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COMMON FIXED POINT THEOREMS IN R-WEAKLY COMMUTING TYPE MAPPING OF FUZZY METRIC SPACES

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Abstract: In this paper, we have common fixed point theorem has generalization of result [2] and the condition for weakly compatible self-mappings f,g,h of complete metric space $(X,\ M,\ *)$ have a unique common fixed point in X.

Keywords: Fuzzy metric space, common fixed point, weakly compatible maps, semi compatible maps.

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1. Introduction

In1965, Zadeh [12] introduced the concept of Fuzzy set as a new way to represent vagueness in our everyday life.

We can divide them into following groups. The first group involves those results in which a fuzzy metric space on a set X is treated as a map where X represets the totality of all fuzzy points of a set and satisfy some axioms which are analogues to the ordinary metric axioms.

On the other hand in second group we keep those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy.

We also discuss result related to R-weakly commuting type mappings.

2. Preliminaries

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Definition 2.1:A binary operation *:[0,1] × [0,1] → [0,1] → [0,1] is continuous t-norm if * satisfy the following conditions:

(i) $*$is commutative and associative
(ii) $*$is continuous
(iii) $a * 1 = a$for all $a \in [0,1]$
(iv) $a * b \leq c * d$ whenever $a \leq c,\ b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 2.2: The 3-tuple $(X, M,*)$ is called a fuzzy metric space if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions

(i) $M\ (x, y, 0) = 0$
(ii) $M\ (x, y, t) = 1$ for all $t > 0$If and only if $x = y$
(iii) $M\ (x, y, t) = M(y, x, t)$
(iv) $M\ (x, y, t) * M(y, z, s) \leq M(x, z, t + s)$triangular inequality
(v) $M(x, y,\ \cdot): [0,1) \to [0,1]$ is left continuous for all $x, y, z \in X$ and $s, t > 0$.

Definition 2.3: Let $(X, M,*)$ be a fuzzy metric space, then

(a) a sequence $\{x_n\}$ in X is said to
  (i) be a Cauchy sequence if

  $lim_{n\to\infty} M\left(x_{n+p}, x_n, t\right) = 1$for all $t > 0$ and $n, p \in N$

  (ii) be convergent to a point $x \in X$ if
  $lim_{n\to\infty} M(x_n, x, t) = 1$for all $t > 0$
(b) $X$is said to be complete if every Cauchy sequence in X converges to some point in X.

Definition 2.4:A self-mappings$f, g\ and\ h$ of a fuzzy metric space $(X, M,*)$ is said to be commuting if

$M(ghx, hgx, t) = 1,$

And $M(fgx, gfx, t) = 1$ for all $x \in X$.

Definition 2.5:A self-mappings(f, g, h) of a fuzzy metric space $(X, M,*)$ is said to be weakly commuting if

$M(fgx, gfx, t) \geq M(fx, gx, t)andM(ghx, hgx, t) \geq M(gx, hx, t)$ for all $x \in X$ and $t > 0$.

Definition 2.6:A self-mappings$f, g\ and\ h$ of a fuzzy metric space $(X, M,*)$ is said to be compatible if

$$lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$$

And $lim_{n\to\infty} M(ghx_n, hgx_n, t) = 1$ for all $t > 0$

Whenever $\{x_n\}$ is a sequence in X such that

$lim_{n\to\infty}\ fx_n = lim_{n\to\infty}\ gx_n = lim_{n\to\infty}\ hx_n = u$for some $u \in X.$

3. Fixed Point Theorems

Theorem 3.1: Let $(X, M,*)$ be a complete metric space let f, g and h be weakly compatible self maps of X satisfying

1.1$M(gx, gy, kt) \geq M(fx, fy, t)$

and$M(hx, hy, kt) \geq M(gx, gy, t)$ where $0 < k < 1$

1.2 $g(X) \subseteq f(X), h(X) \subseteq g(X)$

If one of $h(X), g(X)\ or\ f(X)$ is complete then f, g and h have a unique common fixed point.

Proof: Let $x_0 \in X$. Since $g(X) \subseteq f(x)$ and $h(X) \subseteq g(x)$ Choose $x_1, \in X$

Such that $h(x_0) = g(x_1)$ and $g(x_0) = f(x_1)$

In general

Choose x_{n+1} such that

$$y_n = fx_{n+1} = gx_n = hx_{n-1}$$

Then by 3.1 we have

$$M(fx_n, fx_{n+1}, t) = M(gx_{n-1}, gx_n, t) = M(hx_{n-2}, hx_{n-1}, t) \geq M(gx_{n-1}, gx_n, \frac{t}{k})$$

$$= M(hx_{n-2}, hx_{n-1}, \frac{t}{k}) \geq \cdots M(fx_0, fx_1, \frac{t}{k^n})$$

Therefore, for any p

$$M(fx_n, fx_{n+p}, t) = M(gx_{n-1}, gx_n, t) = M(hx_{n-2}, hx_{n-1}, t)$$

$$\geq M(fx_n, fx_{n+1}, \frac{t}{p})\ \geq \cdots \geq M(fx_{n+p-1}, fx_{n+p}, \frac{t}{p})$$

$$\geq M(fx_0, fx_1, \frac{t}{pk^n})\ \geq \cdots \geq M(fx_0, fx_1, \frac{t}{pk^{n+p-1}})$$

As $n \to \infty, \{fx_n\} = \{y_n\} = \{gx_n\} = \{hx_n\}$

is a Cauchy sequence and so by completeness of X.

$\{y_n\} = \{fx_n\} = \{gx_n\} = \{hx_n\}$is convergent.

call the limit zThen

$$lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = lim_{n\to\infty} \{hx_n\} = z$$

As $f(X)$ is complete.

So there exist a point$r$ in X such that $fr = gr = hr = z$ now from 1.1

$M_{n\to\infty}(hr, hx_n, kt) \geq M(gr, gx_n, t)$

$M(hr, z, kt) \geq M(gr, z, t)$

$M(hr, z, kt) \geq M(z, z, t)$

$M(hr, z, kt) \geq 1$

$M(hr, z, kt) = 1$

$\Rightarrow hr = z = gr$

Similarly

$M_{n\to\infty}(gr, gx_n, kt) \geq M(fr, fx_n, t)$

$M(gr, z, kt) \geq M(fr, z, t)$

$M(gr, z, kt) \geq M(z, z, t)$

$M(gr, z, kt) \geq 1$

$M(gr, z, kt) = 1$

$\Rightarrow gr = z = fr$

Therefore

$hr = gr = fr = z$

As f, g and h are weakly compatible therefore

$fgr = gfr = ghr = hgr$

i.e.$fz = gz = hz$

now we show that z is fixed point of $f, g \;\; and \;\; h$ from (3.1)

$M_{n\to\infty}(hz, hx_n, kt) \geq M(gz, gx_n, t)$

$M(hz, z, kt) \geq M(gz, z, t)$

$M(hz, z, kt) \geq M(hz, z, t)$

$\Rightarrow hz = z = gz$

Similarly

$M_{n\to\infty}(gz, gx_n, kt) \geq M(fz, fx_n, t)$

$M(gz, z, kt) \geq M(fz, z, t)$

$M(gz, z, kt) \geq M(gz, z, t)$

$\Rightarrow gz = z = fz$

Therefore $fz = gz = hz = z$

Hence z is a common fixed point of $f, g \ and\ h$

For uniqueness

Let z_0 be another fixed point of $f, g\ and\ h$ then by (1.1)

$$M(hz, hz_0, kt) \geq M(gz, gz_0, t)$$

$$M(z, z_0, kt) \geq M(z, z_0, t)$$

$$\Rightarrow z = z_0$$

Similarly$M(gz, gz_0, kt) \geq M(fz, fz_0, t)$

$$M(z, z_0, kt) \geq M(z, z_0, t)$$

$$\Rightarrow z = z_0$$

Therefore z is unique common fixed point of f, g and h

Theorem 3.2: Let $(X, M, *)$ be a fuzzy metric space let f, g and h be weakly compatible self maps of X satisfying conditions $\{1.1\}$ and $\{1.2\}$.

If one of $h(X), g(X)\ or\ f(X)$ is complete then f, g and h have a unique common fixed point.

Proof: From the proof of above theorem we conclude that $\{gx_n\} = \{y_n\}$ is a Cauchy sequence in X. Now, suppose that $g(X)$ is complete sub space of X then the subsequence of $\{y_n\}$ must get a limit in $g(X)$. call it be a and $g(b) = a$. As $\{y_n\}$ is a Cauchy sequence containing a convergent subsequence therefore the sequence $\{y_n\}$ also converges implying thereby the convergence of subsequence of the convergent sequence. Now from {1.1}

$M_{n\to\infty}(hb, hx_n, kt) \geq M(gb, gx_n, t)$

$M(hb, a, kt) \geq M(gb, a, t)$

$M(hb, a, kt) \geq M(a, a, t)$

$M(hb, a, kt) \geq 1$

$M(hb, a, kt) = 1$

Therefore
$hb = a = gb$

Similarly

$M_{n\to\infty}(gb, gx_n, kt) \geq M(fb, fx_n, t)$

$M(gb, a, kt) \geq M(fb, a, t)$

$M(gb, a, kt) \geq M(a, a, t)$

$M(gb, a, kt) \geq 1$

$M(gb, a, kt) = 1$

$\Rightarrow gb = a = fb$

Therefore
$hb = gb = fb = a$

Which shows that (f, g, h) has a point of coincidence.

Since $f, g \ and \ h$ are weakly compatible

$fgb = gfb = ghb = hgb$

i.e.$ha = ga = fa$

now, we show that a is a fixed point of $f, g \ and \ h$ from (1.1)

$M_{n\to\infty}(ha, hx_n, kt) \geq M(ga, gx_n, t)$

$M(ha, a, kt) \geq M(ga, a, t)$

$M(ha, a, kt) \geq M(ha, a, t)$

Therefore $ha = a = ga$

Similarly

$M_{n\to\infty}(ga, gx_n, kt) \geq M(fa, fx_n, t)$

$M(ga, a, kt) \geq M(fa, a, t)$

$M(ga, a, kt) \geq M(ga, a, t)$

$\Rightarrow ga = a = fa$

Therefore $ha = ga = fa = a$

Hence a is a fixed point of $f, g \ \ and\ h$

For uniqueness

Let z_1 be another fixed point of $f, g\ and\ h$ then (1.1)

$M(hz, hz_1, kt) \geq M(gz, gz_1, t)$

$M(z, z_1 kt) \geq M(z, z_1, t)$

$\Rightarrow z = z_1$

Therefore z is unique common fixed point of $f, g\ and\ h$.

Conflict of Interests

The authors declare that there is no conflict of interests.

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