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A COINCIDENCE POINT RESULT BY USING ALTERING DISTANCE FUNCTION

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Abstract. In this paper we have used altering distance function to find a coincidence point result and also used weak compatibility to find a common fixed point result. The salient feature of our theorem is that, it contains square root in the inequality. We have also deduced some consequences of our result and supported the result with examples.

Keywords: Contraction, Fixed point, Coincidence point, Altering distance function, Weakly compatible mappings.

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1. Introduction

Banach Contraction Principle [4] is one of the most beautiful and important results in modern mathematics which is widely applied in many other branches of science and technology. This concept has been generalized by many authors and is also an important

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topic of current day's research.

In 1984 Khan, Swaleh and Sessa [14] introduced a new type of contraction with the help of a control function which they called altering distance function. After this a lot of results appeared in the literature which dealt with altering distance function. Some of the results are noted in [2, 9, 15, 16, 17, 18, 19, 20] and [21]. Further, altering distance function has been generalized to two variables, three variables and four variables in [3], [5] and [6] respectively. This is also applied to get fixed point results for fuzzy mappings [7].

The commuting and weakly commuting conditions were introduced by Jungck [10] and Sessa [22] respectively. After that in 1986 Jungck [11] introduced the compatibility of mappings to generalize the commutativity and applied it to find common fixed point results. There are so many results with compatibility conditions and this concept was further generalized by many authors as seen in works like [8, 11] and [24].

In this paper we have used altering distance function to find a coincidence point result and also used weak compatibility to find a common fixed point result. As a consequence of Theorem 3.1, we get results due to Banach, Kanna, Skof etc. We also illustrate some examples in support of our result.

2. Mathematical Preliminaries

Here we have given some definitions, theorem and lemma which are required for the main theorem.

Definition 2.1 (Fixed point [4]**):** Let $f : X \to X$ be a mapping, where (X, ρ) be a metric space. If $fx = x$, for some $x \in X$, x is called a fixed point of f.

Definition 2.2 (Coincidence point [11]**):** Let (X, ρ) be a metric space. Also let f and g be two self-maps on the set X, that is, $f,\ g : X \longrightarrow X$. If $w = fx = gx$, for some $x \in X$, x is called a coincidence point of f and g, w is called a point of coincidence of f and g.

Definition 2.3 (Contraction [4]**):** Let (X, ρ) be a metric space and $f : X \to X$ a mapping. Then f is called a contraction if $\rho(fx, fy) \leq k\rho(x, y)$, where $0 < k < 1$.

Banach gave the following contraction principle in 1922.

Theorem 2.1 (**Banach Contraction Principle** [4]) *Let (X, ρ) be a complete metric space and $f : X \to X$ a mapping such that*

$$\rho(fx, fy) \leq \alpha\rho(x, y),$$

for all $x, y \in X$ and $0 < \alpha < 1$. Then f has a unique fixed point in X.

Definition 2.4 (Kannan type mapping [12, 13]**):** Let (X, ρ) be a metric space and f a mapping on X. The mapping f is called a Kannan type mapping if there exists $0 \leq \alpha < \frac{1}{2}$ such that

$$\rho(fx, fy) \leq \alpha[\rho(x, fx) + \rho(y, fy)], \text{ for all } x, y \in X.$$

Definition 2.5 (Compatible mappings [11]**):** Let (X, ρ) be a metric space. The mappings $f,\ g : X \longrightarrow X$ are said to be compatible if

$$\lim_{n\to\infty} \rho(fgx_n, gfx_n) = 0, \text{ where } \{x_n\} \in X \text{ and } \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x, \text{ for some } x \in X.$$

Definition 2.6 (Weakly compatible mappings [11]**):** Let (X, ρ) be a metric space and f, g two self-maps on the set X, that is, $f,\ g : X \longrightarrow X$. The self-maps f and g are said to be weakly compatible if they commute at their coincidence point, that is, if $fx = gx$, for some $x \in X$, then $fgx = gfx$.

Definition 2.7 (Altering distance function [14]**):** A function $h : [0, \infty) \to [0, \infty)$ is called an altering distance function if

(i) h is monotonic increasing, continuous and

(ii) $h(t) = 0$, if and only if $t = 0$.

The result noted in the following lemma will be used in deriving the fixed point result. For proof of the lemma, refer to the respective work in which it appears.

Lemma 2.1 [1] *Let f and g be two weakly compatible self-maps on the set X. If f and g have a unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.*

3. The Main Result

Theorem 3.1 *Let a, b, c, d be four continuous functions from $[0, \infty)$ into [0,1) which satisfy the property $a(t) + b(t) + c(t) + d(t) < 1$, for all $t \in [0,\ \infty)$. Also let (X, ρ) be a metric space, ψ an altering distance function and $f : X \to X$, $g : X \to X$ two self-mappings, such that $f(X) \subseteq g(X)$, where $g(X)$ is a complete subspace of X. If for all $x,\ y \in X$, f and g satisfy the following inequality:*

$$\psi(\rho(fx, fy)) \leq a(\rho(gx, gy))\psi(\rho(gx, gy)) + b(\rho(gx, gy))\psi(\rho(gx, fx))$$

$$+c(\rho(gx, gy))\psi(\rho(gy, fy)) + d(\rho(gx, gy))\sqrt{\psi(\rho(gx, fy))}\sqrt{\psi(\rho(gy, fx))}, \tag{1}$$

then f and g have a unique point of coincidence in X. Moreover, if f and g are weakly compatible, f and g have a unique common fixed point in X.

Proof: Let x_1 be a point of X. Since $f(X) \subseteq g(X)$, we define $gx_{n+1} = fx_n = y_n$ and $\tau_n = \rho(y_{n-1}, y_n)$, for all $n = 1, 2, ...$.

We first prove that f and g have a coincidence point. We may take $\tau_n > 0$, for all n, because if $\tau_n = 0$, there is a coincidence point of f and g.

Substituting $x = x_n$ and $y = x_{n+1}$ in (1), we get

$$\psi(\tau_{n+1}) \leq a(\tau_n)\psi(\tau_n) + b(\tau_n)\psi(\tau_n) + c(\tau_n)\psi(\tau_{n+1})$$

$$+d(\tau_n)\sqrt{\psi(\rho(y_{n-1}, y_{n+1}))}\sqrt{\psi(\rho(y_n, y_n))}\ . \tag{2}$$

Hence we get

$$\psi(\tau_{n+1}) \leq \frac{a(\tau_n) + b(\tau_n)}{1 - c(\tau_n)}\psi(\tau_n). \tag{3}$$

Since $a(t) + b(t) + c(t) + d(t) < 1$, implies $\frac{a(t)+b(t)}{1-c(t)} < 1$, for all $t \in [0,\ \infty)$, we get

$$\psi(\tau_{n+1}) < \psi(\tau_n). \tag{4}$$

Since ψ is an increasing function we get from (4), that $\{\tau_n\}$ is a monotone decreasing sequence of non-negative real numbers. Hence there exists a $\tau \geq 0$ such that

$$\lim_{n\to\infty} \tau_n = \tau. \tag{5}$$

Letting $n \to \infty$ in (3), using continuity of ψ, we have

$$\psi(\tau) \leq \frac{a(\tau) + b(\tau)}{1 - c(\tau)}\psi(\tau). \tag{6}$$

Since $\frac{a(\tau)+b(\tau)}{1-c(\tau)} < 1$, (6) is a contradiction unless $\tau = 0$.
Therefore,

$$\lim_{n\to\infty} \tau_n = \lim_{n\to\infty} \rho(y_{n-1}, y_n) = 0. \tag{7}$$

We now prove that $\{y_n\}$ is a Cauchy sequence. Suppose it is not. Then there exists $\varepsilon > 0$ for which we can find sub-sequences $\{y_{m(k)}\}$ and $\{y_{n(k)}\}$ of $\{y_n\}$ with $n(k) > m(k) > k$ such that

$$\rho(y_{m(k)}, y_{n(k)}) \geq \varepsilon. \tag{8}$$

Further, corresponding to $m(k)$, we can choose $n(k)$ in such a way that it is the smallest integer with $n(k) > m(k)$ and satisfies (8). Then

$$\rho(y_{m(k)}, y_{n(k)-1}) < \varepsilon. \tag{9}$$

Let $s_n = \rho(y_{n(k)}, y_{m(k)})$, for all $n \geq 1$. Then we get

$$\varepsilon \leq s_n \leq \rho(y_{n(k)-1}, y_{n(k)}) + \rho(y_{n(k)-1}, y_{m(k)}) < \tau_{n(k)} + \varepsilon. \tag{10}$$

Letting $k \to \infty$ in (10) we get

$$\text{since } \lim_{n\to\infty} \tau_n = 0, \text{ then } \lim_{k\to\infty} s_n = \varepsilon. \tag{11}$$

Note that $k \to \infty$ implies $n \to \infty$.
Also from triangular inequality we get, for all $n \geq 0$,

$$-\tau_{n(k)+1} - \tau_{m(k)+1} + s_n \leq \rho(y_{n(k)+1}, y_{m(k)+1}) \leq \tau_{n(k)+1} + \tau_{m(k)+1} + s_n. \tag{12}$$

Letting $k \to \infty$ in (12) we get

$$\lim_{k\to\infty} \rho(y_{n(k)+1}, y_{m(k)+1}) = \varepsilon. \tag{13}$$

Substituting $x = x_{n(k)+1}$ and $y = x_{m(k)+1}$ in (1), we get

$$\begin{aligned} \psi(\rho(y_{n(k)+1}, y_{m(k)+1})) &\leq a(s_n)\psi(s_n) + b(s_n)\psi(\tau_{n(k)+1}) + c(s_n)\psi(\tau_{m(k)+1}) \\ &\quad + d(s_n)\sqrt{\psi(\rho(y_{n(k)}, y_{m(k)+1}))}\sqrt{\psi(\rho(y_{m(k)}, y_{n(k)+1}))}. \end{aligned} \tag{14}$$

Again applying triangle inequality and monotone property of ψ in (14), we get

$$\begin{aligned} \psi(\rho(y_{n(k)+1}, y_{m(k)+1})) &\leq a(s_n)\psi(s_n) + b(s_n)\psi(\tau_{n(k)+1}) + c(s_n)\psi(\tau_{m(k)+1}) \\ &\quad + d(s_n)\sqrt{\psi(s_{n+1} + \tau_{n(k)+1})}\sqrt{\psi(s_{n+1} + \tau_{m(k)+1})}. \end{aligned} \tag{15}$$

Letting $k \to \infty$ in (15) using continuities of $a,\ b,\ c,\ d$ and ψ, we get

$$\psi(\varepsilon) \leq \{a(\varepsilon) + d(\varepsilon)\}\psi(\varepsilon) < \psi(\varepsilon). \tag{16}$$

Since $a(\varepsilon) + d(\varepsilon) < 1$, (16) is a contradiction. Therefore $\{y_n\}$ is a Cauchy sequence in $g(X)$. Since $g(X)$ is complete, there exists a $z \in g(X)$ such that

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} gx_{n+1} = \lim_{n\to\infty} fx_n = z. \tag{17}$$

Since $z \in g(X)$, we can find $p \in X$ such that $gp = z$.

Now, putting $x = x_{n+1}$ and $y = p$ in (1), we have

$$\begin{aligned} \psi(\rho(fx_{n+1}, fp)) &\leq a(\rho(gx_{n+1}, gp))\psi(\rho(gx_{n+1}, gp)) + b(\rho(gx_{n+1}, gp))\psi(\rho(gx_{n+1}, fx_{n+1})) \\ &\quad + c(\rho(gx_{n+1}, gp))\psi(\rho(gp, fp)) \\ &\quad + d(\rho(gx_{n+1}, gp))\sqrt{\psi(\rho(gx_{n+1}, fp))}\sqrt{\psi(\rho(gp, fx_{n+1}))} \\ &= a(\rho(gx_{n+1}, z))\psi(\rho(gx_{n+1}, z)) + b(\rho(gx_{n+1}, z))\psi(\rho(gx_{n+1}, fx_{n+1})) \\ &\quad + c(\rho(gx_{n+1}, z))\psi(\rho(z, fp)) \\ &\quad + d(\rho(gx_{n+1}, z))\sqrt{\psi(\rho(gx_{n+1}, fp))}\sqrt{\psi(\rho(z, fx_{n+1}))}. \end{aligned}$$

Letting $n \to \infty$ in the above inequality, using (7), (17) and the properties of $a,\ b,\ c,\ d$ and ψ, we have

$$\psi(\rho(z, fp)) \leq c(0)\psi(\rho(z, fp)).$$

Since $c(0) < 1$, it follows that $\psi(\rho(z, fp)) = 0$ which implies that $\rho(z, fp) = 0$, that is, $z = fp$.

Therefore, we get that

$$z = fp = gp.$$

Hence p is a coincidence point and z is a point of coincidence of f and g.

We next establish that the point of coincidence is unique. For this, assume that there exists another point q in X such that $z_1 = fq = gq$ and suppose that $z \neq z_1$. Then, for $x = p$ and $y = q$, we have by (1),

$$\begin{aligned}\psi(\rho(fp,fq)) &\le a(\rho(gp,gq))\psi(\rho(gp,gq))+b(\rho(gp,gq))\psi(\rho(gp,fp))+c(\rho(gp,gq))\psi(\rho(gq,fq)) \\ &\quad + d(\rho(gp,gq))\sqrt{\psi(\rho(gp,fq))}\sqrt{\psi(\rho(gq,fp))} \\ &= a(\rho(z,z_1))\psi(\rho(z,z_1)) + d(\rho(z,z_1))\psi(\rho(z,z_1)). \\ &= [a(\rho(z,z_1)) + d(\rho(z,z_1))]\psi(\rho(z,z_1)).\end{aligned}$$

Since $a(\rho(z,z_1)) + d(\rho(z,z_1)) < 1$, it follows that $\psi(\rho(fp,fq)) < \psi(\rho(z,z_1))$, which is a contradiction.

Therefore, z is the unique point of coincidence of f and g.

Now, if f and g are weakly compatible, by Lemma 2.1, z is the unique common fixed point of f and g. Hence the proof is completed.

4. Some Consequences

Let us assume $\psi(x) = x$ and $g(x) = x$ in Theorem 3.1. Hence by our construction $g(X) = X$ and then (X, ρ) is a complete metric space. Then taking $b = c = d = 0$ and $a(t) = \alpha$, where α is a constant with $0 < \alpha < 1$, in Theorem 3.1, we get the Banach Contraction Principle [4].

Theorem 4.1 *Let (X, ρ) be a complete metric space and $f : X \to X$ a mapping such that*

$$\rho(fx, fy) \le \alpha\rho(x, y),$$

for all $x, y \in X$ and $0 < \alpha < 1$. Then f has a unique fixed point in X.

Considering $g(x) = x$, for all $x \in X$, $b = c = d = 0$ and $a(t) = \alpha$, where α is a constant with $0 < \alpha < 1$, in Theorem 3.1, we get the following theorem, which is proved by Khan et al. (Theorem 2 in [14]).

Theorem 4.2 *Let (X, ρ) be a complete metric space, ψ an altering distance function and also let $f : X \to X$ be a self-mapping which satisfies the following inequality:*

$$\psi(\rho(fx, fy)) \le \alpha\psi(\rho(x, y)),$$

for all $x, y \in X$ and for some $0 < \alpha < 1$. Then f has a unique fixed point.

Considering $g(x) = x$, for all $x \in X$, $d = 0$ and $a(t) = \alpha, b(t) = \beta, c(t) = \gamma$, where α, β, γ are constants with $0 \le \alpha + \beta + \gamma < 1$, in Theorem 3.1, we get the theorem of Skof [23].

Theorem 4.3 *Let (X, ρ) be a complete metric space and ψ an altering distance function. Also let $f : X \to X$ be a self-mapping which satisfies the following inequality:*

$$\psi(\rho(fx, fy)) \leq \alpha\psi(\rho(x, y)) + \beta\psi(\rho(x, fx)) + \gamma\psi(\rho(y, fy)),$$

for all $x, y \in X$, $0 \leq \alpha + \beta + \gamma < 1$. Then f has a unique fixed point.

Considering $g(x) = x$, for all $x \in X$ in Theorem 3.1, we get the following theorem.

Theorem 4.4 *Let a, b, c, d be four continuous functions from $[0, \infty)$ into [0,1) satisfying the property $a(t)+b(t)+c(t)+d(t) < 1$, for all $t \geq 0$. Also let (X, ρ) be a complete metric space, ψ an altering distance function and $f : X \to X$ a self-mapping which satisfies the following two properties:*

$$\psi(\rho(fx, fy)) \leq a(\rho(x, y))\psi(\rho(x, y)) + b(\rho(x, y))\psi(\rho(x, fx)) + c(\rho(x, y))\psi(\rho(y, fy)) \\ +d(\rho(x, y))\sqrt{\psi(\rho(x, fy))}\sqrt{\psi(\rho(y, fx))},$$

where $x, y \in X$. Then f has a unique fixed point.

Considering ψ and g to be identity mappings, $a = d = 0$ and $b(t) = c(t) = \alpha$, where α is a constant with $0 \leq \alpha < \frac{1}{2}$, in Theorem 3.1, we get the theorem of Kannan [12, 13] as follows:

Theorem 4.5 *Let (X, ρ) be a complete metric space and f be a self-mapping on X. If*

$$\rho(fx, fy) \leq \alpha[\rho(x, fx) + \rho(y, fy)],$$

for all $x, y \in X$ and $0 \leq \alpha < \frac{1}{2}$, then f has a unique fixed point.

In theorem 3.1, if we take $d = 0$, $b = c$ and $g(x) = x$, for all $x \in X$, we get the following theorem:

Theorem 4.6 *Let a, b be two continuous functions from $[0, \infty)$ into [0,1) which satisfy the property $a(t) + 2b(t) < 1$, for all $t \geq 0$. Also let (X, ρ) be a complete metric space, ψ an altering distance function and $f : X \to X$ a self-mapping which satisfies the following* *property:*

$$\psi(\rho(fx,fy)) \leq a(\rho(x,y))\psi(\rho(x,y)) + b(\rho(x,y))\{\psi(\rho(x,fx)) + \psi(\rho(y,fy))\},$$

where $x, y \in X$. Then f has a unique fixed point.

In theorem 3.1, if we take $b = c$ and $g(x) = x$, for all $x \in X$, we get the following theorem:

Theorem 4.7 *Let a, b, c be three continuous functions from $[0,\infty)$ into [0,1) which satisfy the property $a(t) + 2b(t) + c(t) < 1$, for all $t \geq 0$. Also let (X,ρ) be a complete metric space, ψ an altering distance function and $f : X \to X$ a self-mapping which satisfies the following property:*

$$\psi(\rho(fx,fy)) \leq a(\rho(x,y))\psi(\rho(x,y)) + b(\rho(x,y))\{\psi(\rho(x,fx)) + \psi(\rho(y,fy))\}$$
$$+c(\rho(x,y))\sqrt{\psi(\rho(x,fy))}\sqrt{\psi(\rho(y,fx))},$$

where $x, y \in X$. Then f has a unique fixed point.

5. Conclusions

In Theorem 3.1, the functions f and g are not necessarily continuous. So result of our theorem is more general compared to some other results in this line of research. It is also a feature of our theorem that in the inequality, there is surd root.

We have given below two examples, one is a continuous case and the other a discontinuous case, which support our theorem.

Example 5.1 Let $X = [0,1]$ and $\rho = |x - y|$, for all $x, y \in X$, that is, ρ is the usual metric defined on X. Then (X,ρ) is a complete metric space.

Let $\psi : [0,\infty) \longrightarrow [0,\infty)$ be defined as follows:

$\psi(t) = t^2$, for all $t \in [0,\infty)$.

Then ψ has the properties mentioned in Theorem 3.1.

Let $f,\ g : X \longrightarrow X$ be defined respectively as follows:

$fx = \frac{x}{32}$ and $gx = \frac{x}{4}$, for all $x \in X$.

Then f and g have the required properties mentioned in Theorem 3.1.

Let $a,\ b,\ c,\ d : [0,\infty) \longrightarrow [0,1)$ be defined respectively as follows:

$$a(t) = b(t) = \tfrac{1}{4} \text{ and } c(t) = d(t) = \tfrac{1}{8}, \text{ for all } t \in [0,\infty).$$

Then $a,\ b,\ c$ and d have the required properties mentioned in Theorem 3.1.

It can be verified that (3.1) is satisfied for all $x,\ y \in X$. The conditions of Theorem 3.1 are satisfied and it is seen that 0 is the unique point of coincidence and also the unique common fixed point of f and g.

Example 5.2 Let $X = [0,1]$ and $\rho = |x-y|$, for all $x, y \in X$, that is, ρ is the usual metric defined on X. Then (X,ρ) is a complete metric space.

Let $\psi : [0,\infty) \longrightarrow [0,\infty)$ be defined as follows:

$$\psi(t) = t^2, \text{ for all } t \in [0,\infty).$$

Then ψ has the properties mentioned in Theorem 3.1.

Let $f,\ g : X \longrightarrow X$ be defined respectively as follows:

$$fx = \begin{cases} 0, & \text{if } \ 0 \le x \le \frac{1}{2}, \\ \\ \frac{1}{32}, & \text{if } \ \frac{1}{2} < x \le 1. \end{cases}$$

and

$$gx = \tfrac{x}{2}, \text{ for all } x \in X.$$

Then f and g have the required properties mentioned in Theorem 3.1.

Let $a,\ b,\ c,\ d : [0,\ \infty) \longrightarrow [0,\ \infty)$ be defined respectively as follows:

$$a(t) = d(t) = \tfrac{1}{8} \text{ and } b(t) = c(t) = \tfrac{1}{4}, \text{ for all } t \in [0,\ \infty).$$

Then $a,\ b,\ c$ and d have the required properties mentioned in Theorem 3.1.

It can be verified that (3.1) is satisfied for all $x,\ y \in X$. The conditions of Theorem 3.1 are satisfied and it is seen that 0 is the unique point of coincidence and also the unique common fixed point of f and g.

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