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COMMON FIXED-POINT THEOREM FOR GENERALIZED CONTRACTIVE TYPE MAPPINGS IN COMPLEX VALUED METRIC SPACES

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Abstract. Recently, Azam et al. introduced new spaces called the complex valued metric spaces and established the existence of fixed point theorems under the contraction condition. In this paper, we extend and improve the condition of contraction of results of Azam et al. for two single-valued mappings in such spaces.

Key words: complex valued metric; families of self mappings; common fixed point.

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1. Introduction

Fixed point theory is the one of the most interesting area of research in the last fifty years for instance research about optimization problem, control theory, differential equations, economics, and etc. The fixed point theorem, generally known as the Banach's contraction mapping principle, appeared in explicit form in Banach's thesis in 1922.

A mapping T: X → X, where (X, d) is a metric space, is said to be a contraction mapping if for all x, y ∈ X,

$$d(Tx,Ty) \le \lambda d(x,y) \text{where } 0 \le \lambda < 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Mapping T satisfying (1) in a complete metric space will have a unique fixed point.

Recently Azam et al. introduced a new space, the so called complex-valued metric space, and established a fixed point theorem for some type of contraction mappings as follows. In this

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paper, we extend and improve the condition of contraction of results of Azam et al. for two single-valued mappings in such spaces.

Theorem 1: [7] Let (X, d) be a complex valued metric space, and let A, B, C, D, and E be the nonnegative real numbers such that A+B+C+2D+2E < 1. Suppose that S, T: X → X are mappings satisfying:

$$d(Sx,Ty) \preccurlyeq Ad(x,y) + \frac{Bd(x,Sy)d(y,Ty)}{1+d(x,y)} + \frac{Cd(y,Sx)d(x,Ty)}{1+d(x,y)} + \frac{Dd(x,Sx)d(x,Ty)}{1+d(x,y)} + \frac{Ed(y,Sx)d(y,Ty)}{1+d(x,y)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

for all x, y ∈ X, then S and T have unique common fixed point in X.

The aim of this paper is to establish some common fixed-point theorems two nonlinear contraction mappings in complex-valued metric spaces. Our results generalized Theorem 1.

2. Preliminaries

Let ℂ the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. We define a partial order ≼ on ℂ as follows:

$z_1 \preccurlyeq z_2$ if and only if Re $(z_1) \le$ Re (z_2) and Im $(z_1) \le$ Im (z_2)

that is $z_1 \preccurlyeq z_2$ if one of the following holds

C1: Re (z_1) = Re (z_2) and Im (z_1) = Im (z_2)

C2: Re (z_1) < Re (z_2) and Im (z_1) = Im (z_2)

C3: Re (z_1) = Re (z_2) and Im (z_1) <Im (z_2)

C4: Re (z_1) < Re (z_2) and Im (z_1) <Im (z_2)

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (C2), (C3), and (C4) is satisfied and we will write $z_1 \prec z_2$ if only (C4) is satisfied.

Definition 2: Let X be a non empty set .A mapping $d: X \times X \to C$ is called a complex valued matrix on X if the following conditions are satisfied:

(CM1) 0 ≼ d(x, y) for all x, y ∈ X and d(x, y) = 0 if and only if x = y;

(CM2) d(x, y) = d(y, x) for all x, y∈ X;

(CM3) d(x, y) ≼ d(x, z) + d (z, y), for all x, y, z ∈ X.

Then d is called a complex valued metric space.

Definition 3: Let (X, d) be a complex valued metric space.

(i) A point x ∈ X is called interior point of set A ⊆ X whenever there exist 0 ≺ r ∈ ℂ such that B(x, r) ≔{γ∈ X | d(x, y) ≺r} ⊆ A

(ii) A point x ∈ X is called a limit of A whenever for every 0 ≺r ∈ ℂ, B(x, r) ∩ (A – X) ≠ Ø.

(iii) A subset A⊆ X is called open whenever each element A is an interior point of A.

(iv) A sub set A⊆ X is called closed whenever each limit point of A belongs to A.

(v) A sub-basis for a Hausdorff topology τ on X is a family F = {B(x, r) │x ∈ X and 0 ≺ r}.

Definition 4: Let (x, d) be a complex valued metric space, {x_n} be a sequence in X and x∈ X.

(i) If for every c∈ C, with 0 ≺ c there is N∈ ℕsuch that for all n> N, d (x_n, x) ≺ c, then {x_n} is said to be convergent, {x_n} converges to x and x is the limit point of {x_n}, we denote this by $\lim_{n\to\infty} x_n = x$ (or) {x_n} →x as n→ ∞.

(ii) If for every c∈ C, with 0 ≺ c there is N∈ ℕsuch that for all n> N, d (x_n, x_{n+m}) ≺ c, where m ∈ ℕ, then {x_n} is said to be Cauchy sequence.

(iii) If for every Cauchy sequence in X is convergent, then (x, d) is said to be a complete complex valued metric space.

Lemma 5: [2] Let (x, d) be a complex valued metric space and let {x_n} be a sequence in X. Then {x_n} converges to x if and only if │d (x_n, x) │→ 0. As $n \to \infty$.

Lemma 6: [2] Let (x, d) be a complex valued metric space and, let {x_n} be a sequence in X. Then {x_n} is a Cauchy sequence if and only if │d (x_n, x_{n+m}) │→ 0. as n→ ∞, where$m$ ∈ ℕ.

Definition 7: Two families of self-mappings$\{T_i\}_{i=1}^{m}$ and $\{S_i\}_{i=1}^{n}$are said to be pairwise commuting if:

(i) $T_iT_j = T_jT_i, i, j \in \{1,2, \dots m\}$.

(ii) $S_iS_j = S_JS_{i_j}, i, j \in \{1,2, \dots n\}$.

(iii) $T_iS_j = S_jT_i$, $i \in \{1,2, \dots m\}, j \in \{1,2, \dots n\}$.

Definition 8: (i) A point $x \in X$ is said to be a fixed point of T if $Tx = x$.

ii) A point $x \in X$ is said to be a common fixed point of T and S if $Tx = Sx = x$.

Remark 9: We obtain the following statements hold.

(i) If $z_1 \precsim z_2$and $z_2 \precsim z_3$ then $z_1 \precsim z_3$.

(ii) If $z \in \mathbb{C}$, a, b∈ ℝ, and a ≤ b, then az≾ bz.

(iii) If $0 \precsim z_1 \precsim z_2$, then │$z_1$│≾│$z_2$│.

3. Main Results

In this section, we will prove some common fixed-point theorems for the generalized contractive mappings in complex-valued metric space.

Theorem 10: If Sand T are self-mapping defined on a complex valued metric space (X, d) satisfying the following condition

$$d(Sx, Ty) \precsim Ad(x, y) + \frac{Bd(x, Sx)d(y, Ty)}{1 + d(x, y)} + \frac{Cd(y, Sx)d(x, Ty)}{1 + d(x, y)} + \frac{Dd(x, Sx)d(x, Ty)}{1 + d(x, y)}$$

$$+\frac{Ed(y,Sx)d(y,Ty)}{1+d(x,y)}+\frac{Fd(y,Sy)d(x,Tx)}{1+d(x,y)}+\frac{Gd(y,Sy)d(y,Tx)}{1+d(x,y)}\ldots.\quad(3)$$

for all x, y ∈ X, where L, M, N, O, and E are nonnegative with A+B+C+2D+2E+2F+2G< 1, then S and T have unique common fixed point.

Proof: Let x_n be an arbitrary in X. Since S(X)⊆ X and T(X) ⊆ X, we construct the sequence $\{x_n\}$ in X such that $x_{2k+1} = Sx_{2k}$ and $x_{2k+2} = Tx_{2k+1}$, for all k ≥0. From the definition of $\{x_k\}$ and [3], we obtain that

$$d(x_{2k+1},x_{2k+2})=d(Sx_{2k},Tx_{2k+1})\preccurlyeq Ad(x_{2k},x_{2k+1})$$
$$+\frac{Bd(x_{2k},Sx_{2k})d(x_{2k+1},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}+\frac{Cd(x_{2k+1},Sx_{2k})d(x_{2k},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}$$
$$+\frac{Dd(x_{2k},Sx_{2k})d(x_{2k},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}+\frac{\mathrm{Ed}(\mathrm{x}_{2k+1},\mathrm{Sx}_{2k})\mathrm{d}(\mathrm{x}_{2k+1},\mathrm{Tx}_{2k+1})}{1+\mathrm{d}(\mathrm{x}_{2k},\mathrm{x}_{2k+1})}$$
$$+\frac{\mathrm{Fd}(\mathrm{x}_{2k+1},\mathrm{Sx}_{2k+1})\mathrm{d}(\mathrm{x}_{2k},\mathrm{Tx}_{2k})}{1+\mathrm{d}(\mathrm{x}_{2k},\mathrm{x}_{2k+1})}+\frac{Gd(x_{2k+1},Sx_{2k+1})d(\mathrm{x}_{2k+1},\mathrm{Tx}_{2k})}{1+d(\mathrm{x}_{2k},\mathrm{x}_{2k+1})}\ldots\ldots\ldots.(4)$$

$$d(x_{2k+1},x_{2k+2})\preccurlyeq Ad(x_{2k},x_{2k+1})$$
$$+\frac{Bd(x_{2k},x_{2k+1})d(x_{2k+1},x_{2k+2})}{1+d(x_{2k},x_{2k+1})}+\frac{Cd(x_{2k+1},x_{2k+1})d(x_{2k},x_{2k+2})}{1+d(x_{2k},x_{2k+1})}$$
$$+\frac{Dd(x_{2k},x_{2k+1})d(x_{2k},x_{2k+2})}{1+d(x_{2k},x_{2k+1})}+\frac{Ed(x_{2k+1},x_{2k+1})d(x_{2k+1},x_{2k+2})}{1+d(x_{2k},x_{2k+1})}$$
$$+\frac{Fd(x_{2k+1},x_{2k+2})d(x_{2k},x_{2k+1})}{1+d(x_{2k},x_{2k+1})}+\frac{Gd(x_{2k+1},x_{2k+2})d(\mathrm{x}_{2k+1},\mathrm{x}_{2k+1})}{1+d(\mathrm{x}_{2k},\mathrm{x}_{2k+1})}$$

$$|d(x_{2k+1},x_{2k+2})|\leq A|d(x_{2k},x_{2k+1})|+\frac{B|d(x_{2k},x_{2k+1})||d(x_{2k+1},x_{2k+2})|}{|1+d(x_{2k},x_{2k+1})|}$$
$$+\frac{D|d(x_{2k},x_{2k+1})||d(x_{2k},x_{2k+2})|}{|1+d(x_{2k},x_{2k+1})|}+\frac{F|d(x_{2k+1},x_{2k+2})||d(x_{2k},x_{2k+1})|}{|1+d(x_{2k},x_{2k+1})|}\ldots\ldots\ldots\ldots(5)$$

$$=A|d(x_{2k},x_{2k+1})|+B|d(x_{2k+1},x_{2k+2})|\left[\frac{|d(x_{2k},x_{2k+1})|}{|1+d(x_{2k},x_{2k+1})|}\right]$$
$$+D|d(x_{2k},x_{2k+2})|\left[\frac{|d(x_{2k},x_{2k+1})|}{|1+d(x_{2k},x_{2k+1})|}\right]+F|d(x_{2k+1},x_{2k+2})|\left[\frac{|d(x_{2k},x_{2k+1})|}{|1+d(x_{2k},x_{2k+1})|}\right]\ldots(6)$$

$$|d(x_{2k+1},x_{2k+2})|\preccurlyeq A|d(x_{2k},x_{2k+1})|+B|d(x_{2k+1},x_{2k+2})|+D|d(x_{2k},x_{2k+1})|$$
$$+D|d(x_{2k+1},x_{2k+2})|+F|d(x_{2k+1},x_{2k+2})|\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(7)$$
$$|d(x_{2k+1},x_{2k+2})[1-(B+D+F)]\preccurlyeq[A+D]d(x_{2k},x_{2k+1})|$$

it follows that

$$|d(x_{2k+1},x_{2k+2})|\preccurlyeq\frac{[A+D]}{[1-(B+D+F)]}|d(x_{2k},x_{2k+1})|$$

Similarly, we get

$$d(x_{2k+2},x_{2k+3}) = d(x_{2k+3},x_{2k+2}) = d(Sx_{2k+2},Tx_{2k+1}) \preccurlyeq Ad(x_{2k+2},x_{2k+1})$$
$$+\frac{Bd(x_{2k+2},Sx_{2k+2})d(x_{2k+1},Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})}+\frac{Cd(x_{2k+1},Sx_{2k+2})d(x_{2k+2},Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})}$$
$$+\frac{Dd(x_{2k+2},Sx_{2k+2})d(x_{2k+2},Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})}+\frac{Ed(x_{2k+1},Sx_{2k+2})d(x_{2k+1},Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})}$$
$$+\frac{\mathrm{Fd}(\mathrm{x}_{2k+1},\mathrm{Sx}_{2k+1})\mathrm{d}(\mathrm{x}_{2k+2},\mathrm{Tx}_{2k+2})}{1+\mathrm{d}(\mathrm{x}_{2k+2},\mathrm{x}_{2k+1})}+\frac{Gd(\mathrm{x}_{2k+1},S\mathrm{x}_{2k+1})d(\mathrm{x}_{2k+1},T\mathrm{x}_{2k+2})}{1+d(\mathrm{x}_{2k+2},\mathrm{x}_{2k+1})} \ldots\ldots (8)$$

$$d(x_{2k+2},x_{2k+3}) \preccurlyeq Ad(x_{2k+2},x_{2k+1})$$
$$+\frac{Bd(x_{2k+2},x_{2k+3})d(x_{2k+1},x_{2k+2})}{1+d(x_{2k+2},x_{2k+1})}+\frac{Ed(x_{2k+1},x_{2k+3})d(x_{2k+1},x_{2k+2})}{1+d(x_{2k+2},x_{2k+1})}$$
$$+\frac{\mathrm{Fd}(\mathrm{x}_{2k+1},\mathrm{x}_{2k+2})\mathrm{d}(\mathrm{x}_{2k+2},\mathrm{x}_{2k+3})}{1+\mathrm{d}(\mathrm{x}_{2k+2},\mathrm{x}_{2k+1})}+\frac{Gd(\mathrm{x}_{2k+1},\mathrm{x}_{2k+2})d(\mathrm{x}_{2k+1},\mathrm{x}_{2k+3})}{1+d(\mathrm{x}_{2k+2},\mathrm{x}_{2k+1})}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$|d(x_{2k+2},x_{2k+3})| \preccurlyeq A|d(x_{2k+2},x_{2k+1})|$$
$$+B|d(x_{2k+2},x_{2k+3})|\left[\frac{|d(x_{2k+2},x_{2k+1})|}{|1+d(x_{2k+2},x_{2k+1})|}\right]+E|d(x_{2k+1},x_{2k+3})|\left[\frac{|d(x_{2k+2},x_{2k+1})|}{|1+d(x_{2k+2},x_{2k+1})|}\right]$$
$$+\mathrm{F}|\mathrm{d}(\mathrm{x}_{2k+2},\mathrm{x}_{2k+3})|\left[\frac{|d(x_{2k+2},x_{2k+1})}{|1+d(x_{2k+2},x_{2k+1})|}\right]+\ Gd(\mathrm{x}_{2k+1},\mathrm{x}_{2k+3})\left[\frac{|d(x_{2k+2},x_{2k+1})}{|1+d(x_{2k+2},x_{2k+1})|}\right]$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$(10)|d(x_{2k+2},x_{2k+3})| \preccurlyeq A|d(x_{2k+2},x_{2k+1})| + B|d(x_{2k+2},x_{2k+3})| + E|d(x_{2k+1},x_{2k+2})|$$
$$+E|d(x_{2k+2},x_{2k+3})| + \mathrm{F}|\mathrm{d}(\mathrm{x}_{2k+2},\mathrm{x}_{2k+3})| + G|d(x_{2k+1},x_{2k+2})|$$
$$+G|d(x_{2k+2},x_{2k+3})| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

it follows that $|d(x_{2k+2},x_{2k+3})|\big(1-(B+E+F+G)\big) \preccurlyeq (A+E+G)|d(x_{2k+2},x_{2k+1})|$

$$|d(x_{2k+2},x_{2k+3})| \preccurlyeq \frac{(A+E+G)}{(1-(B+E+F+G))}|d(x_{2k+2},x_{2k+1})| \ldots\ldots\ldots\ldots\ldots (12)$$

Putting k = max $\{[(A+D)/(1-(B+D+F))], [(A+E+G)/(1-(B+E+F+G)]\}$ we obtain that

$$\left|d\left(x_{n,}x_{n+1}\right)\right| \le k\left|d\left(x_{n,}x_{n+1}\right)\right| \le k^2\left|d\left(x_{n,}x_{n+1}\right)\right| \ldots \le k^n\left|d\left(x_{n,}x_{n+1}\right)\right| \ \forall n \ldots\ldots\ldots (13)$$

Thus, for any $n \in \mathbb{N}$, we have

$$d(x_{n,}x_m)| \le |d(x_{n,}x_{n+1})| \le d(x_{n+1,}x_{n+2})| \ldots \le |d(x_{m-1,}x_m)|$$
$$\le (k^n + k^{n+1} + k^{n+2} + \cdots \ k^{m-1})|d(x_{0,}x_1)|$$
$$\le (\frac{k^n}{1-k})|d(x_{0,}x_1)|$$

it follows that $\left|d\left(x_{n,}x_m\right)\right| \le \left(\frac{k^n}{1-k}\right)\left|d\left(x_{0,}x_1\right)\right| \to 0 \ as\ n \to \infty$.

By lemma 6, the sequence $\{x_n\}$ is Cauchy. Since X is complete, there exists a point $z \in X$ such that $x_n \to z$ as $n \to \infty$.

Next we will show that $Sz = z$. By the notion complete complex valued metric d, we have

$$d(z,Sz) \precsim d(z,x_{2k+2}) + d(x_{2k+2,}Sz) = d(z,x_{2k+2}) + d(Sz,Tx_{2k+1})$$
$$\precsim d(z,x_{2k+2}) + Ad(Sz,x_{2k+1}) + \frac{Bd(z,Sz)d(x_{2k+1},Tx_{2k+1})}{1+d(z,x_{2k+1})}$$
$$+\frac{Cd(x_{2k+1},Sz)d(z,Tx_{2k+1})}{1+d(z,x_{2k+1})} + \frac{Dd(z,Sz)d(z,Tx_{2k+1})}{1+d(z,x_{2k+1})}$$
$$+\frac{Ed(x_{2k+1},Sz)d(x_{2k+1},Tx_{2k+1})}{1+d(z,x_{2k+1})} + \frac{Fd(x_{2k+1},Sx_{2k+1})d(z,Tz)}{1+d(z,x_{2k+1})}$$
$$+\frac{Gd(x_{2k+1},Sx_{2k+1})d(x_{2k+1},Tz)}{1+d(z,x_{2k+1})} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (14)$$

Which implies that

$$d(z,Sz)| \precsim |d(z,x_{2k+2})| + |d(x_{2k+2,}Sz)| = |d(z,x_{2k+2})| + |d(Sz,Tx_{2k+1})|$$
$$\precsim |d(z,x_{2k+2})| + A|d(Sz,x_{2k+1})| + \frac{B|d(z,Sz)||d(x_{2k+1},x_{2k+2})|}{|1+d(z,x_{2k+1})|}$$
$$+\frac{C|d(x_{2k+1},Sz)||d(z,x_{2k+2})|}{1+d(z,x_{2k+1})} + \frac{D|d(z,Sz)||d(z,x_{2k+2})|}{|1+d(z,x_{2k+1})|}$$
$$+\frac{E|d(x_{2k+1},Sz)||d(x_{2k+1},x_{2k+2})|}{|1+d(z,x_{2k+1})|} + \frac{F|d(x_{2k+1},x_{2k+2})||d(z,Tz)|}{|1+d(x_{2k},x_{2k+1})|}$$
$$+\frac{Gd(x_{2k+1},Sx_{2k+1})d(x_{2k+1},Tz)}{1+d(z,x_{2k+1})} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (15)$$

Taking $k \to \infty$, we have$|d(z,Sz)| = 0$; it is obtained that $d(z,Sz) = 0$. Thus, $Sz = z$.it follows that similarly $Tz = z$. Therefore, z is common fixed point of S and T.

Finally, to prove the uniqueness of common fixed point, let $z^* \in X$ be another common fixed point of Sand T such that $Sz^* = Tz^* = z^*$.

Consider

$$d(z,z^*) = d(Sz,Tz^*) \precsim Ad(z,z^*) + \frac{Bd(z,Sz)d(z^*,Tz^*)}{1+d(z,z^*)} + \frac{Cd(z^*,Sz)d(z,Tz^*)}{1+d(z,z^*)}$$
$$+\frac{Dd(z,Sz)d(z,Tz^*)}{1+d(z,z^*)} + \frac{Ed(z^*,Sz)d(z^*,Tz^*)}{1+d(z,z^*)} + \frac{Fd(z^*,Sz^*)d(z,Tz)}{1+d(z,z^*)}$$
$$+\frac{Gd(z^*,Sz^*)d(z^*,Tz)}{1+d(z,z^*)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (16)$$

So that

$$d(z,z^*)| \precsim A|d(z,z^*)| + \frac{C|d(z^*,Sz)||d(z,Tz^*)|}{|1+d(z,z^*)|}$$

$$|d(z,z^*)| = A|d(z,z^*)| + C|d(z^*,Sz)|\frac{|d(z,z^*)|}{|1+d(z,z^*)|} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (17)$$

Since$|1 + d(z,z^*)| > d(z,z^*)|$,

therefore

$|d(z,z^*)| <$ A $|d(z,z^*)|$+$C|d(z^*,z)| = (A + C)|d(z,z^*)|$.

This is contraction to $A + C < 1$. Hence $z = z^*$. There z is a unique common fixed point of S and T.

Corollary 11: If T is a self-mapping defined on a complete complex-valued metric space (X, d) satisfying the condition

$$d(Tx,Ty) \preccurlyeq Ad(x,y) + \frac{Bd(x,Tx)d(y,Ty)}{1+d(x,y)} + \frac{Cd(y,Tx)d(x,Ty)}{1+d(x,y)} + \frac{Dd(x,Tx)d(x,Ty)}{1+d(x,y)}$$
$$+ \frac{Ed(y,Tx)d(y,Ty)}{1+d(x,y)} + \frac{Fd(y,Ty)d(x,Tx)}{1+d(x,y)} + \frac{Gd(y,Ty)d(y,Tx)}{1+d(x,y)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

for all x, y∈X,where A, B, C, D, E, and F are non negative with A+B+C+2D+2E+2F+2G<1, then T has a unique fixed point.

Proof: We can prove this result by applying Theorem 10 by putting T = S.

Corollary 12: If S and T are self-mappings defined on a complete complex valued metric space (X, d) satisfying the condition

$$d(Sx,Ty) \preccurlyeq Ad(x,y) + \frac{Bd(x,Sx)d(y,Ty)}{1+d(x,y)} + \frac{Cd(y,Sx)d(x,Ty)}{1+d(x,y)} + \frac{Dd(x,Sx)d(x,Ty)}{1+d(x,y)}$$
$$+ \frac{Ed(y,Sx)d(y,Ty)}{1+d(x,y)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots (19)$$

for all x, y ∈X, where A, B, C, D, and E are non negative with A+B+C+2D+2E <1, then T has a unique fixed point.

Proof: We can prove this result by applying Theorem 10 by putting F = G = 0.

Corollary 13: If T is a self-mappings defined on a complete complex valued metric space (X, d) satisfying the condition

$$d(Tx,Ty) \preccurlyeq Ad(x,y) + \frac{Bd(x,Tx)d(y,Ty)}{1+d(x,y)} + \frac{Cd(y,Tx)d(x,Ty)}{1+d(x,y)} + \frac{Dd(x,Tx)d(x,Ty)}{1+d(x,y)}$$
$$+ \frac{Ed(y,Tx)d(y,Ty)}{1+d(x,y)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (20)$$

for all x, y ∈X, where A, B, C, D, and E are non negative with A+B+C+2D+2E <1, then T has a unique fixed point.

Proof: We can prove this result by applying corollary 12 by putting T = S and F = G = 0.

Corollary 14: If S and T are self-mapping defined on a complete complex-valued metric space (X, d) satisfying the condition

$$d(Sx,Ty) \preccurlyeq Ad(x,y) + \frac{Bd(x,Sx)d(y,Ty)}{1+d(x,y)} + \frac{Cd(y,Sx)d(x,Ty)}{1+d(x,y)}$$

$$+ \frac{Dd(x,Sx)d(x,Ty)}{1+d(x,y)} + \frac{Fd(y,Sy)d(x,Tx)}{1+d(x,y)} + \frac{Gd(y,Sy)d(y,Tx)}{1+d(x,y)} \dots\dots\dots\dots (21)$$

for all x, y ∈X, where A, B, C, D, and F are non negative with A+B+C+2D+2F+2G<1, then T has a unique fixed point.

Proof: We can prove this result by applying Theorem 10 by putting E = 0.

Corollary 15: If T is a self-mappings defined on a complete complex valued metric space (X, d) satisfying the condition

$$d(Tx,Ty) \preccurlyeq Ad(x,y) + \frac{Bd(x,Tx)d(y,Ty)}{1+d(x,y)} + \frac{Cd(y,Tx)d(x,Ty)}{1+d(x,y)} + \frac{Dd(x,Tx)d(x,Ty)}{1+d(x,y)}$$

$$+ \frac{Fd(y,Ty)d(x,Tx)}{1+d(x,y)} + \frac{Gd(y,Ty)d(y,Tx)}{1+d(x,y)} \dots\dots\dots\dots\dots\dots\dots (22)$$

for all x, y ∈X, where A, B, C, D, and F are non negative with A+B+C+2D+2F+2G<1, then T has a unique fixed point.

Proof: We can prove this result by applying Corollary 14 by putting T = S.

Remark 16: (i) By choosing D = E = F = G = 0 in theorem 10, we get theorem 1 of [6]

(ii) By choosing D = E = F = G = 0 and S = T in theorem 10, we get corollary 3 of [6].(iii) By choosing C= D = E = F = G = 0 in theorem 10, we get theorem 4 of Azam et al. [2]

(iv) By choosing C= D = E = F = G = 0 and S = T in theorem 10, we get corollary 5 of Azam et al. [2]

Theorem17: If$\{T_i\}_{i=1}^{m}$ and $\{S_i\}_{i=1}^{n}$are two finite pairwise commuting finite families of self-mapping defined on complete complex-valued metric space (X, d) such that the mappings and S and T with $T = T_1, T_2, T_{3,} \dots T_m$and $S = S_1, S_2, S_{3,} \dots S_n$ satisfy condition (3), then the component masps of the two families $\{T_i\}_{i=1}^{m}$ and $\{S_i\}_{i=1}^{n}$ have unique common fixed point.

Proof: By theorem (10), one can infer that T and S have a unique common fixed point Z ie., Tz = Sz = z).

Now we will show that z is common fixed point of all the component maps of both families. In view of pairwise commutativity of the families $\{T_i\}_{i=1}^{m}$ and$\{S_i\}_{i=1}^{n}$, for every 1 ≤ k ≤ m, we can write $T_k z = T_k Sz = ST_k z,\ T_k z = T_k Tz = TT_k z \ \dots\dots\dots (23)$

It implies that $T_k z$ for all k is also a common fixed point of T and S. By using the uniqueness of common fixed point, we have $T_k z = z$ for all k.

Hence, z is a common fixed point of the family $\{T_i\}_{i=1}^{m}$. Similarly, we can show that z is a common fixed point of the family $\{S_i\}_{i=1}^{n}$. This completes the proof of the theorem.

Corollary 18: If F and G are self-mappings defined on a complex valued metric space (X, d) satisfying the condition

$$d(F^m x, G^n y) \precsim Ad(x,y) + \frac{Bd(x,F^m x)d(y,G^n y)}{1+d(x,y)} + \frac{Cd(y,F^m x)d(x,G^n y)}{1+d(x,y)} + \frac{Dd(x,F^m x)d(x,G^n y)}{1+d(x,y)} + \frac{Ed(y,F^m x)d(y,G^n y)}{1+d(x,y)} + \frac{Fd(y,F^m y)d(x,G^n x)}{1+d(x,y)} + \frac{Gd(y,F^m y)d(y,G^n x)}{1+d(x,y)} \ldots\ldots\ldots\ldots\ldots\ldots..(24)$$

for all x, y ∈X, where A, B, C, D, E,F, and G are nonnegative with

A+B+C+D+2E+2F+2G< 1, then F and G have a unique common fixed point .

Proof: We can prove this result by applying Theorem 17 by setting

$T_1 = T_2 = \cdots = T_m = F$ and $S_1 = S_2 = \cdots S_n = G$.

Corollary 19: If t is a self-mapping defined on a complete complex valued metric space (X, d) satisfying the condition

$$d(T^n x, T^n y) \precsim Ad(x,y) + \frac{Bd(x,T^n x)d(y,T^n y)}{1+d(x,y)} + \frac{Cd(y,T^n x)d(x,T^n y)}{1+d(x,y)} + \frac{Dd(x,T^n x)d(x,T^n y)}{1+d(x,y)} + \frac{Ed(y,T^n x)d(y,G^n y)}{1+d(x,y)} + \frac{Fd(y,T^n y)d(x,T^n x)}{1+d(x,y)} + \frac{Gd(y,T^n y)d(y,T^n x)}{1+d(x,y)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

for all x, y ∈X, where A, B, C, D, E,F, and G are non negative with A + B + C + 2D + 2E + 2F + 2G< 1, then T has a unique fixed point.

Proof: We can prove this result by applying corollary 18 by F = G = T.

Remark 20: (i) By choosing D = E = F = G = 0 in theorem 17, we get theorem 1 of [6]

(ii) By choosing D = E = F = G = 0 in corollary 18, we get corollary 6 of [6].

(iii) By choosing D = E = F = G = 0 in theorem 19, we get corollary 7 of [6]

(iv) By choosing C= D = E = F = G = 0 in corollary 19, we get corollary 6 of Azam et al. [2]

Corollary 21: [5] T: X → X is a mapping defined on a complete complex-valued metric space (X, d) satisfying the condition $d(T^n x, T^n y) \precsim \lambda d(x,y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(26)$

for all x, y ∈X, where λ is nonnegative real number$\lambda < 1$, then T has unique fixed point.

Conflict of Interests

The author declares that there is no conflict of interests.

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