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THE OPTIMIZING WHITENING DIFFERENTIAL EQUATION OF GM(1,1) BASED NON-HOMOGENOUS EXPONENTIAL SERIES

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Abstract: Based on the criteria that is proposed by Mr. Deng Ju-long, this paper starting from non-homogenous exponential series $x^{(0)}(k)$ and the corresponding $1-AGO$ derivative expression $\frac{dx^{(1)}}{dt}|_{t=k}$, to explore the relationship between $x^{(0)}(k)$ and $\frac{dx^{(1)}}{dt}|_{t=k}$, getting the optimizing whitening differential equation of GM(1,1) based non-homogenous exponential series which is equivalent to the original gray differential equation, and through the example verification, the model of this paper has a good effect, which has certain practical value.

Key words: non-homogenous exponential series; GM(1,1); whitening differential equation; optimization.

2010 AMS Subject Classification: 62B10.

1. Introduction

Because with the characteristics of lack data and poor information, Gray system theory is widely used in all aspects of national production[1]. Among them, the GM(1,1) has the advantages of simple operation, high accuracy of prediction, become the focus of many scholars to study [3-12].

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But the founder of the grey system theory Mr. Deng Ju-long pointed out that in the reference [2]: GM(1,1) whitening differential equation itself, and all derived from the whitening differential equation, which is invalid when it has contradiction with the definition.

Based on this idea, this paper starting from non-homogenous exponential series $x^{(0)}(k)$ and the corresponding $1-AGO$ derivative expression $\frac{dx^{(1)}}{dt}\big|_{t=k}$, to explore the relationship between $x^{(0)}(k)$ and $\frac{dx^{(1)}}{dt}\big|_{t=k}$, getting the optimizing whitening differential equation of GM(1,1) based non-homogenous exponential series which is equivalent to the original gray differential equation.

2. Main Results

2.1. To construct the new whitening differential equation

Let the once accumulated of non-homogenous exponential series $x^{(0)}(k)$ is $x^{(1)}(k)$, and $x^{(1)}(k) = Be^{Ak} + Ck + D$ (A、B、C、D is parameters; specially, when C=0, $x^{(0)}(k)$ is a homogenous exponential series), then $x^{(0)}(k) = B(e^A - 1)e^{A(k-1)} + C$, among them, $A = \ln\frac{\alpha^{(1)}x^{(0)}(k)}{\alpha^{(1)}x^{(0)}(k-1)}$,

$B = \frac{\alpha^{(1)}x^{(0)}(k)}{(e^A - 1)^2 e^{A(t-2)}}$, $C = x^{(0)}(k) - \frac{e^A \bullet \alpha^{(1)}x^{(0)}(k)}{e^A - 1}$,

$$D = x^{(1)}(k) - k \bullet x^{(0)}(k) - \frac{k \bullet e^A \bullet \alpha^{(1)}x^{(0)}(k)}{e^A - 1} - \frac{e^{2A} \bullet \alpha^{(1)}x^{(0)}(k)}{(e^A - 1)^2}$$

($\alpha^{(1)}x^{(0)}(k) = x^{(0)}(k) - x^{(0)}(k-1) = B(e^A - 1)^2 e^{A(k-2)}$)

Theorem 1 If $x^{(1)}(k) = Be^{Ak} + Ck + D$ (A、B、C、D is parameters; specially, when C=0, $x^{(0)}(k)$ is a homogenous exponential series), the whitening differential equation which match with original gray differential equation $x^{(0)}(k) + ax^{(1)}(k) = b$ is $\frac{e^A - 1}{Ae^A} \bullet \frac{dx^{(1)}}{dt} + ax^{(1)}(k) = b + C(\frac{e^A - 1}{Ae^A} - 1)$

Proof: Let $x^{(1)}(k)=Be^{Ak}+Ck+D$ (A、B、C、D is parameters), $x^{(0)}(k)=Be^{A(k-1)}(e^{A}-1)+C$,

then $\frac{dx^{(1)}(t)}{dt}=ABe^{At}+C$

Because the whitening differential equation match with the original gray differential equation, that is $\frac{dx^{(1)}(t)}{dt}=x^{(0)}(k)$,

Operate it:

$$x^{(0)}(k)=\frac{e^{A}-1}{Ae^{A}}\bullet\frac{dx^{(1)}}{dt}-C(\frac{e^{A}-1}{Ae^{A}}-1)\ \ldots\ldots.(1),$$

Let (1) into the original gray differential equation $x^{(0)}(k)+ax^{(1)}(k)=b$:

$$\frac{e^{A}-1}{Ae^{A}}\bullet\frac{dx^{(1)}}{dt}-C(\frac{e^{A}-1}{Ae^{A}}-1)+ax^{(1)}(k)=b$$

That is: $\frac{e^{A}-1}{Ae^{A}}\bullet\frac{dx^{(1)}}{dt}+ax^{(1)}(k)=b+C(\frac{e^{A}-1}{Ae^{A}}-1)$

Theorem 2 Let $x^{(0)}=\left\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right\}$ is a non-negative quasi smooth series, $x^{(1)}$ is a once accumulation series of $x^{(0)}$, and $x^{(1)}(t)=Ce^{At}+D$ (among them, A、C、D are parameters).

$\hat{a}=(a,b)^{T}$, $Y=\begin{pmatrix}x^{(0)}(2)\\x^{(0)}(3)\\\vdots\\x^{(0)}(n)\end{pmatrix}$, $B=\begin{pmatrix}-x^{(1)}(2)&1\\-x^{(1)}(3)&1\\\vdots&\vdots\\-x^{(1)}(n)&1\end{pmatrix}$, then The least square parameter estimation series of the gray differential equation $x^{(0)}(k)+ax^{(1)}(k)=b$ meet the conditions: $\hat{a}=(a,b)^{T}=\left(B^{T}B\right)^{-1}B^{T}Y$.

(1) The continuous solutions of the new optimizing whitening differential equation are

$$x^{(1)}(t)=[x^{(0)}(1)-\frac{b+C(\frac{e^{A}-1}{Ae^{A}}-1)}{a}]e^{\frac{-a\cdot(t-1)\cdot A\cdot e^{A}}{e^{A}-1}}+\frac{b+C(\frac{e^{A}-1}{Ae^{A}}-1)}{a};$$

(2) The discrete solution of the new optimizing whitening differential equation are $x^{(1)}(k)=[x^{(0)}(1)-\frac{b+C(\frac{e^A-1}{Ae^A}-1)}{a}]e^{\frac{-a\cdot(k-1)\cdot A\cdot e^A}{e^A-1}}+\frac{b+C(\frac{e^A-1}{Ae^A}-1)}{a}$, $k=2$, 3, $\cdots$ among them,

$C=x^{(0)}(k)-\frac{e^A\bullet\alpha^{(1)}x^{(0)}(k)}{e^A-1}$;

(3) The fitting data: $\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)$, $k=2$, 3, $\cdots$;

2.2. The example analysis

$x^{(0)}(k)$=[2.718, 7.389, 20.086, 54.598, 148.41, 403.43, 1096.6] is a non-homogeneous high growth index series. Let original GM(1,1) model is Model 1 ,the optimization model in reference [13] is Model 2, the modeling method in this Model 3, to simulate the first six data of $x^{(0)}(k)$, and predict the last data.

Table 1 Simulation and prediction accuracy comparison

	Model 1		Model 2		Model 3	
Real value	Simulated value	Relative error	Simulated value	Relative error	Simulated value	Relative error
20.086	16.4677	18.0142	20.8128	3.6158	20.0845	0.0074
54.598	41.4978	23.9939	56.572	3.6156	54.6023	0.0078
148.41	104.573	29.5380	153.7704	3.6119	148.3782	0.0214
403.43	263.519	34.6805	417.9689	3.6038	402.9162	0.1274
Average Relative errors	26.5567		3.6124		0.0410	
Real value	Predicted values	Relative error	Predicted values	Relative error	Predicted values	Relative error
1096.6	1673.4	52.6	1136.096	3.6017	1095.734	0.0790

From the Table 1, for the non-homogeneous high growth index series $x^{(0)}(k)$, the average relative errors of Model 1 is 26.5567%, even the Predicted relative errors is 52.6%, has lost the modeling value. The simulated and prediction precision of Model 2 has a greatly improved, which are 3.6124%、3.6017%, but it still can be optimization. The simulated error of Model3 only 0.041%, the predicted error just 0.079%. So, the Model 3 from this paper is the best.

3. Conclusion

Based on the criteria: GM(1,1) whitening differential equation itself, and all derived from the whitening differential equation, which is invalid when it has contradiction with the definition, this paper starting from non-homogenous exponential series $x^{(0)}(k)$ and the corresponding $1-AGO$ derivative expression $\frac{dx^{(1)}}{dt}|_{t=k}$, to explore the relationship between $x^{(0)}(k)$ and $\frac{dx^{(1)}}{dt}|_{t=k}$, getting the optimizing whitening differential equation of GM(1,1) based non-homogenous exponential series which is equivalent to the original gray differential equation, and through the example verification, the model of this paper has a good effect, which has certain practical value.

Conflict of Interests

The author declares that there is no conflict of interests.

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