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FINITE DIFFERENCE APPROXIMATIONS FOR THE MODIFIED EQUAL WIDTH WAVE (MEW) EQUATION

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Abstract. In this paper, the modified equal width wave (MEW) equation is solved numerically using the finite difference method. The stability analysis using Von-Neumann technique shows the schemes are unconditionally stable. Also the local truncation error of the method is investigated. Three invariant of motion are evaluated to determine the conservation properties of the problem, and the numerical scheme leads to accurate and efficient results. Moreover, interaction two and three solitary waves are studied. The development of the Maxwellian initial condition into solitary waves is also shown, and we shown that the number of solitons which are generated from the Maxwellian initial condition can be determined.

Keywords: modified equal width wave (MEW) equation; finite difference,stability; conservation properties, solitons.

2010 AMS Subject Classification: 47H17, 47H05, 47H09.

1. Introduction

In this paper we consider the numerical solution of the modified equal width wave (MEW) equation based upon the equal width wave (EW) equation [1, 2], the (MEW) equation suggested by Morrison et al. [3] in the form

$$u_t+\varepsilon u^2 u_x-\mu u_{xxt}=0, \tag{1}$$

where subscripts x and t denote differentiation and ε, μ are positive parameter with boundary conditions $u\to 0$ as $x\to\pm\infty$. The analytic solution of the (MEW) equation can be expressed in the form

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$$u(x,t)=\sqrt{2c}\,\operatorname{sech}\,(k(x-ct-x_0)), \tag{2}$$

where, $k=\frac{1}{\sqrt{\mu}}$ and c are constants.

This equation is related with the modified regularized long wave (MRLW) equation [4] and modified Korteweg-de Vries (MKdV) equation [5]. Geyikli and Battal Gazi Karakoc [6, 7] solved the MEW equation by a collocation method using septic B-spline finite elements and using a petrov- Galerkin finite element method with weight functions quadratic and element shape functions which are cubic B-splines. Esen applied a lumped Galerkin method based on quadratic B-spline finite elements which have been used for solving the EW and MEW equation [8, 9]. Saka proposed algorithms for the numerical solution of the MEW equation using quintic B-spline collocation metod [10]. Zaki considered the solitary wave interactions for the MEW equation by collocation method using quintic B-spline finite elements [11] and obtained the numerical solution of the EW equation by using least- squares method [12]. Wazwaz investigated an analytic solution to the MEW equation and two of its variants by the tanh and the sine-cosine methods [13]. Moreover, a solution based on a collocation method incorporated cubic B-spline is investigated by Saka and Dag [14]. The variational iteration method is applied to solve the MEW equation by Lu [15]. Evans and Raslan [16] studied the generalized EW equation by using collocation method based on quadratic B-spline to obtain the numerical solutions of a single solitary waves and the birth of solitons. The exact solitary wave solutions of the generalized EW equation is derived by Hamdi et al. [17] using Maple software. Esen and Kutluay studied a linearized implicit finite difference method in solving the MEW equation [18].

In the present work we solve the MEW equation numerically by the finite difference method. Moreover, interaction of solitary waves and other properties of the MEW equation are also studied.

2. The Proposed Finite Difference Schemes

In this section we will introduce three different schemes using finite difference method to tackle the problem under investigation. Numerical solution will be obtained as well as stability and error analysis will be studied.

2.1. First finite difference schemes

A finite difference scheme is produced when the partial derivatives in the partial differential equation(s) governing a physical phenomenon are replaced by a finite difference approximation. The result is a single algebraic or a system of algebraic equation which, when solved, provides an approximation to the solution of the original partial differential equation at selected points of a solution grid. The solution grid (also referred to as computational grid or numerical grid) is originated by dividing the axes representing the independent variables in the solution domain into a number of intervals. The extreme points of the interval will represent points in the solution grid. If we draw lines perpendicular to a given axes passing through the extreme points of the intervals, the resulting grid is the computational grid.

To apply the finite difference method for solving the MEW equation, firstly we present the following notations for the derivatives

$$
\begin{aligned}
&(u_t)_j^n \cong \frac{u_j^{n+1} - u_j^n}{k}, \\
&(u^2 u_x)_j^n \cong (u_j^n)^2 \frac{\theta(u_{j+1}^{n+1} - u_{j-1}^{n+1}) + (1-\theta)(u_{j+1}^n - u_{j-1}^n)}{2h}, \qquad (3)\\
&(u_{xxt})_j^n \cong \frac{(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}) - (1-\theta)(u_{j+1}^n - 2u_j^n + u_{j-1}^n)}{kh^2},
\end{aligned}
$$

where $x_j = jh,\ t_n = nk,\quad j = 0,1,...$ and $n = 0,1,...$ where superscript n denotes quantity associated with time level t_n and subscript j denotes a quantity associated with space mesh point x_j, and θ is selected to be between 0 and 1. Thus when $\theta = 0.5$ the two time steps, namely n and $n+1$ have equal weight. Also, when $\theta = 0$ the resulting finite difference approximation corresponds to time t_n and when $\theta = 1$ the finite difference approximation corresponds to time t_{n+1}. The second-order finite difference approximation shown above is known as the Crank-Nicholson formulation.

Now, we assume that u_j^n is the exact solution at the grid point (x_j, t_n) and U_j^n is the approximate numerical values at the same point. Then the finite difference scheme for the Eq. (1) becomes

$$(U_j^n)_t + \varepsilon (U_j^n)^2 (U_j^n)_x - \mu (U_j^n)_{xxt} = 0, \qquad (4)$$

By using the difference approximation given by relations (3) in Eq. (4) we have

$$
\begin{aligned}
&(\mu + \theta P)U_{j-1}^{n+1} - (2\mu + h^2)U_j^{n+1} + (\mu - \theta P)U_{j+1}^{n+1} \\
&\qquad = (\mu - (1-\theta)P)U_{j-1}^n - (2\mu + h^2)U_j^n + (\mu + (1-\theta)P)U_{j+1}^n, \qquad (5)
\end{aligned}
$$

where

$$P = \frac{\varepsilon k h}{2} (U_j^n)^2 \ . \qquad (6)$$

2.1.1 Stability Analysis of First Difference Scheme

In this subsection we investigate stability of the scheme using the Von-Neumann stability analysis after linearizing the nonlinear term in (5) through the following stated and proved Lemmas.

Lemma 2.1. The finite difference scheme (5) is unconditionally stable.

Proof. The Von-Neumann stability theory will be applied to investigate the stability of the finite difference scheme as

$$U_j^n = \xi^n e^{ikhj}, \qquad i = \sqrt{-1}, \tag{7}$$

where k is the mode number and h is the element size.

Substitute the Fourier mode (7) into the linearized recurrence relationship Eq. (5) shows that

$$(A - i\,\theta B)\xi^{n+1} = (A + i(1-\theta)B)\xi^n \tag{8}$$

where A and B are as follows

$$\begin{aligned} A &= 2\mu\cos kh - 2\mu - h^2, \\ B &= 2P\sin kh, \end{aligned} \tag{9}$$

The amplification factor for mode k is:

$$g = \frac{\xi^{n+1}}{\xi}, \tag{10}$$

Using Eq. (8) and Eq. (10) we get:

$$g = \frac{(A + i(1-\theta)B)}{(A - i\,\theta B)}, \tag{11}$$

The stability will be discussed for three cases

(1) Explicit scheme,

Put $\theta = 0,$ in Eq. (11) so the amplification factor g takes the form

$$g = \frac{(A + iB)}{A}, \tag{12}$$

So the explicit scheme is unstable since $|g| > 1$

(2) Crank-Nicolson scheme

Put $\theta = \frac{1}{2},$ in Eq. (11) so the amplification factor g takes the form

$$g = \frac{(A + i\frac{B}{2})}{(A - i\frac{B}{2})}, \tag{13}$$

So the Crank-Nicolson scheme is marginally stable since in this case $|g| = 1,$

(3) The fully implicit scheme

Put $\theta = 1,$ in Eq. (11) so the amplification factor g takes the form

$$g = \frac{A}{A - iB}, \tag{14}$$

So the fully implicit scheme is unconditionally stable since in this case $|g| < 1.$

2.1.2 Error Analysis of First Difference Scheme

Lemma 2.2. The truncation error T_j^n of the finite difference scheme (5) is of order $(k + h^2).$

Proof. To study the accuracy of scheme (5) we use a Taylor's series expansion of all terms about the point $(x_j, t_n),$ when $h = x_{j+1} - x_i$ and $k = t_{n+1} - t_n,$ then the local truncation error can be written in the form

$$\begin{aligned} T_j^n &= \left(\frac{\partial u}{\partial t} + \varepsilon u^2 \frac{\partial u}{\partial x} - \mu \frac{\partial^3 u}{\partial x^2 \partial t}\right)_j^n \\ &\quad + \frac{k}{2}\left(\frac{\partial^2 u}{\partial t^2} + 2\varepsilon\theta u^2 \frac{\partial^2 u}{\partial x \partial t}\right)_j^n + \frac{\varepsilon h^2}{6}\left(u^2 \frac{\partial^3 u}{\partial x^3}\right)_j^n + \dots \end{aligned} \tag{15}$$

The first term is zero by Eq.(1), and so we end with local truncation error

$$\left[\frac{k}{2}\left(\frac{\partial^2 u}{\partial t^2} + 2\varepsilon\theta u^2 \frac{\partial^2 u}{\partial x \partial t}\right) + \frac{\varepsilon h^2}{6} u^2 \frac{\partial^3 u}{\partial x^3}\right)_j^n] + \dots, \tag{16}$$

Hence, we get

$$T_j^n = O(k + h^2) \tag{17}$$

2.2. Second finite difference schemes

The 2nd finite difference scheme for MEW equation considered is given by

$$\begin{aligned} (u_t)_j^n &\cong \frac{u_j^{n+1} - u_j^n}{k}, \\ (u^2 u_x)_j^n &\cong \frac{1}{2}(u_{j-1}^n)^2 \frac{(u_{j+1}^{n+1} - u_{j-1}^{n+1})}{2h} + \frac{1}{2}(u_j^n)^2 \frac{(u_{j+1}^n - u_{j-1}^n)}{2h}, \\ (u_{xxt})_j^n &\cong \frac{(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}) - (u_{j+1}^n - 2u_j^n + u_{j-1}^n)}{kh^2}, \end{aligned} \tag{18}$$

By using Eq. (18) in Eq. (4) we have

$$\begin{aligned} &(\mu + P_1)U_{j-1}^{n+1} - (2\mu + h^2)U_j^{n+1} + (\mu - P_1)U_{j+1}^{n+1} \\ &\quad = (\mu - P_1)U_{j-1}^n - (2\mu + h^2)U_j^n + (\mu + P_1)U_{j+1}^n, \end{aligned} \tag{19}$$

where

$$P_1 = \frac{\varepsilon k h}{4}(U)^2, \tag{20}$$

2.2.1 Stability Analysis of Second Difference Scheme

Lemma 2.1. The finite difference scheme (19) is unconditionally stable.

Proof. The Von-Neumann stability analysis will be applied to investigate the stability of the 2^{nd} scheme by substitute the Fourier mode (7) into the linearized recurrence relationship (19) shows that

$$g = \frac{A_1 + i\,B_1}{A_1 - i\,B_1}, \tag{21}$$

where A_1 and B_1 are as follows

$$\begin{aligned} A_1 &= 2\mu\cos kh - 2\mu - h^2, \\ B_1 &= 2P_1\sin kh, \end{aligned} \tag{22}$$

So that from Eq. (21) it is clear that $|g| = 1$ thus the second scheme is marginally stable.

2.2.2 Error Analysis of Second Difference Scheme

We use a Taylor's series expansion of all terms in (19) about the point (x_j, t_n)

Lemma 2.2. The truncation error T_j^n of the finite difference scheme (19) is of order $(k+h)$.

Proof. Using n^{th} Taylor polynomial in two variables x and t of all terms in scheme (19) about the point (x_j, t_n), then the local truncation error can be written in the form

$$\begin{aligned} T_j^n &= \left(\frac{\partial u}{\partial t} + \varepsilon u^2 \frac{\partial u}{\partial x} - \mu \frac{\partial^3 u}{\partial x^2 \partial t}\right)_j^n \\ &\quad + \frac{k}{2}\left(\frac{\partial^2 u}{\partial t^2} + \varepsilon u^2 \frac{\partial^2 u}{\partial x \partial t}\right)_j^n - h\varepsilon\left(\left(u\frac{\partial u}{\partial x}\right)^2\right)_j^n + \dots, \end{aligned} \tag{23}$$

The first term is zero by Eq. (1), and so we end with local truncation error

$$\left[\frac{k}{2}\left(\frac{\partial^2 u}{\partial t^2} + \varepsilon u^2 \frac{\partial^2 u}{\partial x \partial t}\right) - h\varepsilon\left(u\frac{\partial u}{\partial x}\right)^2\right]_j^n + \dots, \tag{24}$$

Hence, we get

$$T_j^n = O(k+h) \tag{25}$$

2.3. Third finite difference schemes

The 3^{rd} finite difference scheme for the MEW equation considered is given by

$$(u_t)_j^n \cong \frac{u_j^{n+1} - u_j^{n-1}}{2k},$$
$$(u^2 u_x)_j^n \cong (u^2)_j^n \frac{(u_{j+1}^n - u_{j-1}^n)}{2h}, \tag{26}$$
$$(u_{xxt})_j^n \cong \frac{(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}) - (u_{j+1}^n - 2u_j^n + u_{j-1}^n)}{2kh^2},$$

where we use central-difference operator in time t and in space x, then by using Eq. (26) in Eq. (4) we have

$$\begin{aligned}\mu U_{j-1}^{n-1} - (2\mu + h^2)U_j^{n-1} + \mu U_{j+1}^{n-1} + P_2(U_{j+1}^n - U_{j-1}^n) \\ = \mu U_{j-1}^{n+1} - (2\mu + h^2)U_j^{n+1} + \mu U_{j+1}^{n+1},\end{aligned} \tag{27}$$

where,

$$P_2 = \varepsilon k h\, U^2 \tag{28}$$

2.3.1 Stability Analysis of Third Difference Scheme

Lemma3.1. The finite difference scheme (27) is unconditionally stable

Proof. Using Fourier method, assuming that u in the nonlinear term is locally constant. In case of applying the Von Neumann stability theory, the growth of Fourier mode takes the form

$$U_j^n = \xi^n e^{ikhj}, \qquad i = \sqrt{-1} \tag{29}$$

where k is a mode number and h is the element size.

Now, substituting Eq. (29) into scheme (27), and we use the amplification factor for mode k

$$\xi^{n+1} = g^2\, \xi^{n-1}, \tag{30}$$

where g is the amplification factor, and from Eqs. (29) and (32) we get

$$g^2 - 2g\, i \sin\varphi - 1 = 0, \tag{31}$$

where, $\sin\varphi = \dfrac{P_2 \sin kh}{2\mu \cos kh - 2\mu - h^2}$,

Eq. (31) yields $|g_1| = |g_2| = 1$, therefore the finite difference scheme is marginaly stable.

2.3.2 Error Analysis of Third Difference Scheme

Lemma 3.2. The truncation error T_j^n of the finite difference scheme (27) is of order $(h^2 + k^2)$.

Proof. Using n[th] Taylor polynomial in two variables x and t of all terms in scheme (27) about the point (x_j, t_n), then the local truncation error can be written in the form

$$T_j^n = \left(\frac{\partial u}{\partial t} + \varepsilon u^2 \frac{\partial u}{\partial x} - \mu \frac{\partial^3 u}{\partial x^2 \partial t}\right)_j^n + \left(\frac{k^2}{6}\frac{\partial^3 u}{\partial t^3} + \varepsilon \frac{h^2}{6} u^2 \frac{\partial^3 u}{\partial x^3}\right)_j^n + \ldots, \tag{32}$$

The first term is zero by the equation (1), and so we end with local truncation error

$$\left[\frac{k^2}{6}\frac{\partial^3 u}{\partial t^3} + \varepsilon \frac{h^2}{6} u^2 \frac{\partial^3 u}{\partial x^3}\right]_j^n + \ldots \tag{33}$$

Hence, we get

$$T_j^n = O(h^2 + k^2). \tag{34}$$

3. Numerical Tests and Results of MEW equation

In this section we present some numerical tests of our schemes and other methods for the solution of MEW equation for single solitary waves in addition to determining the solution of two and three solitary waves interaction at different time levels. The numerical solutions must preserve the conservation laws during propagation as discuss the three invariant conditions which correspond to conversation of mass, momentum, and energy [2] respectively

$$\mathrm{I}_1 = \int_a^b u\,dx, \qquad \mathrm{I}_2 = \int_a^b (u^2 + \mu (u_x)^2)\,dx, \qquad \mathrm{I}_3 = \int_a^b u^4\,dx \tag{35}$$

The accuracy of the method is measured using the following error norms

$$L_2 = \left\|u^E - u^N\right\| = \sqrt{h\sum_{i=0}^{N}(u_i^E - u_i^N)^2}, \quad L_\infty = \max_i \left|(u_i^E - u_i^N)\right|, \quad i = 1,2,\ldots,N-1, \tag{36}$$

Where u^E is the exact solution u and u^N is the approximations solution U_N.

we consider Eq. (1) with the boundary condition $u \to 0$ as $x \to \pm\infty$ and initial condition:

$$u(x,0) = A\operatorname{sec}h[k(x - x_0)]. \tag{37}$$

An analytical solution of this problem is given by

$$u(x,t) = A\operatorname{sec}h[k(x - ct - x_0)] \tag{38}$$

which represents the motion of a single solitary wave with amplitude $A = \sqrt{2c}$, $k = \sqrt{\frac{1}{\mu}}$.

3.1. Single Solitary Wave

To illustrate the validity of our scheme in case of a single soliton, we use the L_2-norm and L_∞-norm to test accuracy, also quantities I_1, I_2 and I_3 are shown to measure conservation laws for the scheme. we choose $h = k = 0.1$, $c = \frac{1}{32}$, $x_0 = 30$, with range [0,80], the simulations are done up to t = 1.0 and the vale of θ in the first scheme is chosen to be $\frac{1}{2}$. The invariants I_1, I_2 and I_3 change from their initial values by less than respectively, $2x10^{-5}$ and $1.2x10^{-6}$ respectively, during the time of running, for the first scheme and less than $1.7x10^{-3}$, $6x10^{-4}$ and $3.8x10^{-5}$ respectively for the second scheme, however the the invariants I_1, I_2 and I_3 for the third scheme approach to zero throughout. Our results are recorded in Table 1 and the motion of the solitary wave is plotted at different time levels in Fig. 1. Now we choose $h = k = 0.1$, $c = \frac{1}{32}$, $x_0 = 30$, with range [0,80], where the simulations are done up to t =5. The invariants I_3 changed by less than $1x10^{-8}$ and the changes for the invariants I_1, I_2 approach to zero throughout. Our results are recorded in Table 2 Errors in L_2 and L_∞ -norms are satisfactorily small as L_2 -error = $8.84979x10^{-5}$ and L_∞ -error $= 5.47289x10^{-5}$.

Table (1) Invariant and error norms for single solitary waves with $h = k = 0.1$, [0,80]

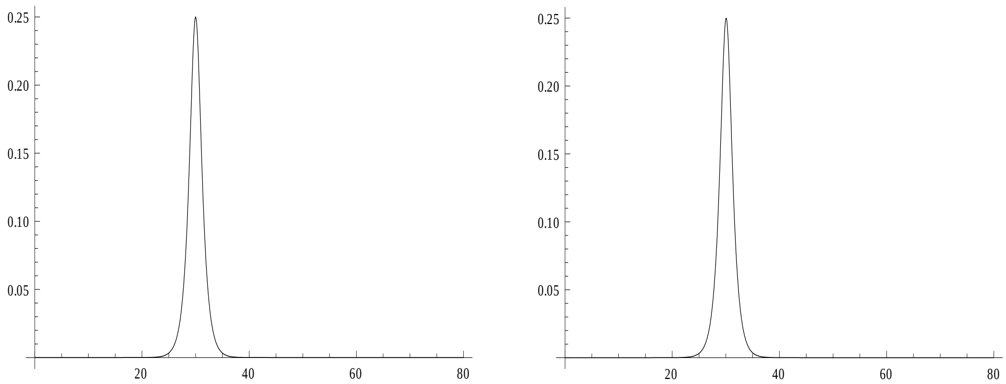
Schemes	t	I_1	I_2	I_3	L_2	L_∞
First scheme $\theta = \frac{1}{2}$	0.0	0.785398	0.166473	0.00520833	0.00000000	0.00000000
	0.2	0.785387	0.166469	0.00520809	5.88283E-6	4.65771E-6
	0.4	0.785375	0.166465	0.00520785	1.17852E-5	9.33685E-6
	0.6	0.785364	0.785364	0.00520761	1.77071E-5	1.40367E-5
	0.8	0.785352	0.166457	0.00520736	2.36487E-5	1.87566E-5
	1.0	0.785341	0.166453	0.00520712	2.96099E-5	2.35070E-5
Second scheme	0.0	0.785398	0.166478	0.00520833	0.00000000	0.00000000
	0.2	0.785753	0.166598	0.00521584	1.45237E-4	8.25630E-5
	0.4	0.786107	0.166718	0.00522337	2.90569E-4	9.83229E-5
	0.6	0.786462	0.166839	0.00523090	4.36003E-4	9.38439E-5
	0.8	0.786818	0.166959	0.00523845	5.81542E-4	9.32127E-5
	1.0	0.787173	0.167079	0.00524600	7.27192E-4	4.1172E-4
Third scheme	0.0	0.785398	0.166473	0.00520833	0.00000000	0.00000000
	0.2	0.785398	0.166473	0.00520833	3.54262E-6	2.16087E-6
	0.4	0.785398	0.166473	0.00520833	7.08503E-5	4.32575E-6
	0.6	0.785398	0.166473	0.00520833	1.06272E-5	6.49287E-6
	0.8	0.785398	0.166473	0.00520833	1.41692E-5	8.66232E-6
	1.0	0.785398	0.166473	0.00520833	1.77110E-5	1.08337E-5

Table (2) Invariant and error norms for single solitary waves with

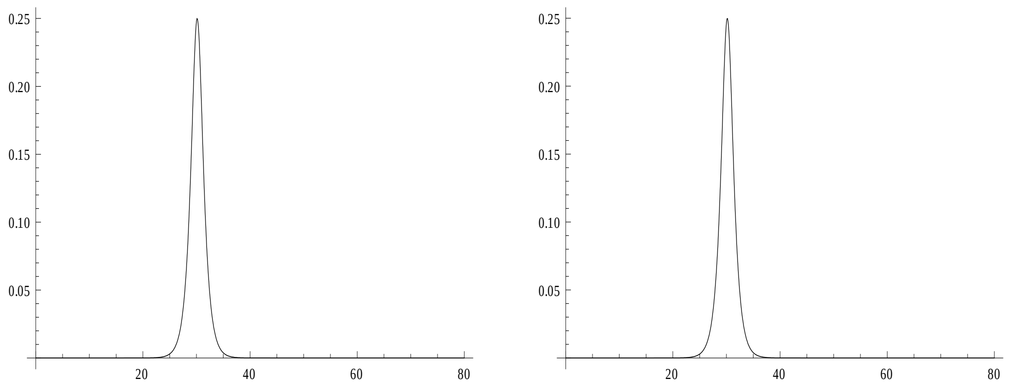
$h = k = 0.1, c = \frac{1}{32}$ [0,80]

t	I_1	I_2	I_3	L_2	L_∞
0.0	0.785398	0.166473	0.00520833	0.00000000	0.00000000
0.5	0.785398	0.166473	0.00520833	7.08583 E-6	4.33198 E-6
1.0	0.785398	0.166473	0.00520833	1.77110E-5	1.08337E-5
1.5	0.785398	0.166473	0.00520834	2.48037E-5	1.52009E-5
2.0	0.785398	0.166473	0.00520833	3.54166E-5	2.17056 E-5
2.5	0.785398	0.166473	0.00520834	4.25260E-5	2.60877 E-5
3.0	0.785398	0.166473	0.00520832	5.31165E-5	3.26262 E-5
3.5	0.785398	0.166473	0.00520834	6.02522E-5	3.71323 E-5
4.0	0.785398	0.166473	0.00520832	7.08104E-5	4.36745 E-5
4.5	0.785398	0.166473	0.00520835	7.79818E-5	4.82113 E-5
5.0	0.785398	0.166473	0.00520832	8.84979E-5	5.47289 E-5
5.0 [20]	0.7853960	0.1666662	0.0052083	2.04838E-5	5.47289 E-5
5.0 [21]	0.7853982	0.1666667	0.0052083	0.00007 E-3	0.00008 E-3

In the Table 2, The invariants I_1, I_2 approach to zero throughout, where I_3 changed by less than $1\text{x}10^{-7}$. A comparison with Petrov–Galerkin method with cubic B-spline functions [20] and with Quintic B-spline collocation method [21], shows not better results in terms of the L_2 and L_∞ error norms because we treat us the problem by the finite difference.



(a) (b)



(c) (d)

Fig. 1. Single solitary wave with $h = k = 0.1, c = \frac{1}{32}$ $[0,80]$ **at times: t = 0, t = 2, t = 4 and t = 5.**

3.2. Interaction of two solitary waves

The interaction of two MEW solitary waves having different amplitudes and traveling in the same directions illustrated. We consider the MEW equation with initial conditions given by the linear sum of two well separated solitary waves of various amplitudes

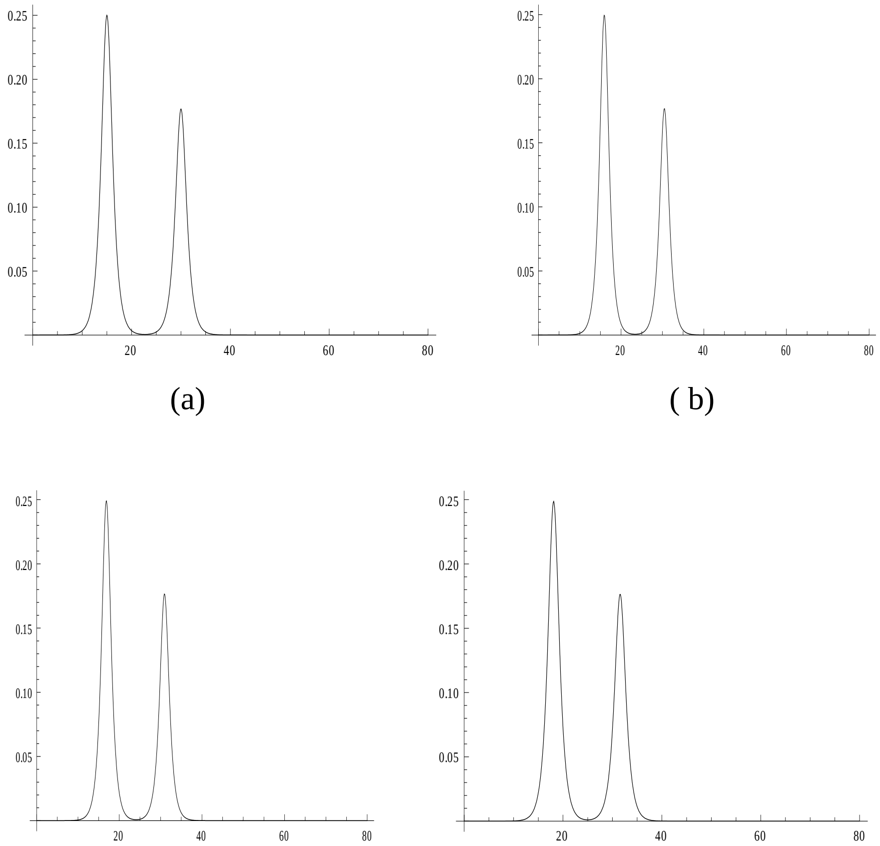
$$u(x,0) = \sum_{i=1}^{2} \sqrt{2c_i} \operatorname{sec}h\ (k(x - x_i)). \tag{39}$$

In our computational work, we choose $c_1 = \frac{1}{32}, c_2 = \frac{1}{64}$, $x_1 = 15, x_2 = 30,$ and $\mu = 1,\ h = 0.1 = \Delta t$ through the interval [0, 80]. And the change in I_1, I_2 and I_3 as seen in Table (3), are $3.4\text{x}10^{-4}$, $1.09\text{x}10^{-4}$ and $6.44\text{x}10^{-6}$ respectively, also Figure (2), shows the computer plot of the interaction of these solitary waves at different time levels.

Table (3) Invariant and error norms for two solitary waves with

$h = 0.1 = k, c_1 = \frac{1}{32}, c_2 = \frac{1}{64}, x_1 = 15, x_2 = 30$ [0,80]

t	I_1	I_2	I_3
0	1.34076	0.249709	0.00651044
1	1.34069	0.249688	0.00650915
2	1.34062	0.249666	0.00650786
3	1.34056	0.249644	0.00650657
4	1.34049	0.249622	0.00650529
5	1.34042	0.249600	0.00650400



(a) (b)

(c) (d)

Fig. 2. Two solitary waves with $h = k = 0.1, c_1 = \frac{1}{32}, c_2 \frac{1}{64}, x_1 = 15, x_2 = 30$ [0,80],

at times: t = 0, t = 30, t = 60 and t = 100

3.3. Interaction of three solitary waves

The interaction of three MEW solitary waves having different amplitudes and traveling in the same directions illustrated. We consider the MEW equation with initial conditions given by the linear sum of two well separated solitary waves of various amplitudes

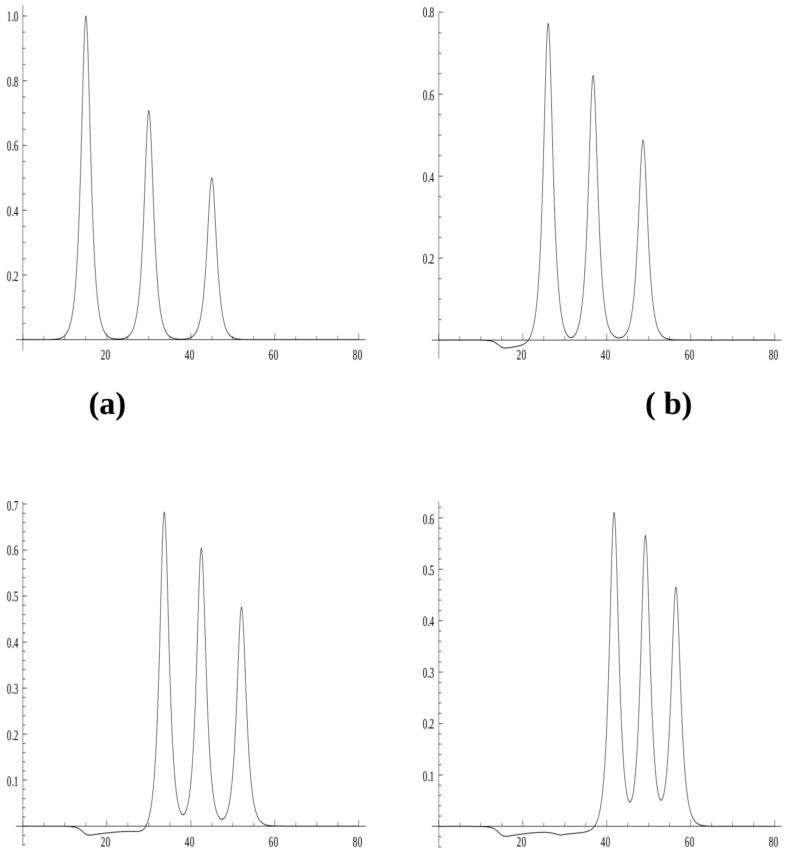
$$u(x,0) = \sum_{i=1}^{3} \sqrt{2c_i}\, \operatorname{sec}h\ (k(x - x_i)), \tag{39}$$

in our computational work, we choose $c_1 = \frac{1}{32}, c_2 = \frac{1}{64}, c_3 = \frac{1}{128},\ x_1 = 15, x_2 = 30, x_3 = 45,$ and $\mu = 1,\ h = 0.1 = \Delta t$ through the interval [0, 80]; and the change in I_1, I_2 and I_3 as seen in Table (4), are 3.5x10^{-4}, 1.11x10^{-4} and 6.64x10^{-6} respectively, also Figure (3), shows the computer plot of the interaction of these solitary waves at different time levels.

Table (4) Invariant and error norms for three solitary waves with

$h = k = 0.1, c_1 = \frac{1}{32}, c_2 = \frac{1}{64}, c_3 = \frac{1}{128}, x_1 = 15, x_2 = 30, x_3 = 45 \quad [0,80]$

t	I_1	I_2	I_3
0	1.73346	0.291328	0.00683596
1	1.73339	0.291306	0.00683467
2	1.73332	0.291284	0.00683338
3	1.73325	0.291261	0.00683209
4	1.73318	0.291239	0.00683079
5	1.73311	0.291217	0.00682950



(a) (b)

(c) (d)

Fig. 3. Three solitary waves with

$h = k = 0.1, c_1 = \frac{1}{32}, c_2 = \frac{1}{64}, c_3 = \frac{1}{128}, x_1 = 15, x_2 = 30, x_3 = 45,$ [0,80],

at times: t = 0, t = 30, t = 60 and t = 100

4. The Maxwellian Initial Condition

In final series of numerical experiments, the development of the Maxwellian initial condition

$$u(x,0) = \exp(-(x-7)^2), \tag{40}$$

into a train of solitary waves is examined. We apply it to the problem for different cases: (I) $\mu = 0.5$, (II) $\mu = 0.05$, $\mu = 0.1$, (III) $\mu = 0.02$, and (IV) $\mu = 0.005$. When μ is large such as case (I), only single soliton is generated as shown in Fig (4). However, when μ is reduced more and more such as case (II) two single soliton is generated as shown in Fig (5), and for case (III) three soliton is generated as shown in Fig (6), for the forth case (IV), the Maxwellian initial condition has decayed into four stable solitary waves as shown in Fig (7). The peaks of the well developed wave lie on a straight line so that their velocities are linearly dependent on their amplitudes and we observe a small oscillating tail appearing behind the last wave as shown in the figure (8) and all states at T =5, the values of the quantities I_1, I_2 and I_3 for the cases: $\mu = 0.05$, $\mu = 0.02$, and $\mu = 0.005$, are given in Table (5).

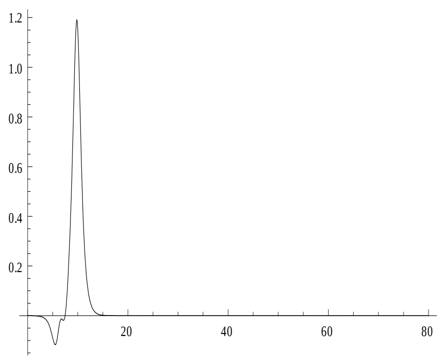
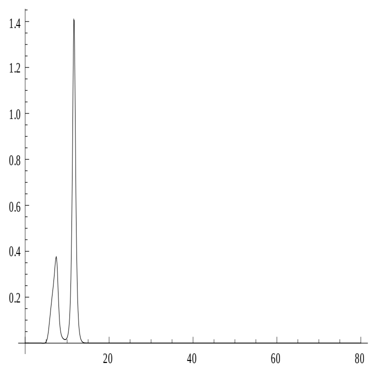
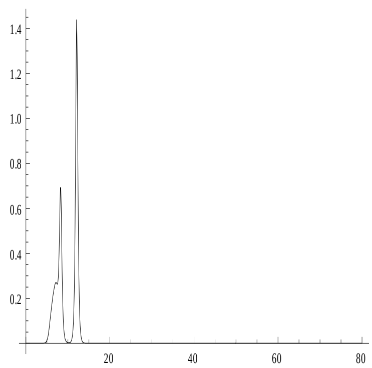


Fig (4): The Maxwellian initial condition at $\mu = 0.5$ and $t = 5$.



(a)



(b)

Fig (5): The Maxwellian initial condition at $t = 5$:

(a) $\mu = 0.1$ **(b)** $\mu = 0.05$.

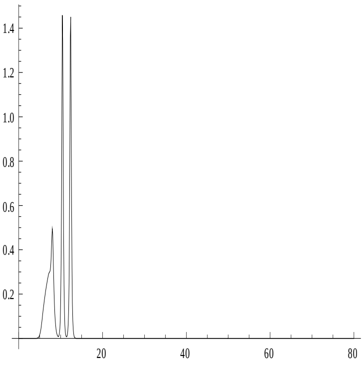


Fig (6): The Maxwellian initial condition at $\mu = 0.02,$ and $t = 5$.

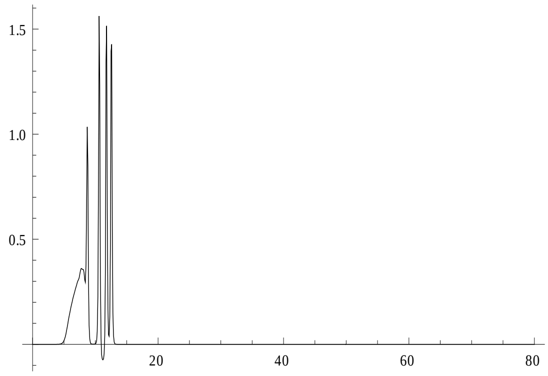


Fig (7): The Maxwellian initial condition at $\mu = 0.005,$ and $t = 5$.

Table (5) Invariant for solitary waves for $\mu = 0.05,\ 0.02,\ 0.005$

μ	t	I_1	I_2	I_3
0.05	0.2	1.77091	1.30815	0.873066
	0.4	1.77076	1.30304	0.864641
	0.6	1.77310	1.30112	0.864484
	0.8	1.77933	1.30526	0.878146
	1.0	1.79069	1.31865	0.910931
0.02	0.2	1.76500	1.26087	0.858357
	0.4	1.75859	1.24416	0.834843
	0.6	1.75385	1.22755	0.819448
	0.8	1.75369	1.21682	0.824998
	1.0	1.76203	1.22152	0.863547
0.005	0.2	1.76109	1.23501	0.847997
	0.4	1.74909	1.20773	0.812565
	0.6	1.73544	1.17258	0.786662
	0.8	1.72899	1.14804	0.807576
	1.0	1.74210	1.15998	0.893848

5. Conclusion

In this paper, the finite difference method is efficiently applied to solve modified equal width wave (MEW) equation. Also, the method is examined solitary waves where the schemes are unconditionally stable. We compared between the three schemes and then tested the best scheme through single solitary wave in which the analytic solution is known, and then extend it to study the interaction of solitons where no analytic solution is known during the interaction. The Maxwellian initial condition is used and a relation between μ and the number of waves is explored. To show how good and accurate the numerical solutions we have calculated the error norm L_2 and L_∞ . Moreover, despite the fact that the wave doesn't change, results show that the interaction results a tail of small amplitude in two and clearly three soliton interactions, and the conservation laws were satisfactorily satisfied. The appearance of such tail can be beneficial in further study.

Conflict of Interests

The author declares that there is no conflict of interests.

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