1}_{E_1}) = \tilde{1}_{E_2}$ *if f_{pu} is surjective.*

3. $f^{-1}_{pu}((F, A)^C) = (f^{-1}_{pu}(F, A))^C$.

Proof.

Obvious.

Definition 3.6. [3] Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu} : (U, E_1, \tau_1) \rightarrow (V, E_2, \tau_2)$ is called fuzzy soft continuous if $f^{-1}_{pu}((G, B)) \in \tau_1$ for all $(G, B) \in \tau_2$.

Theorem 3.4. *Let f_{pu} be a fuzzy soft continuous function carrying a compact fuzzy soft topological space (U, E_1, τ_1) onto a fuzzy soft topological space (V, E_2, τ_2). Then (V, E_2, τ_2) is fuzzy soft compact.*

Proof.

Let $\psi = \{(G, B)_\alpha : \forall \alpha \in \Lambda\}$ be a cover of $\tilde{1}_{E_2}$ by fuzzy soft open sets. Since f_{pu} is fuzzy soft continuous, the family of all fuzzy soft sets of the form $f^{-1}_{pu}((G, B)_\alpha)$ for $(G, B)_\alpha \in \psi$, is a fuzzy soft open cover of $\tilde{1}_{E_1}$ which has a finite subcover (since (U, E_1, τ_1) is compact). However, since f_{pu} is surjective, then it is easily seen that $f_{pu}(f^{-1}_{pu}(G, B)) = (G, B)$ for any fuzzy soft set (G, B) over (V, E_2). Thus, the family of images of members of the subcover is a finite subfamily of ψ which covers $\tilde{1}_{E_2}$. Consequently, (V, E_2, τ_2) is fuzzy soft compact.

4. Fuzzy Soft Semi-compact Sets, Fuzzy Soft Semi-closed Sets

In this section, we introduce fuzzy soft semi-open and fuzzy soft semi-closed sets. Various notions related to these structures are studied and characterized.

Definition 4.1. Let (U, E, τ) be a fuzzy soft topological space. A fuzzy soft set (F, A) in (U, E, τ) is said to be fuzzy soft semi-open set if there exists a fuzzy soft open set (G, B) such that $(G, B) \,\tilde{\subseteq}\, (F, A) \,\tilde{\subseteq}\, \overline{(G, B)}$.

Definition 4.2. A fuzzy soft set (H, A) in a fuzzy soft topological space (U, E, τ) is said to be a fuzzy soft semi-closed set if there exists a fuzzy soft closed set (G, B) such that $(G, B)^o \,\tilde{\subseteq}\, (H, A) \,\tilde{\subseteq}\, (G, B)$.

Remark 4.1. Every fuzzy soft open (closed) set is a fuzzy soft semi-open (semi-closed) set but not conversely.

Remark 4.2. $\tilde{0}_E$ and $\tilde{1}_E$ are always fuzzy soft semi-closed and fuzzy soft semi-open.

From now on, we shall denote the family of all fuzzy soft semi-open sets and fuzzy soft semi-closed sets over (U, E) by $FSSOS(U, E)$ and $FSSCS(U, E)$ respectively.

Theorem 4.1. *Arbitrary unions of fuzzy soft semi-open sets are fuzzy soft semi-open.*

Proof.

Let $\{(G, A)_\alpha : \forall \alpha \in \Lambda\}$ be an arbitrary collection of fuzzy soft semi-open sets of a fuzzy soft topological space (U, E, τ). Then there exists fuzzy soft open sets $\{(H, A)_\alpha : \forall \alpha \in \Lambda\}$ such that $(H, A)_\alpha \tilde{\subseteq} (G, A)_\alpha \tilde{\subseteq} \overline{(H,A)_\alpha}$ for each $\alpha \in \Lambda$. Hence $\tilde{\bigcup}_{\alpha\in\Lambda} (H, A)_\alpha \tilde{\subseteq} \tilde{\bigcup}_{\alpha\in\Lambda} (G, A)_\alpha \tilde{\subseteq} \overline{\tilde{\bigcup}_{\alpha\in\Lambda}(H,A)_\alpha}$ and $\tilde{\bigcup}_{\alpha\in\Lambda} (H, A)_\alpha$ is a fuzzy soft open set. Hence $\tilde{\bigcup}_{\alpha\in\Lambda} (G, A)_\alpha$ is a fuzzy soft semi-open set.

Remark 4.3. Arbitrary intersections of fuzzy soft semi-closed sets are fuzzy soft semi-closed sets.

Theorem 4.2. *If a fuzzy soft semi-open set (G, A) is such that* $(G, A) \tilde{\subseteq} (K, A) \tilde{\subseteq} \overline{(G,A)}$, *then (K, A) is also fuzzy soft semi-open.*

Proof.

Since (G, A) is fuzzy soft semi-open, there exists a fuzzy soft open set (H, A) such that $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \overline{(H,A)}$. Then by hypothesis $(H, A) \tilde{\subseteq} (K, A)$ and $\overline{(G,A)} \tilde{\subseteq} \overline{(H,A)}$. This implies $(K, A) \tilde{\subseteq} \overline{(G,A)} \tilde{\subseteq} \overline{(H,A)}$. So $(H, A) \tilde{\subseteq} (K, A) \tilde{\subseteq} \overline{(H,A)}$. Consequently, ($K$, A) is a fuzzy soft semi-open set.

Theorem 4.3. *If a fuzzy soft semi-closed set (G, A) is such that* $(G, A)^o \tilde{\subseteq} (K, A) \tilde{\subseteq} (G, A)$, *then (K, A) is also fuzzy soft semi-closed.*

Proof.

As (G, A) is fuzzy soft semi-closed there exists a fuzzy soft closed set (H, A) such that $(H, A)^o \tilde{\subseteq} (G, A) \tilde{\subseteq} (H, A)$. Then by hypothesis $(K, A) \tilde{\subseteq} (H, A)$ and $(H, A)^o \tilde{\subseteq} (G, A)^o$. This implies $(H, A)^o \tilde{\subseteq} (G, A)^o \tilde{\subseteq} (K, A)$. So $(H, A)^o \tilde{\subseteq} (K, A) \tilde{\subseteq} (H, A)$. Consequently, ($K$, A) is a fuzzy soft semi-closed set.

Definition 4.3. Let (U, E, τ) be a fuzzy soft topological space and (G, A) be a fuzzy soft set over (U, E). Then

1. the fuzzy soft semi-closure of (G, A) is a fuzzy soft set, denoted by $fsscl(G, A)$, defined as the intersection of all fuzzy soft semi-closed sets which contain (G, A). That is, $fsscl(G, A) = \tilde{\cap}\{(S, A) : (G, A) \tilde{\subseteq} (S, A), \text{ where } (S, A) \in FSSCS(U, E)\}$.
2. the fuzzy soft semi-interior of (G, A) is a fuzzy soft set, denoted by $fssint(G, A)$, defined as the union of all fuzzy soft semi-open sets contained in (G, A). That is, $fssint(G, A) = \tilde{\cup}\{(S, A) : (S, A) \tilde{\subseteq} (G, A), \text{ where } (S, A) \in FSSOS(U, E)\}$.

Clearly, $fsscl(G, A)$ is the smallest fuzzy soft semi-closed set containing (G, A) and $fssint(G, A)$ is the largest fuzzy soft semi-open set contained in (G, A).

Theorem 4.4. *Let (U, E, τ) be a fuzzy soft topological space. Let (G, A) and (K, A) be two fuzzy soft sets over (U, E). Then*

1. $(G, A) \in FSSCS(U, E) \Leftrightarrow (G, A) = fsscl(G, A)$.
2. $(G, A) \in FSSOS(U, E) \Leftrightarrow (G, A) = fssint(G, A)$.
3. $(fsscl(G, A))^C = fssint(G, A)^C$.
4. $(fssint(G, A))^C = fsscl(G, A)^C$.
5. $(G, A) \tilde{\subseteq} (K, A) \Rightarrow fssint(G, A) \tilde{\subseteq} fssint(K, A)$.
6. $(G, A) \tilde{\subseteq} (K, A) \Rightarrow fsscl(G, A) \tilde{\subseteq} fsscl(K, A)$.
7. $fsscl\,\tilde{0}_E = \tilde{0}_E$ *and* $fsscl\,\tilde{1}_E = \tilde{1}_E$.
8. $fssint\,\tilde{0}_E = \tilde{0}_E$ *and* $fssint\,\tilde{1}_E = \tilde{1}_E$.
9. $fsscl((G, A) \tilde{\cup} (K, A)) = fsscl(G, A) \tilde{\cup} fsscl(K, A))$.
10. $fssint((G, A) \tilde{\cap} (K, A)) = fssint(G, A) \tilde{\cap} fssint(K, A))$.
11. $fsscl((G, A) \tilde{\cap} (K, A)) \tilde{\subseteq} fsscl(G, A) \tilde{\cap} fsscl(K, A))$.
12. $fssint((G, A) \tilde{\cup} (K, A)) \tilde{\subseteq} fssint(G, A) \tilde{\cup} fssint(K, A))$.
13. $fsscl(fsscl(G, A)) = fsscl(G, A)$.
14. $fssint(fssint(G, A)) = fssint(G, A)$.

Proof.

Let (G, A) and (K, A) be two fuzzy soft sets over (U, E).

(1) Let (G, A) be a fuzzy soft semi-closed set. Then (G, A) is the smallest fuzzy soft semi-closed set containing itself and hence $(G, A) = fsscl(G, A)$.

Conversely, if $(G, A)=fsscl(G, A)$ and $fsscl(G, A) \in FSSCS(U, E)$, then $(G, A) \in FSSCS(U, E)$.

(2) Similar to (1).

(3) $(fsscl(G, A))^C = (\tilde{\bigcap}\{(S, A) : (G, A) \tilde{\subseteq} (S, A)$, where $(S, A) \in FSSCS(U, E)\})^C$

$= \tilde{\bigcup}\{(S, A)^C : (G, A) \tilde{\subseteq} (S, A)$, where $(S, A) \in FSSCS(U, E)\}$

$= \tilde{\bigcup}\{(S, A)^C : (S, A)^C \tilde{\subseteq} (G, A)^C$, where $(S, A)^C \in FSSOS(U, E)\}$

$= fssint(G, A)^C$.

(4) Similar to (3).

(5) Follows from the definition.

(6) Follows from the definition.

(7) Since $\tilde{0}_E$ and $\tilde{1}_E$ are fuzzy soft semi-closed sets, we have $fsscl\,\tilde{0}_E = \tilde{0}_E$ and $fsscl\,\tilde{1}_E = \tilde{1}_E$.

(8) Since $\tilde{0}_E$ and $\tilde{1}_E$ are fuzzy soft semi-open sets, we have $fssint\,\tilde{0}_E = \tilde{0}_E$ and $fssint\,\tilde{1}_E = \tilde{1}_E$.

(9) We have $(G, A) \tilde{\subseteq} (G, A) \tilde{\bigcup} (K, A)$ and $(K, A) \tilde{\subseteq} (G, A) \tilde{\bigcup} (K, A)$. Then by (6), $fsscl(G, A) \tilde{\subseteq} fsscl((G, A) \tilde{\bigcup} (K, A))$ and $fsscl(K, A) \tilde{\subseteq} fsscl((G, A) \tilde{\bigcup} (K, A))$ which implies $fsscl(K, A) \tilde{\bigcup} fsscl(G, A \tilde{\subseteq} fsscl((G, A) \tilde{\bigcup} (K, A))$. Now $fsscl(K, A)$, $fsscl(G, A) \in FSSCS(U, E)$. Then $(G, A) \tilde{\subseteq} fsscl(G, A)$ and $(K, A) \tilde{\subseteq} fsscl(K, A)$. So $(G, A) \tilde{\bigcup} (K, A) \tilde{\subseteq} fsscl(G, A) \tilde{\bigcup} fsscl(K, A)$. That is, $fsscl(G, A) \tilde{\bigcup} fsscl(K, A)$ is a fuzzy soft semi-closed set containing $(G, A) \tilde{\bigcup} (K, A)$. But $fsscl((G, A) \tilde{\bigcup} (K, A))$ is the smallest fuzzy soft semi-closed set containing $(G, A) \tilde{\bigcup} (K, A)$. Hence $fsscl((G, A) \tilde{\bigcup} (K, A)) \tilde{\subseteq} fsscl(G, A) \tilde{\bigcup} fsscl(K, A)$. Therefore, $fsscl((G, A) \tilde{\bigcup} (K, A))$ $= fsscl(G, A) \tilde{\bigcup} fsscl(K, A)$.

(10) Similar to (9).

(11) We have $(G, A) \tilde{\bigcap} (K, A) \tilde{\subseteq} (G, A)$ and $(G, A) \tilde{\bigcap} (K, A) \tilde{\subseteq} (K, A)$. So $fsscl((G, A) \tilde{\bigcap} (K,$

$A)) \tilde{\subseteq} fsscl(G, A)$ and $fsscl((G, A) \tilde{\cap} (K, A)) \tilde{\subseteq} fsscl(K, A)$. Hence $fsscl((G, A) \tilde{\cap} (K, A)) \tilde{\subseteq} fsscl(G, A) \tilde{\cap} fsscl(K, A)$.

(12) Similar to (11).

(13) Since $fsscl(G, A) \in FSSCS(U, E)$ so by (1) $fsscl(fsscl(G, A)) = fsscl(G, A)$.

(14) Since $fssint(G, A) \in FSSOS(U, E)$ so by (2) $fssint(fssint(G, A)) = fssint(G, A)$.

Theorem 4.5. *If (G, A) is any fuzzy soft set in a fuzzy soft topological space (U, E, τ), then the following are equivalent.*

(1) *(G, A) is a fuzzy soft semi-closed set.*

(2) $(\overline{(G, A)})^o \tilde{\subseteq} (G, A)$.

(3) $\overline{((G, A)^C)^o} \tilde{\supseteq} (G, A)^C$.

(4) *$(G, A)^C$ is a fuzzy soft semi-open set.*

Proof.

$(1) \Rightarrow (2)$ If (G, A) is a fuzzy soft semi-closed set, then there exists a fuzzy soft closed set (H, A) such that $(H, A)^o \tilde{\subseteq} (G, A) \tilde{\subseteq} (H, A)$ and therefore, $(H, A)^o \tilde{\subseteq} (G, A) \tilde{\subseteq} \overline{(G, A)} \tilde{\subseteq} (H, A)$. By the property of interior, we have $(\overline{(G, A)})^o \tilde{\subseteq} (H, A)^o \tilde{\subseteq} (G, A)$.

$(2) \Rightarrow (3)$ $(\overline{(G, A)})^o \tilde{\subseteq} (G, A)$ implies $(G, A)^C \tilde{\subseteq} ((\overline{(G, A)})^o)^C$ implies $\overline{((G, A)^C)^o} \tilde{\supseteq} (G, A)^C$.

$(3) \Rightarrow (4)$ $((G, A)^C)^o$ is a fuzzy soft open set such that $((G, A)^C)^o \tilde{\subseteq} (G, A)^C \tilde{\subseteq} \overline{((G, A)^C)^o}$. So $(G, A)^C$ is a fuzzy soft semi-open set.

$(4) \Rightarrow (1)$ As $(G, A)^C$ is fuzzy soft semi-open set there exists a fuzzy soft semi-open set (H, A) such that $(H, A) \tilde{\subseteq} (G, A)^C \tilde{\subseteq} \overline{(H, A)}$. Then $(H, A)^C$ is fuzzy soft closed set such that $(G, A) \tilde{\subseteq} (H, A)^C$ and $(G, A)^C \tilde{\subseteq} \overline{(H, A)}$ and therefore, $((H, A)^C)^o \tilde{\subseteq} (G, A)$. Hence (G, A) is a fuzzy soft semi-closed set.

5. Fuzzy Soft Semi-continuous Functions

In this section the concepts of various fuzzy soft functions between two fuzzy soft topological spaces are introduced and several properties of these functions are studied.

Definition 5.1. Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft function $f: (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is said to be

1. fuzzy soft semi-continuous if for each fuzzy soft open set (G, B) over (V, E_2), $f^{-1}((G, B))$ is a fuzzy soft semi-open set over (U, E_1).
2. fuzzy soft irresolute if for each fuzzy soft semi-open set (G, B) over (V, E_2), $f^{-1}((G, B))$ is a fuzzy soft semi-open set over (U, E_1).
3. fuzzy soft semi-open function if for each fuzzy soft open set (G, B) over (U, E_1), $f((G, B))$ is a fuzzy soft semi-open set over (V, E_2).
4. fuzzy soft semi-closed function if for each fuzzy soft closed set (F, A) over (U, E_1), $f((F, A))$ is a fuzzy soft semi-closed set over (V, E_2).

Remark 5.1. (1) A fuzzy soft function $f: (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-continuous if for each fuzzy soft closed set (F, B) over (V, E_2), $f^{-1}((F, B))$ is a fuzzy soft semi-closed set over (U, E_1).

(2) A fuzzy soft semi-continuous function is fuzzy soft irresolute.

Theorem 5.1. *A fuzzy soft function $f: (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-continuous if and only if $f(fsscl(F, A)) \tilde{\subseteq} \overline{f((F, A))}$ for every fuzzy soft set (F, A) over (U, E_1).*

Proof.

Let a fuzzy soft function $f: (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ be fuzzy soft semi-continuous. Now $\overline{f((F, A))}$ is a fuzzy soft closed set over (V, E_2). So, by fuzzy soft semi-continuity of f, $f^{-1}(\overline{f((F, A))})$ is fuzzy soft semi-closed and $(F, A) \tilde{\subseteq} f^{-1}(\overline{f((F, A))})$. But $fsscl(F, A)$ is the smallest fuzzy soft semi-closed set containing (F, A). So $fsscl(F, A) \tilde{\subseteq} f^{-1}(\overline{f((F, A))})$. Hence $f(fsscl(F, A)) \tilde{\subseteq} \overline{f((F, A))}$.

Conversely, let (F, B) be any fuzzy soft closed set over (V, E_2). Then $f^{-1}((F, B)) \in (U, E_1, \tau_1)$ which implies $f(fsscl(f^{-1}((F, B))) \tilde{\subseteq} \overline{f(f^{-1}((F, B)))}$. So $f(fsscl(f^{-1}((F, B))) \tilde{\subseteq} \overline{(F, B)} = (F, B)$. Therefore, $fsscl(f^{-1}((F, B)) = f^{-1}((F, B))$. That is, $f^{-1}((F, B))$ is fuzzy soft semi-closed over

(U, E_1). Hence f is fuzzy soft semi-continuous.

Theorem 5.2. *A fuzzy soft function $f : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-continuous if and only if $f^{-1}((H, B))^o \,\tilde{\subseteq}\, fssint(f^{-1}((H, B)))$ for every fuzzy soft set (H, B) over (V, E_2).*

Proof.

Let a fuzzy soft function $f : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-continuous. Let (G, A) be a fuzzy soft set over (U, E_1). Now $(f((G, A)))^o$ is a fuzzy soft open set over (V, E_2). Let $f((G, A)) = (H, B)$ so that $(G, A) = f^{-1}(H, B)$. Now by fuzzy semi-continuity of f, $f^{-1}((H, B))^o$ is fuzzy soft semi-open over (U, E_1) and $f^{-1}((H, B))^o \,\tilde{\subseteq}\, (G, A)$. As $fssint((G, A))$ is the largest fuzzy soft semi-open set contained in (G, A), $f^{-1}((H, B))^o \,\tilde{\subseteq}\, fssint((G, A))$. That is, $f^{-1}((H, B))^o \,\tilde{\subseteq}\, fssint(f^{-1}(H, B))$.

Conversely, let us consider a fuzzy soft open set (G, B). Then $f^{-1}((G, B))^o \,\tilde{\subseteq}\, fssint(f^{-1}((G, B)))$ implies $f^{-1}(G, B) \,\tilde{\subseteq}\, fssint(f^{-1}((G, B)))$. Hence $f^{-1}(G, B)$ is fuzzy soft semi-open and consequently f is fuzzy soft semi-continuous.

Theorem 5.3. *A fuzzy soft function $f : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-open if and only if $f((F, A)^o) \,\tilde{\subseteq}\, fssint(f((F, A)))$ for every fuzzy soft set (F, A) over (U, E_1).*

Proof.

Let a fuzzy soft function $f : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-open. Then $f((F, A)^o) = fssint(f((F, A)^o)) \,\tilde{\subseteq}\, fssint(f((F, A)))$.

Conversely, let (F, A) be a fuzzy soft open set over (U, E_1). Then by hypothesis, $f((F, A)) = f((F, A)^o) \,\tilde{\subseteq}\, fssint(f((F, A)))$ which implies $f((F, A))$ is a fuzzy soft semi-open set over (V, E_2). Hence f is fuzzy soft semi-open.

Theorem 5.4. *Let a fuzzy soft function $f : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is fuzzy soft semi-open. If (K, B) is a fuzzy soft set and (F, A) is a fuzzy soft closed set containing $f^{-1}(K, B)$ then there exists a fuzzy soft semi-closed set (H, B) such that $(K, B) \,\tilde{\subseteq}\, (H, B)$ and $f^{-1}(H, B) \,\tilde{\subseteq}\, (F, A)$.*

Proof.

Let $(H, B) = (f((F, A)^C))^C$. Now $f^{-1}(K, B) \tilde{\subseteq} (F, A)$ implies $f((F, A)^C) \tilde{\subseteq} (K, B)^C$. Then $(F, A)^C$ is fuzzy soft open. So $f((F, A)^C)$ is fuzzy soft semi-open. Therefore, (H, B) is fuzzy soft semi-closed and $(K, B) \tilde{\subseteq} (H, B)$ and $f^{-1}(H, B) \tilde{\subseteq} (F, A)$.

Theorem 5.5. *A fuzzy soft function* $f : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ *is fuzzy soft semi-closed if and only if* $fsscl(f((F, A))) \tilde{\subseteq} f(\overline{(F, A)})$ *for every fuzzy soft set* (F, A) *over* (U, E_1).

6. Fuzzy Soft Semicompact Topological Spaces

In this section the notion of fuzzy soft semi-compact topological space is introduced along with its characterizations.

Definition 6.1. A cover of a fuzzy soft set is said to be a fuzzy soft semi-open cover if every member of the cover is a fuzzy soft semi-open set.

Definition 6.2. A fuzzy soft topological space (U, E, τ) is said to be fuzzy soft semi-compact if each cover of $\tilde{1}_E$ by fuzzy soft semi-open sets over (U, E) has a finite subcover.

Remark 6.1. Every fuzzy soft compact space is fuzzy soft semi-compact.

Theorem 6.1. A fuzzy soft topological space (U, E, τ) is fuzzy soft semi-compact if and only if each family of fuzzy soft semi-closed sets with finite intersection property (FIP) has a nonempty intersection.

Proof: Let $\{(F, A)_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy soft semi-closed sets with finite intersection property (*FIP*). If possible let $\tilde{\bigcap}_{\alpha\in\Lambda} (F, A)_\alpha = \tilde{0}_E$. Then $\tilde{\bigcup}_{\alpha\in\Lambda} ((F, A)_\alpha)^C = \tilde{1}_E$. So, the collection $\{((F, A)_\alpha)^C : \alpha \in \Lambda\}$ forms a fuzzy soft semi-open cover of $\tilde{1}_E$ which is fuzzy soft semi-compact. So, there exists a finite subcollection Λ_o of Λ for which $\tilde{\bigcup}_{\alpha\in\Lambda_o} ((F, A)_\alpha)^C = \tilde{1}_E$ which implies $\tilde{\bigcap}_{\alpha\in\Lambda_o} (F, A)_\alpha = \tilde{0}_E$ a contradiction.

Conversely, if possible, let (U, E, τ) is not fuzzy soft semi-compact. Then there exists a cover $\{(G, A)_\alpha : \alpha \in \Lambda\}$ of $\tilde{1}_E$ by fuzzy soft semi-open sets such that for every finite subcollection Λ_o of Λ, we have $\tilde{\bigcup}_{\alpha\in\Lambda_o} (G, A)_\alpha \neq \tilde{1}_E$. This implies $\tilde{\bigcap}_{\alpha\in\Lambda_o} ((G, A)_\alpha)^C \neq \tilde{0}_E$. Hence $\{((G, A)_\alpha)^C : \alpha \in \Lambda\}$ has the finite intersection property (*FIP*). So, by hypothesis, $\tilde{\bigcap}_{\alpha\in\Lambda} ((G, A)_\alpha)^C \neq \tilde{0}_E$. This implies $\tilde{\bigcup}_{\alpha\in\Lambda} (G, A)_\alpha \neq \tilde{1}_E$, a contradiction, since $\{(G, A)_\alpha : \alpha \in \Lambda\}$ is a cover of $\tilde{1}_E$ by fuzzy soft semi-open sets.

Theorem 6.2. *A fuzzy soft topological space (U, E, τ) is fuzzy soft semi-compact if and only if every family ψ of fuzzy soft sets with the finite intersection property (FIP) is such that* $\tilde{\bigcap}_{(G,A)\in\psi} fsscl(G, A) \neq \tilde{0}_E$.

Proof.

Let (U, E, τ) be fuzzy soft semi-compact and if possible, let $\tilde{\bigcap}_{(G,A)\in\psi} fsscl(G, A) = \tilde{0}_E$ for some family ψ of fuzzy soft sets with the finite intersection property (*FIP*). So $\tilde{\bigcup}_{(G,A)\in\psi} (fsscl(G, A))^C = \tilde{1}_E$. This implies $\gamma = \{(fsscl(G, A))^C : (G, A) \in \psi\}$ is a cover of $\tilde{1}_E$ by fuzzy soft semi-open sets over (U, E). Then by fuzzy soft semi-compactness of $\tilde{1}_E$, there exists a finite subcover λ (say) of γ. That is, $\tilde{\bigcup}_{(G,A)\in\lambda} (fsscl(G, A))^C = \tilde{1}_E$ which implies $\tilde{\bigcup}_{(G,A)\in\lambda} (G, A)^C = \tilde{1}_E$. So $\tilde{\bigcap}_{(G,A)\in\lambda} (G, A) = \tilde{0}_E$, a contradiction. Hence $\tilde{\bigcap}_{(G,A)\in\psi} fsscl(G, A) \neq \tilde{0}_E$.

Conversely, we have $\tilde{\bigcap}_{(G,A)\in\psi} fsscl(G, A) \neq \tilde{0}_E$, for every family ψ of fuzzy soft sets with the finite intersection property (*FIP*). Let us assume that (U, E, τ) is not fuzzy soft semi-compact. Then there exists a family γ of fuzzy soft semi-open sets which covers $\tilde{1}_E$ without a finite subcover. So for every finite subfamily λ (say) of γ we have that, $\tilde{\bigcup}_{(G,A)\in\lambda} (G, A) \neq \tilde{1}_E \Rightarrow \tilde{\bigcap}_{(G,A)\in\lambda} (G, A)^C \neq \tilde{0}_E$ implies $\{((G, A))^C : (G, A) \in \gamma\}$ is a family of fuzzy soft sets with the finite intersection property (*FIP*). Now $\tilde{\bigcup}_{(G,A)\in\gamma} (G, A) = \tilde{1}_E$ implies $\tilde{\bigcap}_{(G,A)\in\gamma} (G, A)^C = \tilde{0}_E$ implies $\tilde{\bigcap}_{(G,A)\in\gamma} (fsscl(G, A)^C) = \tilde{0}_E$, a contradiction.

Theorem 6.3. *Fuzzy soft semi-continuous image of a fuzzy soft semi-compact space is fuzzy soft compact.*

Proof.

Let a fuzzy soft function $f: (U, E_1, \tau_1) \rightarrow (V, E_2, \tau_2)$ is fuzzy soft semi-continuous. Let $\{(G, B)_\alpha : \alpha \in \Lambda\}$ be a cover of $\tilde{1}_{E_2}$ by fuzzy soft open sets over (V, E_2). This implies $\{f^{-1}(G, B)_\alpha : \alpha \in \Lambda\}$ forms a cover of $\tilde{1}_{E_1}$ by fuzzy soft semi-open sets over (U, E_1). So there exists a finite subset Λ_o of Λ such that $\{f^{-1}(G, B)_\alpha : \alpha \in \Lambda_o\}$ is a cover of $\tilde{1}_{E_1}$ by fuzzy soft semi-open sets over (U, E_1). Therefore, $\{(G, B)_\alpha : \alpha \in \Lambda_o\}$ forms a finite cover of $\tilde{1}_{E_2}$ by fuzzy soft open sets over (V, E_2).

Conclusion

In this article both the concepts of compactness and semi-compactness in fuzzy soft environment have been sketched and discussed. As the concept of compactness of a space is one of the central and important concept, the notions and results given in this article may lead to some interesting in-depth analytical study and research using lattice-valued approaches to fuzzy soft topology.

Conflict of Interests

The author declares that there is no conflict of interests.

REFERENCES

[1] H. Aktas, N. Cagman, Soft sets and soft groups, Inf. Sci. 1(77) (2007), 2726-2735.

[2] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59(2010), 3458–3463.

[3] S. Atmaca and I. Zorlutuna, On fuzzy soft topological spaces, Annals of Fuzzy, Mathematics and Informatics (2012), pp. 1- 10.

[4] A. AygÄunoglu and H. AygÄun, Introduction to fuzzy soft groups, Comput. Math. Appl. 58 (2009), 1279-1286.

[5] B. Ahmad and A. Kharal, On fuzzy soft sets. Advances in Fuzzy Systems, 2009 (2009), Article ID 586507, 6 pages.

[6] N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European J. Oper.Res. 207 (2010), 848-855.

[7] N. Cagman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl. 62 (2011), 351-358.

[8] N. Cagman, S. Enginoglu and F. Citak, Fuzzy soft set theory and its applications, Iranian Journal of Fuzzy Systems Vol. 8, No. 3 (2011), 137-147.

[9] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, The parameterization reduction of soft sets and its applications. Comput. Math. Appl., 49 (2005), 757-763.

[10] B. Dinda and T. K. Samanta, Relations on intuitionistic fuzzy soft sets, Gen. Math. Notes 1(2) (2010), 74–83.

[11] F. Feng, X. Liu, Soft rough sets with applications to demand analysis, In: Int. Workshop Intell. Syst. Appl., ISA (2009), 23-24.

[12] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appl. 56(10) (2008), 2621-2628.

[13] F. Feng, Y. B. Jun, X. Y. Liu and L. F. Li, An adjustable approach to fuzzy soft set *based decision making*, J. Comput. Appl. Math. 234 (2010), 10-20.

[14] F. Feng, Y. M. Li and N. Cagman, Generalized uni-int decision making schemes based on choice value soft sets. European J. Oper. Res. 220 (2012), 162-170.

[15] T. Herawan, M. Deris, M. U. T. H. O, Malaysia, On multi-soft sets construction in information systems, In: Emerging Intelligent Computing Technology and Applications with Aspects of Artificial Intelligence: 5th Int. Conf. Intell. Comput., ICIC 2009 Ulsan, South Korea, September 16-19, 2009. Springer,101–110.

[16] T. Herawan, A. N. M. Rose, M. M. Deris, Soft set theoretic approach for dimensionality Reduction, In: Database Theory andApplication: International Conference, DTA 2009, Jeju Island, Korea, December 10-12, 2009. Springer,171–178.

[17] Y. Jiang, Y. Tang, Q. Chen, J. Wang, S. Tang, Extending soft sets with description logics, Comput. Math. Appl. 59(6) (2009),2087–2096.

[18] Y. B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56(5) (2008), 1408–1413.

[19] .Y. B. Jun, K. J. Lee, A. Khan, Soft ordered semigroups, Math. Logic Quart. 56 (1) (2010), 42–50.

[20] Y. B. Jun, K. J. Lee, C. H. Park, Soft set theory applied to ideals in d-algebras, Comput. Math. Appl. 57(3) (2009), 367–378.

[21] Y. B. Jun, C. H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178(11) (2008), 2466–2475.

[22] Y. B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008), 1408–1413.

[23] Y. B. Jun, C. H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inf. Sci. 178 (2008), 2466–2475.

[24] Y. B. Jun, C. H. Park, Applications of soft sets in Hilbert algebras, Iran. J. Fuzzy Syst. 6(2) (2009), 55–86.

[25] Y. B. Jun, H. S. Kim, J. Neggersm, Pseudo d-algebras, Inf. Sci. 179 (2009), 1751–1759.

[26] Y. B. Jun, K. J. Lee, A. Khan, Soft ordered semigroups, Math. Logic Q. 56(1) (2010), 42–50.

[27] Y. B. Jun, K. J. Lee, C.H. Park, Soft set theory applied to commutative ideals in BCK- Algebras, J.Appl. Math. Inform. 26(3-4) (2008), 707–720.

[28] Y. B. Jun, K. J. Lee, C. H. Park, Soft set theory applied to ideals in d-algebras, Comput. Math. Appl. 57 (2009), 367–378.

[29] Y. B. Jun, K. J. Lee, C. H. Park, Fuzzy soft set theory applied to BCK/BCI-algebras, Comput. Math. Appl. 59 (2010), 3180–3192.

[30] Y. C. Jiang, Y. Tang, Q. M. Chen, J.Wang and S. Q. Tang, Extending soft sets with description logics, Knowledge-Based Systems, 24 (2011), 1096-1107.

[31] A. Kharal and B. Ahmad, Mapping on fuzzy soft classes, Advances in Fuzzy Systems.2009 (2009).

[32] Z. Kong, L. Gao, L. Wang and S. Li, The normal parameter reduction of soft sets and its Algorithm, Comput. Math. Appl., 56 (2008), 3029-3037.

[33] Z. Kong, L. Gao and L. Wang, Comment on a fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 223 (2009), 540–542.

[34] D. Molodtsov, Soft set theory-First results, Computers and Mathematics with Applications 37(4/5) (1999), 19-31.

[35] P. Majumdar, S. K. Samanta, Generalised fuzzy soft sets, Comput. Math. Appl. 59(4)(2010), 1425–1432.

[36] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45(2003), 555-562.

[37] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math., 9(3) (2001), 589-602.

[38] P. K. Maji, A. R. Roy and R. Biswas, An application of soft sets in desicion making problem, Comput. Math. Appl. 44 (2002), 1077-1083.

[39] M. M. Mushrif, S. Sengupta and A. K. Ray, Texture classification using a novel, soft- set theory based classification algorithm, Lect.Notes Comput. Sci. 3851(2006), 246– 254.

[40] T. J. Neog, D. K. Sut, G. C. Hazarika, Fuzzy soft topological spaces, int. J Latest Trend . Math.Vol-2 No. 1(2012), 54-67

[41] C. H. Park, Y. B. Jun, M. A. Ozturk, Soft WS-algebras, Commun. Korean Math. Soc. 23(3) (2008), 313–324.

[42] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making Problems. J. Comput. Appl. Math. 203(2) (2007), 412-418.

[43] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics, Volume 3, No. 2, (April 2012), 305- 311.

[44] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011), 1786-1799.

[45] Q. M. Sun, Z. L. Zhang, J. Liu, Soft sets and soft modules, In: Wang, G, Li, T, Grzymala-Busse, JW, Miao, D, Skowron, A, Yao, Y (eds.) Proceedings of Rough Sets and Knowledge Technology, Third International Conference, RSKT 2008 Chengdu, China, May (2008), 403–409. Springer, Berlin.

[46] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Computer and Mathematics with Applications 61 (2011), 412–418.

[47] Z. Xiao, K. Gong, Y. Zou, A combined forecasting approach based on fuzzy soft sets, J. Comput. Appl. Math. 228 (1) (2009), 326–333.

[48] Z. Xiao, K. Gong, S. Xia and Y. Zou, Exclusive disjunctive soft sets, Comput. Math. Appl. 59(6) (2009), 2128–2137.

[49] W. Xu, J. Ma, S. Wang and G. Hao, Vague soft sets and their properties, Comput. Math. Appl. 59(2) (2010), 787–794.

[50] X. Yang, T. Y. Lin, J. Yang, Y. Li and D. Yu, Combination of interval-valued fuzzy set and soft set, Comput. Math. Appl. 58(3) (2009), 521–527.

[51] L. A. Zadeh, Fuzzy sets, Inform. Control. 8(1965), 338–353.

[52] P. Zhu, Q. Wen, Probabilistic soft sets, In: IEEE Conference on Granular Computing, GrC 2010, San Jose, USA, August (2010), 14-16.

[53] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics 3(2) (2011), 171-185.

[54] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowl.-Based Syst. 21(8) (2008), 941–945.