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SOME NEW FAMILIES OF PAIR SUM GRAPHS

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Abstract. Let G be a graph. An injective map $f: V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ is called a pair sum labeling if the induced edge function, $f_e: E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{(q-1)/2}\} \cup \{k_{(q+1)/2}\}$ according as q is even or odd. In this paper we investigate the pair sum labeling behavior of $P_n \times P_n$ if n is even, Prism $C_m \times P_2$, m is even and some cycle related graphs.

Keywords: Path, Cycle, Prism

2000 AMS Subject Classification:05C78

1. Introduction

The graphs in this paper are finite, undirected and simple.$V(G)$and$E(G)$ will denote the vertex set and edge set of a graph G. $p$and$q$ denote respectively the number of vertices and edges of G. The corona $G_1 \odot G_2$of two graphs $G_1$and$G_2$is defined as the graph obtained by taking one copy of G_1(with p_1 vertices) and p_1 copies of G_2 and then joining thei^{th}vertex of G_1 to all the vertices in the i^{th} copy of G_2. The product of two graphs $G_1$and G_2 is the graph $G_1 \times G_2$with vertex set $V_1 \times V_2$and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever

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$[u_1 = v_1\ and\ u_2\ adj\ v_2]$ or $[u_2 = v_2\ and\ u_1\ adj\ v_1]$. The graph $P_m \times P_n$ is called planar grid and $C_m \times P_n$ is called prism. Terms not defined here are used in the sense of Harary [1]. Concepts of pair sum labeling have been introduced in [2] and their behaviors are studied in [3,4,5]. In this paper we investigate pair sum labeling behavior of some cycle related graphs.

2. Pair Sum Labeling

Definition 2.1: Let G be a (p,q) graph. A one - one map $f: V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ is said to be pair sum labeling if the induced edge function $f_e: E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{(q-1)/2}\} \cup \{k_{(q+1)/2}\}$ according as q is even or odd. A graph with a pair sum labeling defined on it is called pair sum graph.

Theorem 2.2: The graph $P_n \times P_n$ is a pair sum graph if n is even.

Proof: We now display the structure of the graph$P_n \times P_n$.

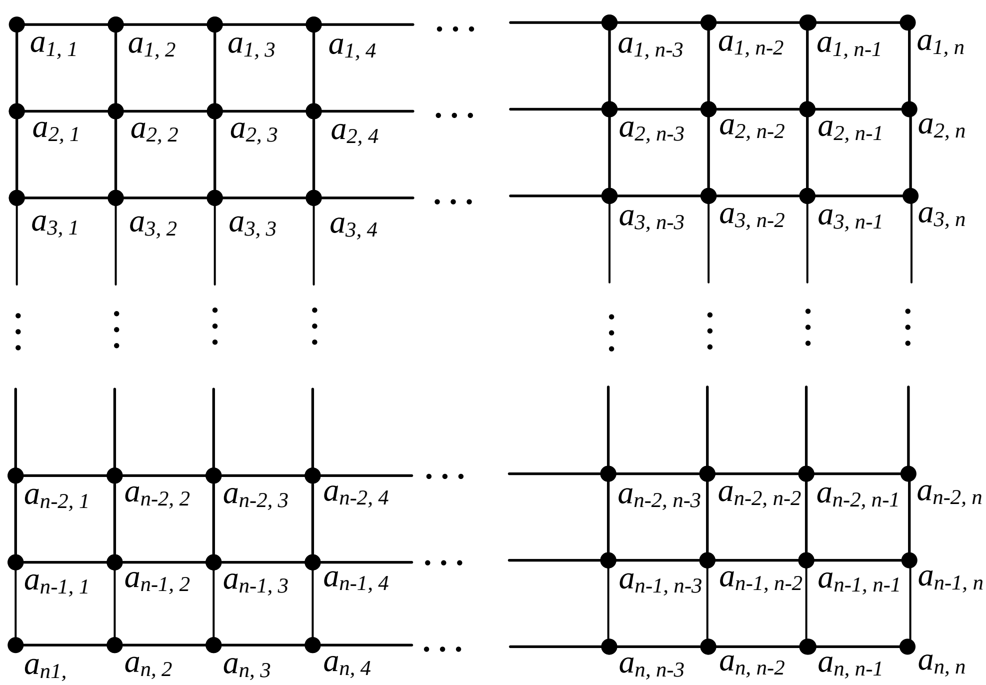

Fig. 1

Define $f: V(P_n \times P_n) \to \{\pm 1, \pm 2, \ldots, \pm n^2\}$ by

$$f(a_{i,j}) = -(2i + 2nj - 2n - 1); 1 \leq i \leq n, 1 \leq j \leq n/2$$

$$f(a_{i,j+n/2}) = n^2 - 2i - 2nj + 2n + 1; 1 \leq i \leq n, 1 \leq j \leq n/2.$$

Here $f_e(E(P_n \times P_n)) = \{(\pm(2n+2), \pm(6n+2), \pm(10n+2), \ldots, \pm(2n^2-6n+2)), (\pm(2n+6), \pm(6n+6), \pm(10n+6), \ldots, \pm(2n^2-6n+6)), \ldots, (\pm(6n-2), \pm(10n-2), \pm(14n-2), \ldots, \pm(2n^2-2n+2))\} \cup \{(\pm 4, \pm 8, \ldots, \pm(4n-4)), (\pm(4n+4), \pm(4n+8), \ldots, \pm(8n-4)), (\pm(8n+4), \pm(8n+8), \ldots, \pm(12n-4)), \ldots \ldots \ldots, (\pm(2n^2-4n+4), \pm(2n^2-4n+8), \ldots, \pm(2n^2-4))\} \cup \{(\pm 2, \pm 6, \pm 10, \ldots, \pm(2n-2))\}$.

Then f is a pair sum labeling. Then $P_n \times P_n$ is a pair sum graph if n is even.

Illustration 1: A pair sum labeling of $P_6 \times P_6$ is

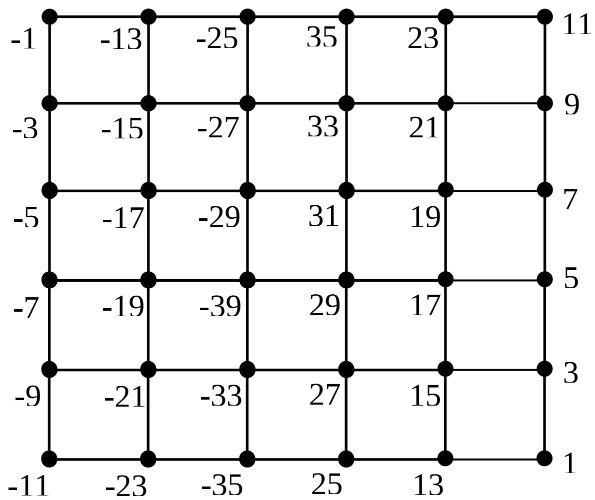

Fig. 2

Theorem 2.3: The Prism $C_n \times P_2$ is a pair sum graph if n is even.

Proof: Let $V(C_n \times P_2) = \{u_i, v_i : 1 \leq i \leq n\}$ and

$E(C_n \times P_2) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$.

Define $f : V(C_n \times P_2) \rightarrow \{\pm 1, \pm 2, \ldots, \pm 2n\}$ as follows.

Case (i) $n = 4m + 2$

Define $f(u_i) = i\ ; 1 \leq i \leq 2m+1$

$f(u_{2m+1+i}) = -i\ ; 1 \leq i \leq 2m+1$

$f(v_i) = 8m - 2i + 6\ ; 1 \leq i \leq 2m+1$

$f(v_{2m+1+i}) = -8m + 2i - 6\ ; 1 \leq i \leq 2m+1$

Here $f_e(E(C_n \times P_2)) = \{(\pm 3, \pm 5, \ldots, \pm(4m+1)\} \cup \{\pm 2m\} \cup \{(\pm(6m+5), \pm(6m+6), \ldots, \pm(8m+5)\}$.

Case (ii) $n = 4m$

$f(u_i) = i\ ; 1 \le i \le 2m - 1$

$f(u_{2m}) = 2m + 1$

$f(u_{2m+i}) = -i\ ; 1 \le i \le 2m - 1$

$f(u_{4m}) = -(2m + 1)$

$f(v_{2m+1-i}) = 8m - 2i + 2\ ; 1 \le i \le 2m$

$f(v_{2m+1}) = -(4m + 2i)\ ; 1 \le i \le 2m$

$f_e(E(C_n \times P_2)) = \{(\pm 3, \pm 5, \ldots, \pm(4m - 3)\} \cup \{\pm 2m, \pm 4m\} \cup \{(\pm(4m + 3), \pm(4m + 6), \ldots, \pm(10m - 3)\} \cup \{\pm(10m + 1)\}$.

Then f is a pair sum labeling.

Illustration 2: A pair sum labeling of $C_{10} \times P_2$ is

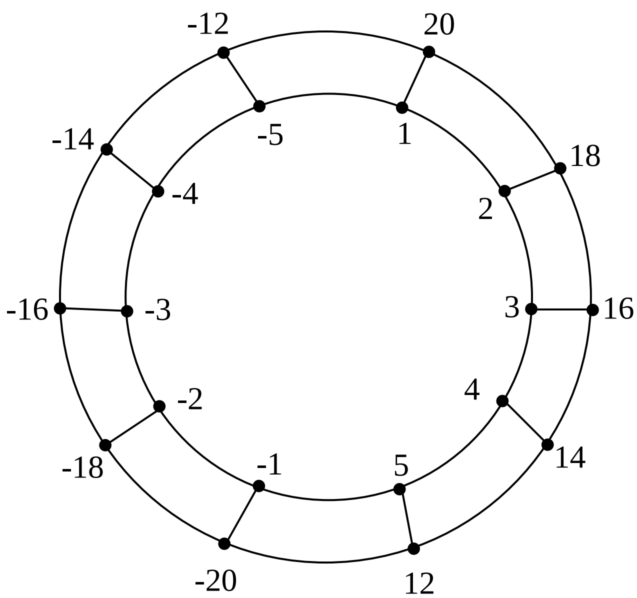

Fig. 3

Notation: We denote the vertex set and edge set of ladder $L_n = P_n \times P_2$ as follows.

Let $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and

$E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n - 1\} \cup \{u_i v_i : 1 \le i \le n\}$.

Theorem 2.4: $L_n \odot K_1$ is a pair sum graph.

Proof: Let $w_{1,} w_{2,} \ldots, w_n$ be the pendant vertices adjacent to $u_{1,} u_{2,} \ldots, u_n$ and $w_{n+1,} w_{n+2,} w_{2n}$

be the pendant vertices adjacent to $v_{1,}v_{2,} \ldots , v_n$.

Case (i) n is odd.

Let $n = 2m + 1$.

Define $f: V(L_n \odot K_1) \to \{\pm 1, \pm 2, \ldots , \pm(8m + 4)\}$ by

$f(u_i) = -4(m + 1) + 2i\,; 1 \leq i \leq m$

$f(u_{m+1}) = -(2m + 1)$

$f(u_{m+1+i}) = 2m + 2i + 2\,; 1 \leq i \leq m$

$f(v_i) = -4m - 3 + 2i\,; 1 \leq i \leq m$

$f(v_{m+1}) = 2m + 2$

$f(v_{m+1+i}) = 2m + 2i + 1\,; 1 \leq i \leq m$

$f(w_i) = -8m - 6 + 2i\,; 1 \leq i \leq m + 1$

$f(w_{2m+2-i}) = 8m + 6 - 2i\,; 1 \leq i \leq m$

$f(w_{2m+1+i}) = -8m - 6 + 2i\,; 1 \leq i \leq m$

$f(w_{2n+1-i}) = 8m + 5 - 2i\,; 1 \leq i \leq m + 1$

Here $f_e\big(E(L_n \odot K_1)\big) = f_e\big(E(L_n)\big) \cup \{\pm 6m, \pm(6m - 4), \pm(6m - 8), \ldots , \pm(4m + 6)\} \cup \{-4m - 1\} \cup \{\pm(6m - 2), \pm(6m - 6), \ldots , \pm(6m + 4)\} \cup \{4m + 1\}$.

Case (ii) n is even.

Let $n = 2m$. Define a map f as follows:

$f(u_{m+1-i}) = -2i; 1 \leq i \leq m$

$f(u_{m+i}) = 2i - 1\,; 1 \leq i \leq m$

$f(u_{m+i}) = 2i\,; 1 \leq i \leq m$

$f(u_{m+1-i}) = -(2i - 1); 1 \leq i \leq m$

$f(w_i) = -8m + 2 + 2i + 1\,; 1 \leq i \leq m$

$f(w_{2m+1-i}) = 8m + 1 - 2i\,; 1 \leq i \leq m$

$f(w_{2m+i}) = -8m - 1 + 2i\,; 1 \leq i \leq m$

$f(w_{4m+1-i}) = 8m - 2 - 2i\,; 1 \leq i \leq m$

Here $f_e\big(E(L_n \odot K_1)\big) = f_e\big(E(L_n)\big) \cup \{\pm 10m, \pm(10m - 4), \pm(10m - 8), \ldots , \pm 6m\} \cup \pm\{(10m - 2), \pm(10m - 6), \ldots , \pm(6m + 2)\}$. Then f is a pair sum labeling.

Illustration 3: A pair sum labeling of $L_7 \odot K_1$ is

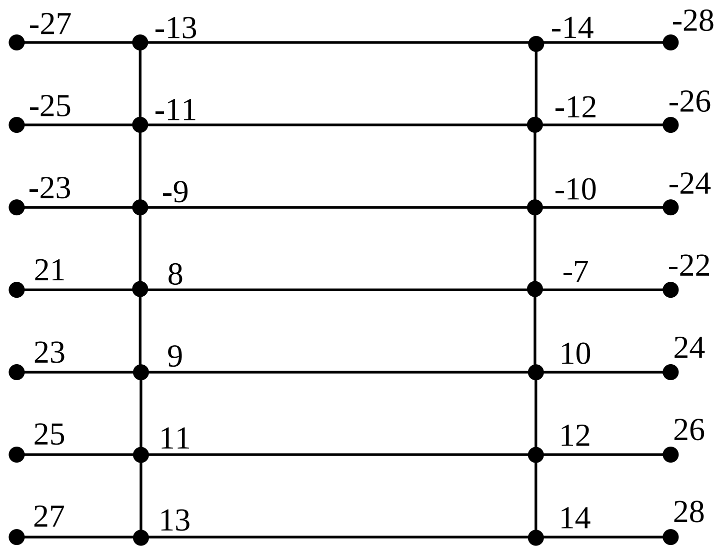

Fig.4

Notation: Two copies of the cycle C_m connected by the path P_n is denoted by $[C_m, P_n]$.

Theorem 2.5: The graph $[C_m, P_m]$ is a pair sum graph.

Proof: Let the first copy of cycle C_m be $u_1u_2 \ldots u_mu_1$ and second copy of cycle $C_m$be $v_1v_2 \ldots v_nv_1$. Let P_m be the path $w_1w_2 \ldots w_m$.Let $V([C_m, P_m]) = V(C_m) \cup V\ (C_m) \cup V(P_m)$ and$E([C_m, P_m]) = E(C_m) \cup E\ (C_m) \cup E(P_m) \cup \{u_1w_1, w_nv_1\}$.

Case (i) m is odd.

Define $f: V([C_m, P_m]) \rightarrow \{\pm 1, \pm 2, \ldots, \pm 3m\}$ by

$f\left(w_{(m-1)/2+i}\right) = i\ ; 1 \leq i \leq (m+1)/2$

$f\left(w_{(m-1)/2-2i+2}\right) = -2-2i\ ; 1 \leq i \leq (m-1)/4\ if\ m \equiv 1(mod4)$

$1 \leq i \leq (m+1)/4\ if\ m \equiv 3(mod4)$

$f\left(w_{(m-3)/2-2i+2}\right) = -2i+1\ ;\ 1 \leq i \leq (m-1)/4\ if\ m \equiv 1(mod4)$

$1 \leq i \leq (m+1)/4\ if\ m \equiv 3(mod4)$

$f(v_i) = -3m-i+1\ ; 1 \leq i \leq m$

$f(u_i) = 2m+2+i\ ; 1 \leq i \leq m-2$

$f(u_{m-1}) = 2m+1$

$f(u_m) = 2m+2.$

Here $f_e(E(C_m, P_m)) = \{(\pm3, \pm5, \ldots, \pm n)\} \cup \{\pm(4m+3), \pm(4m+5), \ldots, \pm(6m-1)\} \cup \{\pm(5m+1), \pm(5m+7/2)\}$.

Case (ii) m is even.

Assign the label to the vertices of path $P_{m-1}: u_2, u_3, \ldots, u_m$ as in case (i).

Label the vertex u_1 by $m/2+1$.

$f(u_i) = 2m+1+i\,; 1 \le i \le m-1$

$f(u_m) = 2m+1$

$f(v_i) = -3m+i\,; 1 \le i \le m-1$

$f(v_m) = -3m$.

Here $f_e(E(C_m, P_m)) = \{(\pm3, \pm5, \ldots, \pm n)\} \cup \{n+1, \pm(3m+1)\} \cup \{\pm(4m+3), \pm(4m+5), \ldots, \pm(6m-1)\} \cup \{\pm(5m+1)\}$.

Then clearly f is a pair sum labeling.

Illustration 4: A pair sum labeling of $[C_5, P_5]$ is

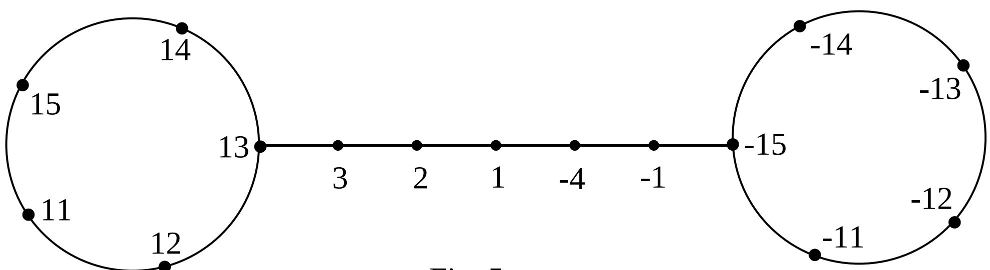

Fig. 5

Theorem 2.6: Let G be the graph with $V(G) = V(C_n) \cup \{v\}$ and $E(G) = E(C_n) \cup \{u_1v, u_3v\}$. Then G is a pair sum graph.

Proof: Let $u_1u_2 \ldots u_mu_1$ be the cycle C_n .

Case (i) $n = 2m+1$.

Define $f: V(G) \to \{\pm1, \pm2, \ldots, \pm(n+1)\}$ by

$f(u_i) = i; 1 \le i \le m+1$

$f(u_{n-2i+2}) = -2-2i\,; 1 \le i \le \lceil m/2 \rceil$

$f(u_{n-2i+1}) = 1-2i\,; 1 \le i \le \lfloor m/2 \rfloor$

$f(v) = -2$.

Here $f_e(E(G)) = \{(\pm3,\pm5,\dots,\pm(2m+1))\} \cup \{2,\pm1\}$ if $n \equiv 1(mod 4)$ and $f_e(E(G)) = \{(\pm3,\pm5,\dots,\pm(2m+1))\} \cup \{-2,\pm1\}$ if $n \equiv 3(mod4)$.

Case (ii) $n = 4m + 2$.

$f(u_1) = -(4m + 2)$

$f(u_{1+i}) = 2i; 1 \le i \le 2m + 1$

$f(u_{2m+2+i}) = -2i; 1 \le i \le 2m$

$f(v) = 2m - 1$.

Here $f_e\big(E(G)\big) = \{(\pm6,\pm10,\dots,\pm(8m+2))\} \cup \{\pm4m, \pm(2m + 3)\}$.

Case (iii)

Sub case (i) $n = 4$.

A pair sum labeling of G with $n = 4$ is

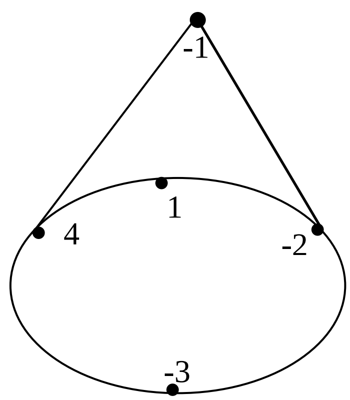

Fig. 6

Sub case (ii) $n = 4m, m > 1$

$f(u_1) = -4m + 1$

$f(u_{1+i}) = 2i - 1; 1 \le i \le 2m$

$f(u_{2m+2+i}) = -2i + 1; 1 \le i \le 2m - 1$

$f(v) = 2m - 2$.

Here $f_e\big(E(G)\big) = \{(\pm4,\pm8,\dots,\pm(8m-4))\} \cup \{\pm(2m + 1), \pm(4m - 2)\}$.

Then f is obviously a pair sum labeling.

Illustration 5: A pair sum labeling of $G$with $n = 9$ is

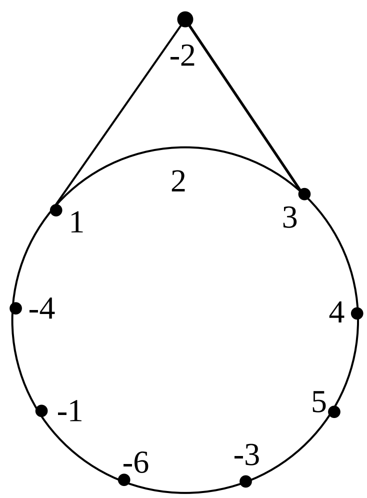

Fig. 7

Notation: Let G_n denotes the graph with vertex set $V(G_n) = V(C_n) \cup \{v_i: 1 \leq i \leq n\}$and edge set $E(G_n) = E(C_n) \cup \{u_i v_i, u_{i(i+1)modn}: 1 \leq i \leq n\}$.

Theorem 2.7: If n is even then G_n is a pair sum graph.

Define $f: V(G_n) \to \{\pm 1, \pm 2, \dots, \pm 2n\}$

$f(u_i) = 2i; 1 \leq i \leq n/2 - 1$

$f(u_{n/2}) = -2 - 2i$

$f(u_{n/2+i}) = -2i; 1 \leq i \leq n/2 - 1$

$f(u_n) = -2n$

$f(v_i) = 2i - 1; 1 \leq i \leq n/2$

$f(v_{n/2+i}) = -(2i - 1); 1 \leq i \leq n/2$.

Here $f_e(E(G)) = \{(\pm 6, \pm 10, \dots, \pm(2n\text{-}6))\} \cup \{\pm(3n-2), \pm(2n-2)\} \cup \{\pm 3, \pm 5, \dots, \pm(2n-3)\} \cup \{\pm(3n-1), \pm(2n-1)\}$.

Then G_n is a pair sum graph.

Illustration 6: A pair sum labeling of G_8 is

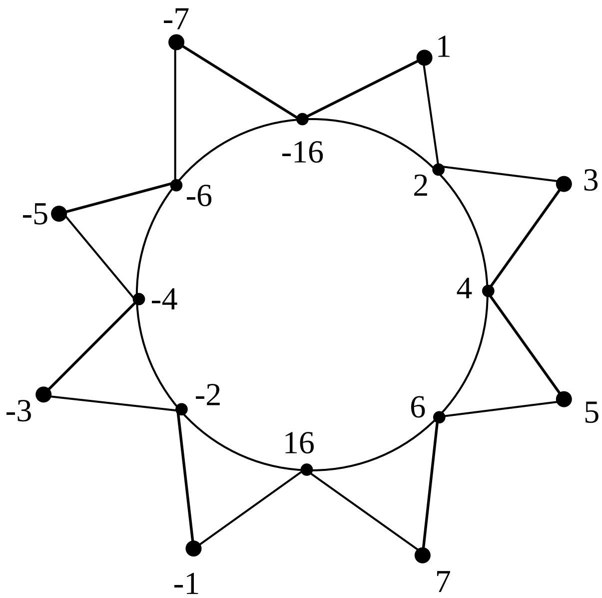

Fig. 8

Notation: Let G_n^* denotes the graph with vertex set $V(G_n^*) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and edge set $E(G_n^*) = E(C_n) \cup \{u_i v_i, u_i w_i v_i w_i : 1 \leq i \leq n\}$.

Theorem 2.8: If n is even then G_n^* is a pair sum graph.

Proof: Define $f: V(G_n^*) \rightarrow \{\pm 1, \pm 2, \dots, \pm 3n\}$ by

$f(u_i) = 6i - 5; 1 \leq i \leq n/2$

$f\left(u_{n/2+i}\right) = -6i + 5; 1 \leq i \leq n/2$

$f(v_i) = 6i - 4; 1 \leq i \leq n/2$

$f\left(v_{n/2+i}\right) = -6i + 4; 1 \leq i \leq n/2$

$f(w_i) = 6i - 3; 1 \leq i \leq n/2$

$f\left(w_{n/2+i}\right) = -6i + 3; 1 \leq i \leq n/2$

Here $f_e(E(G_n^*)) = \{(\pm 8, \pm 20, \pm 32, \dots, \pm(6n - 16))\}$

$\cup \{\pm(3n - 6)\} \cup \{(3,4,5), (15,16,17), \dots, (6n - 9, 6n - 8, 6n - 7)\}$

$\cup \{(-3, -4, -5), (-15, -16, -17), \dots, (-6n + 9, -6n + 8, -6n + 7)\}$.

Then f is a pair sum labeling.

Illustration 7: A pair sum labeling of G_8^* is

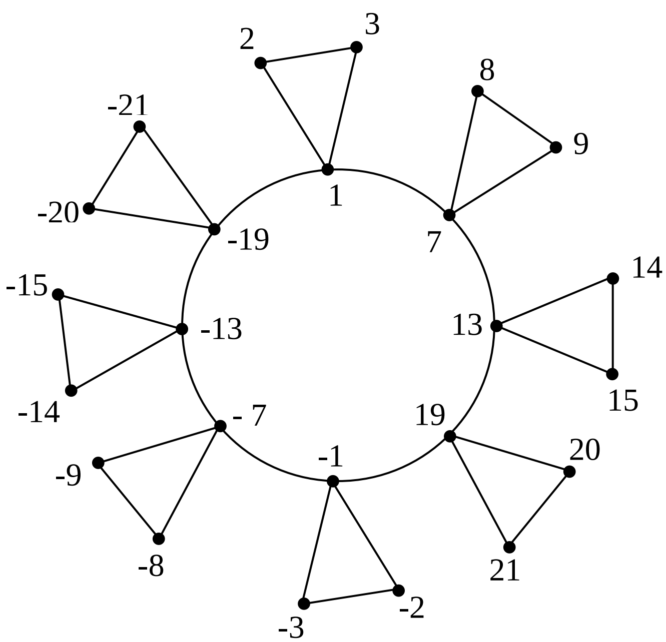

Fig. 9

Conclusion

Here we investigatepair sum labeling behavior of$P_n \times P_n$ (n is even), $C_m \times P_2$ ($m$is even), $L_n \odot K_1$, $[C_m, P_m]$ and some more standard graphs. Investigation ofpair sum labeling behavior of$P_m \times P_n$ $(m \neq n), C_m \times P_n$ $(n \neq 2)$, $[C_m, P_n]$ $(m \neq n)$ are open problems for future research.

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