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A NOTE ON SOME NEW SOFT STRUCTURES

ABDELGHANI TAOUTI, WAHEED AHMAD KHAN*

Department of Mathematics and Statistics, Caledonian College of Engibneering,

PO Box 2322, Seeb 111, Sultanate of Oman

Abstract. Recently, soft rings have been introduced in the literature and have taken part in the growing soft set theory. In this note we introduce some new soft structures: soft integral domains and soft fields. In addition, we also introduce soft rings of fractions (quotients of soft rings).

Keywords: soft rings of fractions; soft fields; soft integral domains.

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1. Introduction

Theory of probability, theory of fuzzy sets [13], theory of intuitionistic fuzzy sets [5], theory of vague sets [7], theory of interval mathematics [8], and theory of rough sets [11] which were considered best mathematical tools for dealing with uncertainties. But all these theories have their own difficulties, and the reasons for these difficulties are may be some imperfections in these theories. To fill loop holes, Molodtsov [10] introduced the concept of soft theory to deal with uncertainties which is free from the above difficulties. In [10], Molodtsov showed that to fix uncertainties, soft set theory works more efficient than any other tool.

*Corresponding author

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In [2], soft groups, soft subgroups and few of their results were discussed. In [1], soft rings, soft ideals of soft rings have been introduced, furthermore the authors also introduced idealistic soft rings.

For basic terminologies of soft set one may consult [10] and for soft rings and soft ideals we refer [1]. However we recall few useful definitions and terminologies. Following [10, definition 2.1] pair (F, E) is called a soft set (over U) if and only if F is a mapping on E into the set of subsets of the set U. Assume that (F, A) and (H, B) are two soft sets over a common universe U . We say that (F, A) is a soft subset of (H, B), if it satisfies: (1) $A \subset B$ and (2) $F(x)$ and $H(x)$ are identical approximations for all $x \in A$[10]. In [1, definition 3.1] authors introduced soft rings i.e., Let (F,A) be a non-null soft set over a ring R. Then (F,A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in A$. Further in [1, definition 4.1] introduce soft ideal of a soft ring i.e., Let (F, A) is a soft ring over R. A non-null soft set (γ, I) over R is called soft ideal of (F, A), if it satisfies: (1) $I \subset A$ and (2) $\gamma(x)$ is an ideal of $F(x)$ for all $x \in Supp(\gamma, I)$. Throughout this paper E is a set of parameters, $P(R)$ is the power set of R , $\mathbb{Z}$ is the ring of integer numbers.

Soft rings have been introduced in the literature, in this note we introduce soft integral domains and soft fields. We also introduce soft rings of fractions.

2. Preliminaries

We recall some useful definitions and terminologies from the literature.

Definition 1. [10] A pair (F, E) is called a soft set over universal set U, where F is a mapping from a set of parameters (E) to power set $P(U)$,*i.e.,* $F : E \to P(U)$.

Definition 2. [9] Let (F, A) and (G, B) be two soft sets over a common universe U, we say that (F, A) is a soft subset of (G, B), if it satisfies: (1) $A \subset B$ and (2) $F(x)$ and $G(x)$ are identical approximations for all $x \in A$. We write it $(F, A) \tilde{\subset} (G, B)$.

Definition 3. [9] Let (F, A) and (G, B) be two soft sets over a common universe U. The intersection of (F, A) and (G, B) is defined as the soft set (H,C) satisfying the following conditions:

(a) $C = A \cap B$

(b) For all $x \in C$, $H(x) = F(x)$ or $G(x)$ (while the two sets are the same).

In this case we write $F(A) \tilde{\cap} G(B)$.

Definition 4. [9] Let (F, A) and (G, B) be two soft sets over a common universe U. The bi-intersection of (F, A) and (G, B) is defined as the soft set (H,C) satisfying the following conditions:

(a) $C = A \cap B$

(b) For all $x \in C$, $H(x) = F(x) \cap G(x)$

In this case we write $H(C) = F(A) \tilde{\sqcap} G(B)$.

Definition 5. [9] Let (F, A) and (G, B) be two soft sets over a common universe U. The union of (F, A) and (G, B) is defined as the soft set (H,C) satisfying the following conditions:

(a) $C = A \cup B$

(b) For all $x \in C$,

$$H(x) = \begin{cases} F(x), \text{ if } x \in A - B \\ G(x), \text{ if } x \in B - A \\ F(x) \cup G(x), \text{ if } x \in A \cap B. \end{cases}$$

In this case we write $H(x) = F(A) \tilde{\cup} G(B)$.

Definition 6. [9] If (F, A) and (G, B) be two soft sets over a common universe U. Then "(F, A) AND (G, B)" denoted by $F(A) \tilde{\wedge} G(B)$ is defined as $F(A) \tilde{\wedge} G(B) = (H, C)$, where $C = A \times B$ and $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in C$.

Definition 7. [3] Let (F,A) and (G,B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted intersection of (F,A) and (G,B) is denoted by $(F,A) \Cap (G,B)$, and is defined as $(F,A) \Cap (G,B) = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 8. [9] If (F, A) and (G, B) be two soft sets over a common universe U. Then "(F, A) OR (G, B)" denoted by $F(A) \tilde{\vee} G(B)$ is defined as $F(A) \tilde{\vee} G(B) = (H, C)$, where $C = A \times B$ and $H(x, y) = F(x) \cup G(y)$ for all $(x, y) \in C$.

Definition 9. [12] Let (F,A) and (G,B) be two soft sets over U, then the Cartesian product of (F,A) and (G,B) is defined as, $(F,A) \times (G,B) = H(A \times B)$ where $H : (A \times B) \to P(U \times U)$ and

$H(a,b) = F(a) \times G(b)$ where $(a,b) \in A \times B$ i.e., $H(a,b) = \{(h_i, h_j) : h_i \in F(a), h_j \in G(b)\}$ is a soft set over $U \times U$.

Definition 10. [12] Let (F, A) and (G, B) are two soft sets (resp. rings) over the same set U (resp. ring). Then $(F,A) \times (G,B)$ is defined as, $H = F \times G : A \times B \to P(U \times U)$ defined by $H(a,b) = F(a) \times G(b)$, where $(a,b) \in A \times B,$ and $H(a,b) = \{(h_a,\ h_b) : h(a) \in F(a),\ h(b) \in G(b)\}$for all $a \in A$, $b \in B$ and $F(a)$, $G(b)$ are soft sets in U.

Definition 11. [2] Let (F, A) and (H, B) be two soft groups over G and K respectively, and let $f : G \to K$ and $g : A \to B$ be two functions. Then we say that (f, g) is a soft homomorphism, and that (F, A) is soft homomorphic to (H, B). The later is written as $(F, A)\tilde{}(H, B)$, if the following conditions are satisfied: (a) f is a homomorphism from G onto K, (b) g is a mapping from A onto B, and (c) $f(F(x)) = H(g(x))$ for all $x \in A$. *If* f is an isomorphism and g is one-one, then we say that (f, g) is a soft isomorphism.

Definition 12. [1] Let (F, A) be a non-null soft set over a ring R. Then (F, A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in A$.

Definition 13. [1] Let (F, A) is a soft ring over R, a non-null soft set (γ, I) over R is called soft ideal of (F, A), and is denoted by $(\gamma, I) \mathrel{\tilde{\triangleleft}} (F, A)$ if it satisfies:

(a) $I \subset A$

(b) $\gamma(x)$ is an ideal of $F(x)$ for all $x \in Supp(\gamma, I)$.

Definition 14. [1] Let (F, A) and (G, B) be non-null soft sets over a ring R. Then (G, B) is called a soft subring of (F, A) if it satisfy the following.

(a) $A \subset B$

(b) $G(x)$ is a subring of $F(x)$, for all $x \in Supp(G,B)$.

Definition 15. [1] Let (F, A) be a non-null soft sets over a ring R. Then (F, A) is called an idealistic soft ring over R, if $F(x)$ is an ideal of R for all $x \in Supp(F,A)$.

Definition 16. [4] Let (F, A) is a soft ring over a ring R. A soft ideal (ϕ, I) of (F, A) is called a prime soft ideal of (F, A), if $\phi(x)$ is a prime ideal in $F(x)$ for all $x \in Supp(\phi, I)$.

Definition 17. [4] Let (F, A) is a soft ring over a ring R. A soft ideal (ϕ, I) of (F, A) is called a maximal soft ideal of (F, A), if $\phi(x)$ is a maximal ideal in $F(x)$ for all $x \in Supp(\phi, I)$.

Definition 18. [6] Let (σ, A) be a soft set over $X \times X$, then (σ, A) is called a soft binary relation over X.

Definition 19. [6] A soft binary relation (σ, A) over a set X is a soft equivalence relation over X if and only if $\sigma(x) \neq \emptyset$ is an equivalence relation on X for all $x \in A$.

Definition 20. [6] Let (σ, A) be a soft binary relation over a semigroup S. Then (σ, A) is said to be (right, left)compatible if $\sigma(x)$ is a (right, left) compatible relation on S for all $x \in A$.

3. Soft Integral domain and soft field

In this section, we introduce soft integral domains and few new relevant terminologies. Following [1, definition 3.1] let (F, A) be a non-null soft set over a ring R. Then (F, A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in A$. By a multiplicatively closed subset of a ring we mean a subset S of a ring R having 1 and whenever $a, b \in S$, it implies $a.b \in S$.

Definition 21. Let (F, A) be a soft ring over R. A non-null soft set (ϕ, B) over ring R is said to be a multiplicative soft subset of soft ring $F(x)$ if it satisfies the following conditions:

(a) $B \subset A$.

(b) $\phi(x)$ is a multiplicatively closed and is contained in soft ring $F(x)$ for all $x \in Supp(\phi, B)$.

Example 1. Let $R = A = Z_6$ and consider the set valued function $F : A \to P(R)$ given by $F(x) = \{y \in R : x.y = 0\}$. Then $F(0) = R$, $F(1) = \{0\}$, $F(2) = \{0, 3\}$, $F(3) = \{0, 2, 4\}$, $F(4) = \{0, 3\}$ and $F(5) = \{0\}$. As we see that all of these sets are subrings of of R, hence (F, A) is a soft ring over R. Consider $B = \{1, 3, 5\}$ be a subset of A and a set valued function $\phi : B \to P(R)$ is defined by $\phi(1) = \{1\} \subset F(0)$, $\phi(5) = \{1, 5\} \subset F(0)$, $\phi(3) = \{1, 3\} \subset F(0)$, clearly each $\phi(x)$ is multiplicatively closed and contained in $F(x)$ and hence (ϕ, B) is a multiplicatively closed soft subset of $F(x)$.

Proposition 1. *Union of two disjoint multiplicatively closed soft subsets of a soft ring (F, A) over a ring R is a multiplicatively closed.*

Proof. Let (G, B) and (H, C) be two multiplicatively closed soft subsets of a soft ring (F, A). Consider $(G, B)\ \tilde{U}\ (H, C) = (K, D)$, where $D = B \cup C$ for all $x \in D$, we define;

$$\begin{aligned} K(x) &= G(x),\ \text{if } x \in B - C \\ &= H(x),\ \text{if } x \in C - B. \end{aligned}$$

Clearly (K, D) is a multiplicatively closed soft subset by definition21. ∎

Proposition 2. *Union of two multiplicatively closed soft subsets of a soft ring over ring R is multiplicatively closed.*

Proof. Let (G, B) and (H, C) be two multiplicatively closed soft subsets of a soft ring (F, A). We have $(G, B) \cup (H, C) = (I, D)$. Then by definition5 and definition21, clearly (I, D) is a multiplicatively closed soft subset of a soft ring (F, A). ∎

Proposition 3. *Bi-intersection of two multiplicatively closed soft subsets of a soft ring* (F, A) *is a multiplicatively closed.*

Proof. Let (F, A) and (G, B) be two multiplicatively closed soft subsets of a soft ring (F, A). The bi-intersection of (F, A) and (G, B) is defined as the soft set (H, C) satisfying the following conditions:

(1) $C = A \cap B$

(2) For all $x \in C$,

$$H(x) = F(x) \cap G(x).$$

Then from definition21, clearly (H, C) is a multiplicatively closed soft set. ∎

In classical ring theory there is a connection between prime ideals and multiplicatively closed subsets. We will see that in a soft set theory the situation is quite same, but first we recall definition.

Definition 22. Let (F, A) is a soft ring over a ring R. A soft ideal (ϕ, I) of (F, A) is called a prime soft ideal of (F, A), if $\phi(x)$ is a prime ideal in $F(x)$ for all $x \in Supp(\phi, I)$ [4, definition 10].

Theorem 4. *Let (F, A) is a soft ring over R and $(\phi, I) \subset (F, A)$ be a prime soft ideal then $F(x) \backslash \phi(x)$ is a multiplicatively closed subset of $F(x)$.*

Proof. Let (F, A) is a soft ring over R and $(\phi, I) \subset (F, A)$ be a prime soft ideal. If (ϕ, I) is a prime soft ideal then $\phi(x)$ is necessary prime ideal in $F(x)$ [4, definition 10]. Consider a soft set $F(x) \backslash \phi(x) = S$ then the result folows by definition of prime soft ideal and definition21. ∎

Example 2. Let $A = R = \mathbb{Z}_4$ and let $I = \{0\}$. Suppose that $F : A \to P(R)$ is a set valued function defined: $F(0) = F(2) = \mathbb{Z}_4$ and $F(1) = F(3) = \{0, 2\}$, clearly (F, A) is a soft ring over R. Consider map $\phi : I \to P(R)$ defined by $\phi(x) = \{y \in \mathbb{Z}_4 : x + 2y = 0\}$, then $\phi(0) = \{0, 2\}$ is a prime ideal of $F(0) = \mathbb{Z}_4$ so (ϕ, I) is a prime soft ideal of a soft ring (F, A). Now consider $S = F(0) \backslash \phi(0) = \{1, 2, 3\} \subset F(2) = \mathbb{Z}_4$, and clearly a multiplicatively closed subset.

Definition 23. Let (F, A) be a soft set over an integral domain R. Then (F, A) is said to be an integral domain if $F(x)$ is a sub-domain of R.

Example 3. Let $A = \mathbb{Z}$ be a soft set over an integral domain $R = \mathbb{Z}$. Consider set-valued function $F : A \to P(\mathbb{Z})$ defined by $F(x) = \{nx\mathbb{Z} : n \in \mathbb{Z}\}$, since $nx\mathbb{Z}$ with 1 is a sub-domain of $\mathbb{Z}$.

Proposition 5. *Let (F, A) and (G, B) be two soft integral domains over the domain R. Then $F(A) \tilde{\cap} G(B)$ is a soft integral domain.*

Proof. Let (F, A) and (G, B) be two soft integral domains over the same integral domain R. Consider $F(A) \tilde{\cap} G(B) = (H, C)$ where $x \in C = A \cap B$ and for all $x \in C$, $H(x) = F(x)$ or $G(x)$. Either $H(x) = F(x)$ or $H(x) = F(x)$, we are done. ∎

Proposition 6. *Let (F, A) and (G, B) be two soft integral domains over the same domain R. Then $(F, A) \tilde{\sqcap} (G, A)$ is a soft integral domain, if it is non-null.*

Proof. Let (F, A) and (G, B) be two soft integral domains over the same domain R. Consider $(F, A) \tilde{\sqcap} (G, B) = (H, C)$ where $C = A \cap B$ and for all $x \in C$, $H(x) = F(x) \cap G(x)$. Since intersection of two integral domains is an integral domain, thus for all $x \in C$, $H(x)$ is an integral domain contained in R, it implies (H, C) is a soft integral. ∎

After introducing soft integral domain we discuss about the construction of quotients of soft integral domains.

Definition 24. Let (F, A) be a soft integral domian over domain R and $X = A\backslash\{0\}$. We will represent the fraction a/b by a tuple $(a, b) \in X \times X$ where $a, b \in X = A\backslash\{0\}$ and $b \neq 0$. We set a soft binary relation (Ψ, A) on $X \times X$ as $\Psi(\alpha) = \{(a, b) \in X \times X : a, b \in X, b \neq 0 \wedge (c, d) \in \Psi(\alpha) \Leftrightarrow ad = bc\}$. We may express this relation as; $(a,\ b) \sim_{\Psi(\alpha)} (c,\ d) \Leftrightarrow ad = bc$.

Remark 1. Each $\Psi(\alpha)$ is reflexive, symmetric and transitive thus $\sim_{\Psi(\alpha)}$ is an equivalence relation, hence $\Psi(\alpha) = \{(a, b) \in X \times X : a, b \in X, b \neq 0 \wedge (c, d) \in \Psi(\alpha) \Leftrightarrow ad = bc\}$ is an equivalence relation. This equivalence relation designate equivalence classes i.e., for each $(a, b) \in \Psi(\alpha)$ there is a class $[a, b]_{\Psi(\alpha)} = \{(a_1, b_1) \in X \times X :\ (a_1, b_1) \sim_{\Psi(\alpha)} (a, b)\}$. The set of all equivalence classes $\{[a, b]_{\Psi(\alpha)} : (a, b) \in \Psi(\alpha) \subset X \times X\}$ will be represented by $Q(F, A)$ or $Q_F(A)$. Let us define a soft binary operation adddition "$+$" and multiplication "." on $Q_F(A)$ as;

$$\begin{aligned} [a,b]_{\Psi(\alpha)} + [c,\ d]_{\Psi(\alpha)} &= [ad + bc,\ ad]_{\Psi(\alpha)} \text{ and,} \\ [a,b]_{\Psi(\alpha)}.\ [c,\ d]_{\Psi(\alpha)} &= [ac, bd]_{\Psi(\alpha)}. \end{aligned}$$

Then $(Q_F(A), +, .)$ is a field and we call it soft quotient field of a soft integral domain (F, A).

Conflict of Interests

The authors declare that there is no conflict of interests.

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