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SUM CORDIAL LABELING OF GRAPHS

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Abstract. In this paper, we investigate the sum cordial labeling of flower graph, web graph, tadpole, triangular snake and shell graph.

Keywords: cordial labeling; sum cordial labeling; sum cordial graph.

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1. Introduction

All graphs $G = (V(G), E(G))$ in this paper are finite, connected and undirected. For any undefined notations and terminology we follow [3]. If the vertices or edges or both of the graph are assigned valued subject to certain conditions it is known as graph labeling. A dynamic survey on graph labeling is regularly updated by Gallian [4]. Labeled graphs have variety of applications in graph theory, particularly for missile guidance code, design good radar type codes and convolution codes with optimal autocorrelation properties. Labeled graphs plays vital role in the study of X-ray crystallography, communication network and to determine optimal

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circuit layouts. A detailed study on variety of applications on graph labeling is carried out in Bloom and Golomb [1].

2. Preliminaries

Definition 1. A mapping $f : V(G) \longrightarrow \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.

The induced edge labeling $f^* : E(G) \longrightarrow \{0,1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Let us denote $v_f(0),\ v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f ad $e_f(0),\ e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 2. A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called cordial if it admits labeling.

The concept of cordial labeling was introduced by Cahit [2] in which he investigated several results on this newly defined concept. Also, some new graphs are investigated as product cordial graphs by Vaidya [6].

Definition 3. A binary vertex labeling of a graph G with induce edge labeling $f^* : E(G) \longrightarrow \{0,1\}$ defined by $f^*(uv) = (f(u) + f(v))(mod 2)$ is called sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is sum cordial if it admits sum cordial labeling.

Shiama [5] investigated the sum cordial labeling and proved that path P_n, cycle C_n, star $K_{1,n}$ etc are some cordial graphs.

Definition 4. The wheel graph W_n is defined as the join of $K_1 + C_n$. The vertex corresponding to K_1 is said to be apex vertex, the vertices corresponding to cycle are called rim vertices. The edges corresponding to cycle are called the rim edges and edges joining apex and vertices of the cycle are called spoke edges.

Definition 5. The web Wb_n is the graph obtained by joining the pendant vertices of a helm H_n to from a cycle and then adding a pendant edge to each vertex of outer cycle.

Definition 6. The tadpole $C_n@P_m$ is formed by joining the end point of a path P_m to a cycle C_n.

Definition 7. The triangular snake T_n is obtained from the path P_n by replacing every edge of a path by a triangle C_3.

Definition 8. The shell S_n is the graph obtained by taking $n-3$ concurrent cords in the cycle C_n.
The vertex at which all the cords are concurrent is called an apex vertex. The shell S_n is also called fan F_{n-1}. Thus, $S_n = F_{n-1} = P_{n-1} + K_1$.

Definition 9. The wheel graph W_n is defined as the join of $K_1 + C_n$. The vertex corresponding to K_1 is said to be apex vertex, the vertices corresponding to cycle are called rim vertices. The edges corresponding to cycle are called the rim edges and edges joining apex and vertices of the cycle are called spoke edges.

Definition 10. The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.

Definition 11. The flower Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm.

3. Main results

Theorem 12. *The Web Wb_n is a sum cordial graph.*

Proof. Let v be an apex vertex and $v_1, v_2, \ldots, v_n$ are vertices of an inner cycle. We denote the pendant vertices and the vertices of an outer cycle by $v_1^{'}, v_2^{'}, \ldots, v_n^{'}$ and $v_1^{''}, v_2^{''}, \ldots, v_n^{''}$. Then $|V(Wb_n)| = 3n+1$ and $|E(Wb_n)| = 5n$. To define $f : V(Wb_n) \longrightarrow \{0,1\}$, we consider the following two cases.

For even n:

$$f(v) = 0$$

$$f(v_i) = f(v_i^{'}) = f(v_i^{''}) = \begin{cases} 1, & i \text{ is odd;} \\ 0, & i \text{ is even.} \end{cases} \quad ;1 \leq i \leq n$$

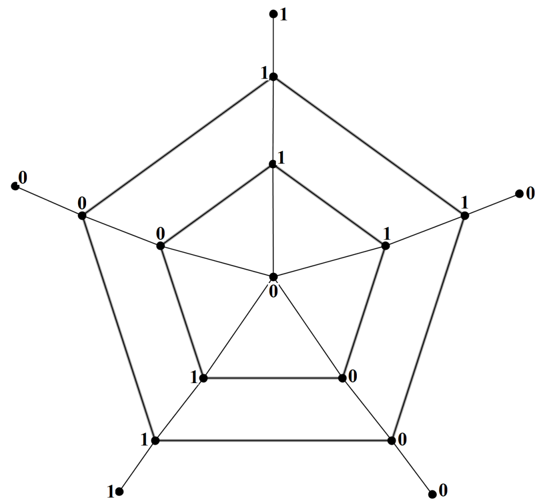

FIGURE 1. Sum cordial labeling of the Web graph Wb_5

In view of the above labeling pattern, we have $v_f(0) = \left\lceil \frac{3n+1}{2} \right\rceil$, $v_f(1) = \left\lfloor \frac{3n+1}{2} \right\rfloor$, $e_f(0) = \frac{5n}{2} = e_f(1)$. Thus, we get $|v_f(0) - v_f(1)| \leq 1,\ |e_f(0) - e_f(1)| \leq 1$.

For odd n:

$$f(v) = 0$$

$$f(v_i) = f(v_i^{'}) = \begin{cases} 1, & i \text{ is odd;} \\ 0, & i \text{ is even.} \end{cases} \quad ; 1 \leq i \leq n$$

$$f(v_i^{''}) = \begin{cases} 1, & i \text{ is odd;} \\ 0, & i \text{ is even.} \end{cases} \quad ; 1 \leq i \leq n-1$$

$$f(v_n^{''}) = 0$$

In view of the above labeling pattern, we have $v_f(0) = \frac{3n+1}{2} = v_f(1)$, $e_f(0) = \left\lceil \frac{5n}{2} \right\rceil$, $e_f(1) = \left\lfloor \frac{5n}{2} \right\rfloor$. Thus, we get $|v_f(0) - v_f(1)| \leq 1,\ |e_f(0) - e_f(1)| \leq 1$.

Hence, Wb_n is a sum cordial graph.

Example 13. The web graph Wb_5 is a sum cordial graph.

Theorem 14. *The shell S_n is a sum cordial graph.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n. Therefore, for the shell S_n, $|V(S_n)| = n$ and $|E(S_n)| = 2n-3$. To define, $f : V(S_n) \to \{0,1\}$, we consider the following cases,

For odd n:

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \text{ or } 2(mod 4); \\ 0, & i \equiv 0 \text{ or } 3(mod 4). \end{cases}$$

Therefore, $v_f(0) = \left\lfloor \frac{n}{2} \right\rfloor$, $v_f(1) = \left\lceil \frac{n}{2} \right\rceil$, $e_f(0) = \left\lfloor \frac{2n-3}{2} \right\rfloor$, $e_f(1) = \left\lceil \frac{2n-3}{2} \right\rceil$. Hence, $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

For even n: $n \equiv 0(mod 4)$**:**

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \text{ or } 2(mod 4); \\ 0, & i \equiv 0 \text{ or } 3(mod 4). \end{cases} \quad ; \ 1 \le i \le n$$

$n \equiv 2(mod 4)$**:**

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \text{ or } 2(mod 4); \\ 0, & i \equiv 0 \text{ or } 3(mod 4). \end{cases} \quad ; \ 1 \le i \le n-2$$

$$f(v_n) = 1$$

$$f(v_{n-1}) = 0$$

Therefore, $v_f(0) = \frac{n}{2} = v_f(1)$ and $e_f(0) = \left\lfloor \frac{2n-3}{2} \right\rfloor$, $e_f(1) = \left\lceil \frac{2n-3}{2} \right\rceil$. So, $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. Hence, the shell S_n is a sum cordial graph.

Example 15. The shell S_6 is a sum cordial graph.

Theorem 16. *The triangular snake T_n is a sum cordial graph except $n \equiv 3(mod 4)$.*

Proof. Let the path P_n having vertices $v_1, v_2, \ldots, v_n$ and edges $e_1, e_2, \ldots, e_{n-1}$. To construct a triangular snake T_n from the path P_n, join v_i and v_{i+1} to new vertex $v_i^{'}$ by edges $e_{2i-1}^{'} = v_i v_i^{'}$ and $e_{2i}^{'} = v_{i+1} v_i^{'}$ for $i = 1, 2, \ldots, n-1$. Then $|V(T_n)| = 2n-1$ and $|E(T_n)| = 3n-3$. To define $f : V(T_n) \to \{0,1\}$, we consider the following cases

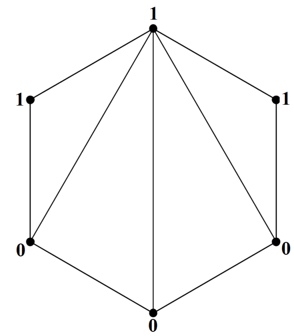

FIGURE 2. Sum cordial labeling of the Shell S_6

For $n \equiv 0,1,2(mod4)$**:**

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \text{ or } 2(mod4); \\ 0, & i \equiv 0 \text{ or } 3(mod4). \end{cases} ;\ 1 \leq i \leq n$$

$$f(v_i^{'}) = \begin{cases} 1, & \text{if } i \text{ is even;} \\ 0, & \text{if } i \text{ is odd.} \end{cases} ;\ 1 \leq i \leq n-1$$

Therefore,

$$v_f(0) = \begin{cases} \left\lceil \frac{2n-1}{2} \right\rceil, & n \equiv 0(mod4); \\ \left\lfloor \frac{2n-1}{2} \right\rfloor, & n \equiv 1 \text{ or } 2(mod4). \end{cases}$$

$$v_f(1) = \begin{cases} \left\lfloor \frac{2n-1}{2} \right\rfloor, & n \equiv 0(mod4); \\ \left\lceil \frac{2n-1}{2} \right\rceil, & n \equiv 1 \text{ or } 2(mod4). \end{cases}$$

$$e_f(0) = \begin{cases} \left\lceil \frac{3n-3}{2} \right\rceil, & n \equiv 0(mod4); \\ \frac{3n-3}{2}, & n \equiv 1(mod4); \\ \left\lfloor \frac{3n-3}{2} \right\rfloor, & n \equiv 2(mod4). \end{cases}$$

$$e_f(1) = \begin{cases} \left\lfloor \frac{3n-3}{2} \right\rfloor, & n \equiv 0(mod4); \\ \frac{3n-3}{2}, & n \equiv 1(mod4); \\ \left\lceil \frac{3n-3}{2} \right\rceil, & n \equiv 2(mod4). \end{cases}$$

Hence, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. So, triangular T_n is a sum cordial for $n \equiv 0,1$ or $2(mod4)$.

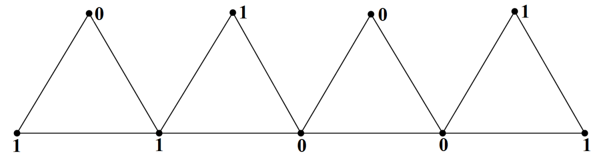

FIGURE 3. Sum cordial labeling of the Triangular snake T_5

For $n \equiv 3(mod4)$: In order to satisfy the vertex condition for the sum cordial graph it is necessary to assign 0 to at least $\left\lfloor \frac{2n-1}{2} \right\rfloor$ vertices out of $2n-1$ vertices. The vertices having label 0 will give rise either at least $\left\lceil \frac{3n-3}{2} \right\rceil$ or at most $\left\lfloor \frac{3n-3}{2} \right\rfloor$ edges with label 0 and at most $\left\lfloor \frac{3n-3}{2} \right\rfloor$ or at least $\left\lceil \frac{3n-3}{2} \right\rceil$ edges with label 1 out of $3n-3$ edges. Therefore, $|e_f(0) - e_f(1)| \geq 2$. Hence the edge condition for the sum cordial graph is not satisfied. hence the triangular T_n is not sum cordial for $n \equiv 3(mod4)$.

Example 17. The triangular snake T_5 is a sum cordial graph.

Theorem 18. *The Flower Fl_n is a sum cordial graph.*

Proof. Let v be an apex vertex and $v_1, v_2, \ldots, v_n$ are rim vertices. We denote the pendant vertices by $v_1', v_2', \ldots, v_n'$. Then $|V(Fl_n)| = 2n+1$ and $|E(Fl_n)| = 4n$. we define $f : V(Fl_n) \rightarrow \{0,1\}$ by $f(v) = 1,\ f(v_i) = 1,\ f(v_i') = 0$ for $1 \leq i \leq n$. In view of the above labeling pattern, we have $v_f(0) = n,\ v_f(1) = n+1,\ e_f(0) = 2n = e_f(1)$. Thus, we get $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Hence, Fl_n is a sum cordial graph.

Example 19. The flower F_5 is a sum cordial graph.

Theorem 20. *The tadpole $C_n@P_m$ is a sum cordial graph.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n and $v_1', v_2', \ldots, v_n'$ be the vertices of path P_m. Therefore $|V(C_n@P_m)| = n+m-1$ and $E(C_n@P_m)| = n+m-1$. Here, we denote $v_f^n(0),\ v_f^n(1)(v_f^m(0),\ v_f^m(1))$ by the number of vertices of a cycle C_n(Path P_m) having labels 0 and 1 respectively under f and $e_f^n(0),\ e_f^n(1)(e_f^m(0),\ e_f^m(1))$ by the number of edges of a cycle C_n(Path P_m) having labels 0 and 1 respectively under f_*. To prove To define $f : V(C_n@P_m) \rightarrow \{0,1\}$, we consider the following cases,

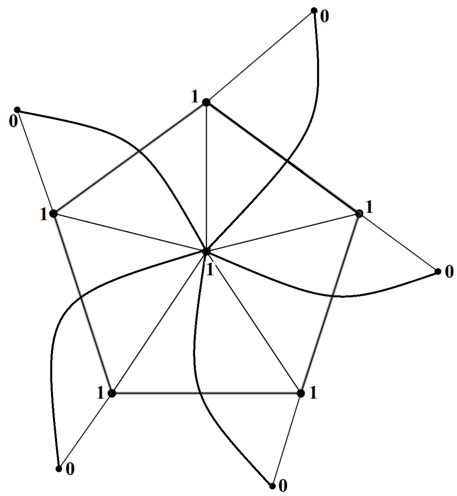

FIGURE 4. Sum cordial labeling of the Flower F_5

$n \equiv 0(mod4)$ **and** $m \in \mathbb{N}$**:**

$$f(v_i) = \begin{cases} 0, & i \equiv 1 \text{ or } 2(mod4); \\ 1, & i \equiv 0 \text{ or } 3(mod4). \end{cases} ;\ 1 \leq i \leq n$$

$$f(v_j^{'}) = \begin{cases} 1, & j \equiv 0 \text{ or } 1(mod4); \\ 0, & j \equiv 2 \text{ or } 3(mod4). \end{cases} ;\ 1 \leq j \leq m-1$$

Therefore, we have $v_f^n(0) = \dfrac{n}{2} = v_f^n(1)$ and $e_f^n(0) = \dfrac{n}{2} = e_f^n(1)$. Also,

$$v_f^m(0) = \begin{cases} \dfrac{m-1}{2}, & m \equiv 1 \text{ or } 3(mod4); \\ \left\lceil \dfrac{m-1}{2} \right\rceil, & m \equiv 0(mod4). \\ \left\lfloor \dfrac{m-1}{2} \right\rfloor, & m \equiv 0(mod4). \end{cases}$$

$$v_f^m(1) = \begin{cases} \dfrac{m-1}{2}, & m \equiv 1 \text{ or } 3(mod4); \\ \left\lfloor \dfrac{m-1}{2} \right\rfloor, & m \equiv 0(mod4). \\ \left\lceil \dfrac{m-1}{2} \right\rceil, & m \equiv 0(mod4). \end{cases}$$

$$e_f^m(0) = \begin{cases} \dfrac{m-1}{2}, & m \equiv 1 \text{ or } 3(mod4); \\ \left\lceil \dfrac{m-1}{2} \right\rceil, & m \equiv 0 \text{ or } 2(mod4). \end{cases}$$

$$e_f^m(1) = \begin{cases} \dfrac{m-1}{2}, & m \equiv 1 \text{ or } 3(mod4); \\ \left\lfloor \dfrac{m-1}{2} \right\rfloor, & m \equiv 0 \text{ or } 2(mod4). \end{cases}$$

Therefore,

$$\begin{aligned} v_f(0) - v_f(1) &= |v_f^n(0) + v_f^m(0) - (v_f^n(1) + v_f^m(1))| \\ &= |v_f^n(0) - v_f^n(1) + v_f^m(0) - v_f^m(1)| \\ &= |\frac{n}{2} - \frac{n}{2} + v_f^m(0) - v_f^m(1)| \\ &\leq 1 \end{aligned}$$

Also,

$$\begin{aligned} |e_f(0) - e_f(1)| &= |e_f^n(0) - e_f^n(1) + e_f^m(0) - e_f^m(1)| \\ &= |\frac{n}{2} - \frac{n}{2} + e_f^m(0) - e_f^m(1)| \\ &\leq 1 \end{aligned}$$

$n \equiv 1(mod4)$ **and** $m \in \mathbb{N}$**:**

$$f(v_i) = \begin{cases} 0, & i \equiv 1 \text{ or } 2(mod4); \\ 1, & i \equiv 0 \text{ or } 3(mod4). \end{cases} \quad 1 \leq i \leq n$$

$$f(v_j^{'}) = \begin{cases} 1, & j \equiv 1 \text{ or } 2(mod4); \\ 0, & j \equiv 0 \text{ or } 3(mod4). \end{cases} \quad 1 \leq j \leq m-1$$

Therefore, $v_f^n(0) = \left\lceil \frac{n}{2} \right\rceil$, $v_f^n(1) = \left\lfloor \frac{n}{2} \right\rfloor$ and $e_f^n(0) = \left\lceil \frac{n}{2} \right\rceil$, $e_f^n(1) = \left\lfloor \frac{n}{2} \right\rfloor$.
Therefore, $v_f^n(0) = v_f^n(1) + 1$ and $e_f^n(0) = e_f^n(1) + 1$. Also,

$$v_f^m(0)=\begin{cases}\dfrac{m-1}{2}, & m\equiv 1(mod4);\\ \dfrac{m-3}{2}, & m\equiv 3(mod4);\\ \dfrac{m-2}{2}, & m\equiv 0 \text{ or } 2(mod4).\end{cases}$$

$$v_f^m(1)=\begin{cases}\dfrac{m-1}{2}, & m\equiv 1(mod4);\\ \dfrac{m+1}{2}, & m\equiv 3(mod4);\\ \dfrac{m}{2}, & m\equiv 0 \text{ or } 2(mod4).\end{cases}$$

$$e_f^m(0)=\begin{cases}\dfrac{m-1}{2}, & m\equiv 1 \text{ or } 3(mod4);\\ \left\lfloor\dfrac{m-1}{2}\right\rfloor, & m\equiv 0 \text{ or } 2(mod4).\end{cases}$$

$$e_f^m(1)=\begin{cases}\dfrac{m-1}{2}, & m\equiv 1 \text{ or } 3(mod4);\\ \left\lceil\dfrac{m-1}{2}\right\rceil, & m\equiv 0 \text{ or } 2(mod4).\end{cases}$$

Therefore,

$$|v_f^m(0)-v_f^m(1)|=\begin{cases}0, & m\equiv 1(mod4);\\ -1, & m\equiv 3(mod4);\\ -1, & m\equiv 0 \text{ or } 2(mod4).\end{cases}$$

$$|e_f^m(0)-e_f^m(1)|=\begin{cases}0, & m\equiv 1 \text{ or } 3(mod4);\\ -1, & m\equiv 0 \text{ or } 2(mod4).\end{cases}$$

Hence,

$$|v_f(0)-v_f(1)|\leq 1$$

$$|e_f(0)-e_f(1)|\leq 1$$

$n\equiv 2(mod4)$ **and** m **is even:**

$$f(v_i)=\begin{cases}0, & i\equiv 1 \text{ or } 2(mod4);\\ 1, & i\equiv 0 \text{ or } 3(mod4).\end{cases}\quad 1\leq i\leq n$$

$$f(v_j^{'})=\begin{cases}1, & j\equiv 1 \text{ or } 2(mod4);\\ 0, & j\equiv 0 \text{ or } 3(mod4).\end{cases}\quad 1\leq j\leq m-1$$

Therefore, $v_f^n(0) = \frac{n}{2}+1,\ v_f^n(1) = \frac{n}{2}-1$ and $e_f^n(0) = \frac{n}{2}+1,\ e_f^n(1) = \frac{n}{2}-1$.
Therefore, $v_f^n(0) - v_f^n(1) = 2$ and $e_f^n(0) - e_f^n(1) = 2$.
Also, $v_f^m(0) = \frac{m}{2}-1,\ v_f^m(1) = \frac{m}{2}$ and $e_f^m(0) = \frac{m}{2}-1,\ e_f^m(1) = \frac{m}{2}$.
Therefore, $v_f^n(0) - v_f^n(1) = -1$ and $e_f^n(0) - e_f^n(1) = -1$.

Hence,

$$|v_f(0) - v_f(1)| \leq 1$$

$$|e_f(0) - e_f(1)| \leq 1$$

$n \equiv 2(mod 4)$ **and** m **is odd:**

$$f(v_i) = \begin{cases} 0, & i \equiv 1 \text{ or } 2(mod4); \\ 1, & i \equiv 0 \text{ or } 3(mod4). \end{cases} \quad 1 \leq i \leq n-2$$

$$f(v_{n-1}) = 1$$

$$f(v_n) = 0$$

$$f(v_j^{'}) = \begin{cases} 1, & j \equiv 0 \text{ or } 1(mod4); \\ 0, & j \equiv 2 \text{ or } 3(mod4). \end{cases} \quad 1 \leq j \leq m-1$$

Therefore, $v_f^n(0) = \frac{n}{2} = v_f^n(1)$ and $e_f^n(0) = \frac{n}{2}+1,\ e_f^n(1) = \frac{n}{2}-1$.
Therefore, $v_f^n(0) - v_f^n(1) = 0$ and $e_f^n(0) - e_f^n(1) = 2$.
Also, $v_f^m(0) = \frac{m-1}{2},\ v_f^m(1) = \frac{m-1}{2}$ and $e_f^m(0) = \left\lfloor \frac{m}{2} \right\rfloor - 1,\ e_f^m(1) = \left\lceil \frac{m}{2} \right\rceil$.
Therefore, $v_f^m(0) - v_f^m(1) = 0$ and $e_f^m(0) - e_f^m(1) = -2$.

Hence,

$$|v_f(0) - v_f(1)| = 0$$

$$|e_f(0) - e_f(1)| = 0$$

$n \equiv 3(mod 4)$ **and** $m \in \mathbb{N}$**:**

$$f(v_i) = \begin{cases} 0, & i \equiv 1 \text{ or } 2(mod4); \\ 1, & i \equiv 0 \text{ or } 3(mod4). \end{cases} ;\ 1 \leq i \leq n$$

$$f(v_j^{'}) = \begin{cases} 1, & j \equiv 1 \text{ or } 2(mod4); \\ 0, & j \equiv 0 \text{ or } 3(mod4). \end{cases} ;\ 1 \leq j \leq m-1$$

Therefore, $v_f^n(0) = \left\lceil \frac{n}{2} \right\rceil$, $v_f^n(1) = \left\lfloor \frac{n}{2} \right\rfloor$ and $e_f^n(0) = \left\lfloor \frac{n}{2} \right\rfloor$, $e_f^n(1) = \left\lceil \frac{n}{2} \right\rceil$.
Therefore, $v_f^n(0) - v_f^n(1) = 1$ and $e_f^n(0) - e_f^n(1) = -1$. Also,

$$v_f^m(0) = \begin{cases} \frac{m-1}{2}, & m \equiv 1(mod4); \\ \frac{m-3}{2}, & m \equiv 3(mod4); \\ \left\lfloor \frac{m-1}{2} \right\rfloor, & m \equiv 0 \text{ or } 2(mod4). \end{cases}$$

$$v_f^m(1) = \begin{cases} \frac{m-1}{2}, & m \equiv 1(mod4); \\ \frac{m+1}{2}, & m \equiv 3(mod4); \\ \left\lceil \frac{m-1}{2} \right\rceil, & m \equiv 0 \text{ or } 2(mod4). \end{cases}$$

$$e_f^m(0) = \begin{cases} \left\lceil \frac{m-1}{2} \right\rceil, & m \equiv 0 \text{ or } 2(mod4); \\ \frac{m+1}{2}, & m \equiv 1 \text{ or } 3(mod4). \end{cases}$$

$$e_f^m(1) = \begin{cases} \left\lfloor \frac{m-1}{2} \right\rfloor, & m \equiv 0 \text{ or } 2(mod4); \\ \frac{m-3}{2}, & m \equiv 1 \text{ or } 3(mod4). \end{cases}$$

Therefore,

$$v_f^m(0) - v_f^m(1) = \begin{cases} 0, & m \equiv 1(mod4); \\ -2, & m \equiv 3(mod4); \\ -1, & m \equiv 0 \text{ or } 2(mod4). \end{cases}$$

$$e_f^m(0) - e_f^m(1) = \begin{cases} 1, & m \equiv 0 \text{ or } 2(mod4); \\ 2, & m \equiv 1 \text{ or } 3(mod4). \end{cases}$$

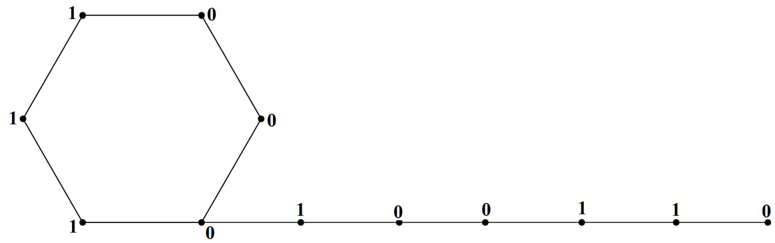

FIGURE 5. Sum cordial labeling of the Tadpole $C_6@P_7$

Hence,

$$|v_f(0) - v_f(1)| \leq 1$$

$$|e_f(0) - e_f(1)| \leq 1$$

Hence, the tadpole is a sum cordial graph.

Example 21. The tadpole $C_6@P_7$ is a sum cordial graph.

Conflict of Interests

The authors declare that there is no conflict of interests.

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