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SOLVING FULLY FUZZY LINEAR PROGRAMMING WITH SYMMETRIC TRAPEZOIDAL FUZZY NUMBERS USING MEHAR'S METHOD

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Abstract: Recently, Kumar and Kaur (2011) proposed a new method, named as Mehar's method, for solving fuzzy linear programming problem in which the element of coefficient matrix of the constrains are represented by real number and rest of the parameters are represented by fuzzy number, with converting to the classical linear programming problem. In this paper, we adopt this method for solving fully fuzzy linear programming problem (FFLPP) involving symmetric trapezoidal fuzzy numbers including both equality and inequality constraints. By using the proposed method the fuzzy optimal solution of FFLPP, occurring in real life situation, can be easily obtained. To illustrate the proposed method numerical examples are solved.

Keywords: fully fuzzy linear programming; symmetric trapezoidal fuzzy number; ranking function.

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1. Introduction

Bellman and Zadeh [1] proposed the basic concepts of fuzzy decision making for the first time. Tanaka et al. [2] adopted these concepts to solve mathematical programming problem. Zimmermann [3] initially proposed fuzzy linear programming formulation by using of both the minimum operator and the product operator. Afterwards, several authors considered different kinds of the fuzzy linear programming problems and proposed several approaches

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for solving them. Ganesan and Veeramani [4] introduced a fuzzy simplex method for solving a kind of fuzzy linear programming problem with symmetric trapezoidal fuzzy numbers. Also Kumar and Kaur [5] proposed a new method, named as Mehar's method, for solving the same type of these problems. Hatami and Kazemipoor [6] proposed a fuzzy Big-M method for solving fuzzy linear programs with trapezoidal fuzzy numbers.

The fuzzy linear programming in which all the parameters as well as variables are represented by fuzzy numbers in known as FFLPP. Buckley and Feuring [7] introduced a method to find the solution for FFLPP by changing the objective function into the multi-objective fuzzy linear programming problems. Dehghan et al. [8] used a computational method to find the exact solution of a FFLPP. Hashemi et al. [9] have introduced the weak duality theorem based on alphabetic order function for FFLPP. Allahviranloo et al. [10] proposed a method for solving FFLPP by using a kind of defuzzification method. Hosseinzadeh Lotfi et al. [11] discussed FFLPP of which all parameters and variables are symmetric triangular fuzzy numbers. Kumar et al. [12] proposed a method to find the fuzzy optimal solution of FFLPP with equality constraints. Khan et al. [13] proposed a novel technique for solving linear programming problems in a fully fuzzy environment. Kaur and Kumar [14] proposed a new method for solving FFLPP in which some or all the parameters are represented by unrestricted L-R fuzzy numbers. Ezzati et al. [15] proposed a novel algorithm for solving the FFLPP by converting it to its equivalent a multi-objective linear programming problem. In this paper, we adopt Mehar's method [5] for solving fully fuzzy linear programming problem (FFLPP) involving symmetric trapezoidal fuzzy numbers. By using the proposed method the fuzzy optimal solution of FFLPP, occurring in real life situation, can be easily obtained.

This paper is organized as follows: In section 2 some basic definitions and arithmetics of between two symmetric trapezoidal fuzzy numbers are presented. A review of formulation of FFLPP is given in section 3. In section 4 a new method is proposed for FFLPP and a numerical example is solved. Finally, conclusions are discussed in section 5.

2. Preliminaries

Here, some necessary definitions and arithmetic operations of fuzzy numbers are reviewed.

Definition 2.1. A fuzzy number $\tilde{a}$ on real $\mathbb{R}$ numbers is said to be a symmetric trapezoidal fuzzy number if there exist real numbers, a^L and a^U, $a^L \le a^U$ and h>0 such that

$$\tilde{a}(x) = \begin{cases} \frac{x-(a^L-h)}{h} & a^L-h \le x \le a^L \\ 1 & a^L \le x \le a^U \\ \frac{-x+(a^U+h)}{h} & a^U \le x \le a^U+h \\ 0 & else \end{cases}$$

A symmetric trapezoidal fuzzy number is denoted as $\tilde{a} = (a^L, a^U, h)$. The set of all symmetric trapezoidal fuzzy numbers on $\mathbb{R}$ by F(R).

Let $\tilde{a} = (a^L, a^U, h)$ and $\tilde{b} = (b^L, b^U, k)$ be two symmetric trapezoidal fuzzy numbers. The arithmetic operations on these fuzzy numbers as follows:

Addition: $\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, h + k)$

Subtraction: $\tilde{a} - \tilde{b} = (a^L - b^U, a^U - b^L, h + k)$

Scalar multiplication: $\lambda\tilde{a} = \begin{cases} (\lambda a^L, \lambda a^U, \lambda h) & \lambda \ge 0 \\ (-\lambda a^U, -\lambda a^L, h) & \lambda < 0 \end{cases}$

Multiplication: $\tilde{a}\tilde{b} = ((\frac{a^L + a^U}{2})(\frac{b^L + b^U}{2}) - w, (\frac{a^L + a^U}{2})(\frac{b^L + b^U}{2}) + w, , |a^U k + b^U h|)$

where $w = \frac{\beta - \alpha}{2}, \alpha = \min\{a^L b^L, a^L b^U, a^U b^L, a^U b^U\}$ and $\beta = \max\{a^L b^L, a^L b^U, a^U b^L, a^U b^U\}$

Definition 2.2. A ranking function is a mapping from a fuzzy set to real numbers. Define the ranks on $\mathbb{R}^n$ as

i. $\tilde{a} \succcurlyeq \tilde{b} \ iff \ \Re(\tilde{a}) \ge \Re(\tilde{b})$

ii. $\tilde{a} \succ \tilde{b} \ iff \ \Re(\tilde{a}) > \Re(\tilde{b})$

iii. $\tilde{a} \approx \tilde{b} \ iff \ \Re(\tilde{a}) = \Re(\tilde{b})$

For ranking symmetric trapezoidal fuzzy numbers, this study employs the ranking function as given below:

$$\Re(\tilde{a}) = \frac{a^L + a^U}{2}$$

Lemma 2.1. Suppose $\tilde{a}, \tilde{b} \in F(R)$. We have

i. $\Re(\tilde{a}\tilde{b}) = \Re(\tilde{a}).\Re(\tilde{b})$

ii. $\Re(\tilde{a} + \tilde{b}) = \Re(\tilde{a}) + \Re(\tilde{b})$

Proof.

$$\Re(\tilde{a}\tilde{b}) = \Re\left(\left(\left[\frac{a^L+a^U}{2}\right]\left[\frac{b^L+b^U}{2}\right] - w, \left[\frac{a^L+a^U}{2}\right]\left[\frac{b^L+b^U}{2}\right] + w, |a^U k + b^U h|\right)\right) =$$

i. $\left[\frac{a^L+a^U}{2}\right]\left[\frac{b^L+b^U}{2}\right] = \Re(\tilde{a}).\Re(\tilde{b})$

$$\Re(\tilde{a} + \tilde{b}) = \Re((a^L + b^L, a^U + b^U, h + k)) = \frac{a^L+a^U+b^L+b^U}{2} = \left[\frac{a^L+a^U}{2}\right] + \left[\frac{b^L+b^U}{2}\right] =$$

ii. $\Re(\tilde{a}) + \Re(\tilde{b})$

3. Fully Fuzzy Linear Prgramming

Let $\tilde{A} = (\tilde{a}_{ij})_{m\times n}, \tilde{c} = (\tilde{c}_j)_{1\times n}, \tilde{b} = (\tilde{b}_i)_{m\times 1}, \tilde{x} = (\tilde{x}_j)_{1\times n}$ where

$\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j, \tilde{x}_j \in F(R)(i = 1,2, \dots, m \quad, j = 1,2, \dots, n)$, $\tilde{b}_i, \tilde{x}_j \succcurlyeq \tilde{0}$ Then

$$\max \tilde{z} \approx \tilde{c}\tilde{x}$$

$$s.t. \tilde{A}\tilde{x} \preccurlyeq, \approx, \succcurlyeq \tilde{b} \qquad (1)$$

$$\tilde{x} \succcurlyeq \tilde{0}$$

is said to be a FFLPP.

Definition 3.1 Any fuzzy vector $\tilde{x} \in (F(R))^n$ is called a fuzzy feasible solution to (1) if $\tilde{x}$ non-negativity restrictions and satisfies the constraints of the problem.

Definition 3.2 A fuzzy feasible solution $\tilde{x}_0$ is said to be a fuzzy optimal solution for (1) if $\tilde{c}\tilde{x}_0 \succcurlyeq \tilde{c}\tilde{x}$ for all fuzzy feasible solution $\tilde{x}$ for (1).

4. The Proposed Method

Here, a new method is proposed to find the fuzzy optimal solution of FFLPP. The FFLPP (1) may be formulated as follows:

$$max\ \tilde{z} \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$$

$$s.t. \sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \preccurlyeq, \approx, \succcurlyeq \tilde{b}_i\ , i = 1, \dots, m \quad (2)$$

$$\tilde{x}_j \succcurlyeq \tilde{0} \quad \forall j$$

The steps of the new method are as follows:

Step 1. If all parameters $\tilde{c}_j, \tilde{x}_j, \tilde{a}_{ij}$ and $\tilde{b}_i$ are represent by symmetric trapezoidal fuzzy numbers (p_j, q_j, β_j) , (x_j, y_j, α_j), $(d_{ij}, e_{ij}, \theta_{ij})$ and (b_i, g_i, γ_i) respectively then the FFLPP (2), may be written as:

$$max\ \tilde{z} \approx \sum_{j=1}^{n} (p_j, q_j, \beta_j)(x_j, y_j, \alpha_j)$$

$$s.t. \sum_{j=1}^{n} (d_{ij}, e_{ij}, \theta_{ij})(x_j, y_j, \alpha_j) \preccurlyeq, \approx, \succcurlyeq (b_i, g_i, \gamma_i)\ , i = 1, \dots, m \quad (3)$$

$$(x_j, y_j, \alpha_j) \succcurlyeq \tilde{0} \quad \forall j$$

Step 2. Using the linearity function and lemma 2.1, the FFLPP, obtained in Step 1, can be written as:

$$max\ \tilde{z} \approx \sum_{j=1}^{n} \Re\left((p_j, q_j, \beta_j)\right) \Re((x_j, y_j, \alpha_j))$$

$$s.t. \sum_{j=1}^{n} \Re\left((d_{ij}, e_{ij}, \theta_{ij})\right) \Re((x_j, y_j, \alpha_j)) \preccurlyeq, \approx, \succcurlyeq \Re((b_i, g_i, \gamma_i))\ , i = 1, \dots, m \quad (4)$$

$$\Re((x_j, y_j, \alpha_j)) \succcurlyeq 0 \quad \forall j$$

$$y_j - x_j \geq 0 \ , \ \alpha_j \geq 0$$

Step 3. Find the optimal solution x_j, y_j and α_j by linear programming problem obtained in step 2.

Step 4. Find the fuzzy optimal solution by putting the values of x_j, y_j and α_j in $\tilde{x}_j \approx (x_j, y_j, \alpha_j)$.

Step 5. Find the fuzzy optimal value of FFLPP by putting the values $\tilde{x}_j$ in $\sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$.

For an illustration of the above method a FFLPP is solved. Consider the following problem:

$$\max \tilde{z} \approx (-2,2,2)\tilde{x}_1 + (-2,0,1)\tilde{x}_2$$

$$s.t. \ \ (2,4,1)\ \tilde{x}_1 + (1,3,2)\tilde{x}_2 \preccurlyeq (-4,4,2)$$

$$(-2,0,1)\tilde{x}_1 + (-2,4,2)\tilde{x}_2 \preccurlyeq (-1,5,3)$$

$$\tilde{x}_1, \tilde{x}_2 \succcurlyeq \tilde{0}$$

Step 1. Assuming $\tilde{x}_1 = (x_1, y_1, \alpha_1)$ and $\tilde{x}_2 = (x_2, y_2, \alpha_2)$ the FFLPP, obtained in problem, can be written as:

$$\max \tilde{z} \approx (-2,2,2)(x_1, y_1, \alpha_1) + (-2,0,1)(x_2, y_2, \alpha_2)$$

$$s.t. \ \ (2,4,1)\ (x_1, y_1, \alpha_1) + (1,3,2)(x_2, y_2, \alpha_2) \preccurlyeq (-4,4,2)$$

$$(-2,0,1)(x_1, y_1, \alpha_1) + (-2,4,2)(x_2, y_2, \alpha_2) \preccurlyeq (-1,5,3)$$

$$\tilde{x}_1, \tilde{x}_2 \succcurlyeq \tilde{0}$$

Step 2. Using step 2 of the proposed method, obtained in step 1, can be written as:

$$\max \Re(\tilde{z}) \approx 0 \times \frac{x_1 + y_1}{2} + (-1) \times \frac{x_2 + y_2}{2}$$

$$s.t. \ \ 3 \times \frac{x_1 + y_1}{2} + 2 \times \frac{x_2 + y_2}{2} \leq 0$$

$$(-1) \times \frac{x_1 + y_1}{2} + 1 \times \frac{x_2 + y_2}{2} \leq 2$$

$$x_1 + y_1 \geq 0$$

$$x_2 + y_2 \geq 0$$

$$x_1 \leq y_1$$

$$x_2 \leq y_2$$

$$\alpha_1 \geq 0$$

$$\alpha_2 \geq 0$$

Step 3. Solving the linear programming problem in step 2, we get:

$$x_1 = 0, y_1 = 0, \alpha_1 = 0$$

$$x_2 = 2, y_2 = 2, \alpha_2 = 0$$

Step 4. Using step 4, the fuzzy optimal solution is given by $\tilde{x}_1 = (0,0,0), \tilde{x}_2 = (2,2,0)$.

Step 5. Using step 5, the fuzzy optimal value of the given FFLPP is $\tilde{Z} \approx (-4,0,2)$.

5. Conclusions

In this paper, we adopted Mehar's method [5] for solving fully fuzzy linear programming problem involving symmetric trapezoidal fuzzy numbers. By using the proposed method the fuzzy optimal solution of fully fuzzy linear programming problem, occurring in real life situation, can be easily obtained. To illustrate the proposed method numerical examples are solved.

Conflict of Interests

The author declares that there is no conflict of interests.

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