Available online at http://scik.org
J. Math. Comput. Sci. 4 (2014), No. 6, 1055-1063
ISSN: 1927-5307

T_2 CONCEPTS IN FUZZY BITOPOLOGICAL SPACES

M. R. AMIN[1,*], D. M. ALI[2] AND M. S. HOSSAIN[2]

[1]Department of Mathematics, Begum Rokeya University, Rangpur, Rangpur-5400, Bangladesh

[2]Department of Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh

Abstract. In this paper, we introduce a notion of fuzzy pairwise-T_2 bitopological space and find relations with other such spaces. We also study some other properties of these concepts.

Keywords: Quasi-coincidence; Q-neighbourhood; Fuzzy Bitopological spaces; Fuzzy pairwise-T_2 bitopological spaces.

2010 AMS Subject Classification: 93C42.

1. INTRODUCTION

The notion of bitopological spaces was initially introduced by Kelly [7] in 1963. Concept of fuzzy pairwise-T_2 (in short FPT_2) bitopological spaces were introduced earlier by Kandil and El-Shafee [5]. Later on several other authors continued investigating such concepts. Fuzzy pairwise-T_2 separation axioms have also been introduced by Abu Sufiya et al. [1] and Nouh [9]. The purpose of this paper is to introduce a definition of fuzzy pairwise-T_2 bitopological space and derive some related results in this area. Also, we investigate that this concept holds good extension property in the sense of [8] due to Lowen.

2. PRELIMINARIES

Now we recall some definitions and concepts which will be used in our work.

---

*Corresponding author

Received September 13, 2014

Definition 2.1. [13] A fuzzy set μ in a set X is a function from X into the closed unit interval $I = [0, 1]$. For every $x \in X,\ \mu(x) \in I$ is called the grade of membership of x. Throughout this paper, I^X will denote the set of all fuzzy sets from X into the closed unit interval I.

Definition 2.2. [3] Let f be a mapping from a set X into a set Y and u be a fuzzy set in X. Then the image of u, written as $f(u)$, is a fuzzy set in Y whose membership function is given by

$$f(u)(y) = \begin{cases} \sup\{u(x)\} & if\ f^{-1}[\{y\}] \neq \emptyset,\ \ x \in X \\ 0 & \text{otherwise} \end{cases}.$$

Definition 2.3. [4] Let f be a mapping from a set X into a set Y and v be a fuzzy set in Y. Then the inverse of v written as $f^{-1}(v)$ is a fuzzy set in X which is defined by $f^{-1}(v)(x) = v(f(x))$, for $x \in X$.

Definition 2.4. [12] A fuzzy set μ in X is called a fuzzy singleton iff $\mu(x) = r, (0 < r \leq 1)$ for a certain $x \in X$ and $\mu(y) = 0$ for all points y of X except x. The fuzzy singleton is denoted by x_r and x is its support. We call x_r is a fuzzy point if $0 < r < 1$. The class of all fuzzy singletons in X will be denoted by $S(X)$.

Definition 2.5. [3] A fuzzy topology t on X is a collection of members of I^X which is closed under arbitrary suprema and finite infima and which contains constant fuzzy sets 1 and 0. The pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t-open (or simply open) fuzzy sets. A fuzzy set μ is called a t- closed (or simply closed) fuzzy set if $1 - \mu \in t$.

Definition 2.6. [3] Let (X, t) and (Y, s) be two fuzzy topological spaces. A mapping $f: (X, t) \longrightarrow (Y, s)$ is called an fuzzy continuous iff for every $v \in s$, $f^{-1}(v) \in t$.

Definition 2.7. [3] Let (X, t) and (Y, s) be two fuzzy topological spaces. A mapping $f: (X, t) \longrightarrow (Y, s)$ is called an fuzzy open iff for every $u \in t$, $f(u) \in s$.

Definition 2.8. [10] Let f be a real valued function on a topological space. If $\{x: f(x) > \alpha\}$ is open for every real $\alpha \in I_1$, then f is called lower semi continuous function.

Definition 2.9. [2] Let X be a nonempty set and T be a topology on X. Let $t = \omega(T)$ be the set of all lower semi continuous functions from (X, T) to I (with usual topology). Thus $\omega(T) = \{\mu \in I^X: \mu^{-1}(\alpha,\ 1] \in T\}$ for each $\alpha \in I_1$. It can be shown that $\omega(T)$ is a fuzzy topology on X.

Let P be a property of topological spaces and FP be its fuzzy topology analogue. Then FP is called a 'good extension' of P "iff the statement $(X,\ T)$ has P iff $(X,\ \omega(T))$ has FP" holds good for every topological space $(X,\ T)$.

Definition 2.10. [6] A fuzzy singleton x_r is said to be quasi-coincident with a fuzzy set μ, denoted by $x_r q\mu$ iff $r+\mu(x)>1$. If x_r is not quasi-coincident with μ, we write $x_r\bar{q}\mu$.

Definition 2.11. [9] A fuzzy set u of (X,t) is called quasi-neighborhood (Q-nbd, in short) of x_r iff there exists $v\in t$ such that $x_r qv$ and $v\subset u$. If x_r is a fuzzy point or a fuzzy singletone, then $N(x_r,t)=\{\mu\in t: x_r\in\mu\}$ is the family of all fuzzy t-open neighborhoods (t-nbds, in short) of x_r and $N_Q(x_r,t)=\{\mu\in t: x_r q\ \mu\}$ is the family of all Q-neighborhoods (Q-nbd, in short) of x_r.

Definition 2.12. [6] A fuzzy bitopological space (fbts, in short) is a triple (X,s,t) where s and t are arbitrary fuzzy topologies on X.

Definition 2.13. [4] Let $(X,s,\ t)$ and (Y,s_1,t_1) be two fuzzy bitopological spaces. A mapping $f\colon(X,s,t)\longrightarrow(Y,s,t)$ is called an fuzzy FP-continuous iff $f\colon(X,s)\longrightarrow(Y,s_1)$ and $f\colon(X,t)\longrightarrow(Y,t_1)$ are both continuous.

Definition 2.14. [4] Let $(X,s,\ t)$ and (Y,s_1,t_1) be two fuzzy bitopological spaces. A mapping $f\colon(X,s,t)\longrightarrow(Y,s,t)$ is called an fuzzy FP-open iff $f\colon(X,s)\longrightarrow(Y,s_1)$ and $f\colon(X,t)\longrightarrow(Y,t_1)$ are both open.

Definition 2.15. [7] A space $(X,S,\ T)$ is said to be pairwise Hausdorff iff for each two distinct points x and y, there are a S-neighbourhood U of x and a T-neighbourhood V of y such that $U\cap V=\emptyset$.

3. FUZZY PAIRWISE T_2-SPACES

Definition 3. 1. An fbts (X,s,t) is called

(a) $FPT_2(i)$ iff for every pair of fuzzy singletons $x_r,\ y_s$ in X with $x\neq y$, there exist fuzzy sets $\mu\in s,\ \lambda\in t$ such that $x_r q\mu$, $y_s q\lambda$ and $\mu\cap\lambda=0$.

(b)[9] $FPT_2(ii)$ iff $(\forall x_r,\ y_s\in S(X), x\neq y)$, $(\exists\mu\in N(x_r,\ s)\left(\exists\ \lambda\in N_Q(y_r,t)\right)(\mu\bar{q}\lambda)$ or $(\exists\mu^*\in N(x_r,\ t)\left(\exists\ \lambda^*\in N_Q(y_r,\ s)\right)(\mu^*\bar{q}\lambda^*)$.

(c)[5] $FPT_2(iii)$ iff for every pair of fuzzy singletons $x_p,\ y_r$ in X such that $x_p\bar{q}y_r$, there exist fuzzy sets $\mu\in s,\ \lambda\in t$ such that $x_p\in\mu$, $y_r\in\lambda$ and $\mu\bar{q}\lambda$.

(d)[1] $FPT_2(iv)$ iff for every pair of fuzzy singletons $x_r,\ y_s$ in X with $x\neq y$, there exist fuzzy sets $\mu\in s,\ \lambda\in t$ such that $x_r\in\mu$, $y_s\in\lambda$ and $\mu\cap\lambda=0$.

(e)[1] $FPT_2(v)$ iff for any two distinct fuzzy points $x_r,\ y_s$ in X, there exist fuzzy sets $\mu\in s$, $\lambda\in t$ such that $x_r\in\mu$, $y_s\in\lambda$ and $\mu\subseteq\lambda^c$.

Theorem 3.2. Let (X,s,t) be an fbts. Then we have the following implications:

(a)$\Leftrightarrow$(d) $\Rightarrow$ (b) $\Rightarrow$ (e) but (b)$\nRightarrow$ (d), (e)$\nRightarrow$ (b), (a)$\nRightarrow$ (c) and (c)$\nRightarrow$ (a).

Proof: (a) ⇒ (d): Let $x_r, y_p \in S(X)$ with $x \neq y$. Since (X, s, t) is FP$T_2(i)$-space, then there exist fuzzy sets $\mu \in s,\ \lambda \in t$ such that $x_{1-r} q\mu$, $y_{1-p} q\lambda$ and $\mu \cap \lambda = 0$.

That is, $\mu(x) > r, \lambda(y) > p$ and $\mu \cap \lambda = 0$.

So, $x_r \in \mu$, $y_p \in \lambda$ and $\mu \cap \lambda = 0$. Hence (X, s, t) is FP$T_2(iv)$-space. Similarly we can show that (d) ⇒ (a).

(d) ⇒ (b): Let $x_r, y_p \in S(X)$ with $x \neq y$. Choose $p^* \in (0, 1)$ such that $p^* > 1 - p$. Since (X, s, t) is FP$T_2(iv)$-space, then there exist fuzzy sets $\mu \in s,\ \lambda \in t$ such that $x_r \in \mu$, $y_{p^*} \in \lambda$ and $\mu \cap \lambda = 0$. That is, $x_r \in \mu, \lambda(y) \geq p^*$ and $\mu\bar{q}\lambda$.

Since $\lambda(y) \geq p^*$ and $p^* > 1 - p$, then we have

$\lambda(y) > 1 - p \Rightarrow \lambda(y) + p > 1$. So, $y_p q\lambda$.

Hence $x_r \in \mu$, $y_p q\lambda$ and $\mu\bar{q}\lambda$. Therefore (X, s, t) is FP$T_2(ii)$-space.

(b) ⇒ (e): Let x_r, y_p be two distinct fuzzy points in X. Since (X, s, t) is FP$T_2(ii)$-space, then there exist fuzzy sets $\mu \in s,\ \lambda \in t$ such that $x_r \in \mu$, $y_{1-p} q\lambda$ and $\mu\bar{q}\lambda$.

That is, $x_r \in \mu, \lambda(y) + 1 - p > 1$ and $\mu \subseteq \lambda^c$.

That is, $x_r \in \mu$, $\lambda(y) > p$ and $\mu \subseteq \lambda^c$. That is, $x_r \in \mu$, $y_p \in \lambda$ and $\mu \subseteq \lambda^c$. Hence (X, s, t) is FP$T_2(v)$.

Example 3.3. Let $X = \{x, y\}$ and λ, u, v be three fuzzy sets defined by

$\lambda(x) = m, \lambda(y) = n$, where $m, n \in (0, 1]$,

$u(x) = 0, u(y) = 0.5$ and $v(x) = 1, v(y) = 0.1$.

Let s and t be two fuzzy topologies on X generated by $\{\lambda, u, v\}$.

Then we can show that (X, s, t) is FP$T_2(ii)$ but not FP$T_2(iv)$ and FP$T_2(i)$. Similarly (X, s, t) is FP$T_2(iii)$ but not FP$T_2(i)$.

Example3.4. Let $X = \{x, y\}$ and $s = \{0, 1, u, v, w\}$,

$u(x) = 1, u(y) = 0, v(x) = 0, v(y) = 0.7$ and $w(x) = 1, w(y) = 0.7$, and

$t = \{u_1,\ v_1,\ w_1\}$, where

$u_1(x) = 0, u_1(y) = 1, v_1(x) = 0.1, v_1(y) = 0$ and $w_1(x) = 0.7, w_1(y) = 1$.

Then we can show that (X, s, t) is FP$T_2(i)$ but not FP$T_2(iii)$. Indeed, if $x, = y$ and $p + r \leq 1$, then $\left(\forall \mu \in N\left(x_p, s\right)\right) (\forall \gamma \in N(y_r,\ t))(\mu q\, \gamma)$.

In general it is true that union of topologies is not a topology. But if union of two topologies is again a topology then we have the following theorem.

Theorem 3.5. If an fbts (X, s, t) is $FPT_2(j)$, then $(X, s \cup t)$ is $FPT_2(j)$, where j=I, ii, iii, iv, v.

Proof: Obvious.

The converse of the above theorem 3.5 is not true in general.

Example 3.6. Let $X = [0, 1]$ and s be the discrete fuzzy topology on X and t be the indiscrete fuzzy topology on X. Then $(X, s \cup t)$ is $FPT_2(j)$, but (X, s, t) is not $FPT_2(j)$, where $j = i, ii, iii, iv, v$.

Now we obtain some tangible features of fuzzy pairwise T_2-spaces.

Theorem 3.7. Let (X, s, t) be a fuzzy bitopological space, $A \subset X$ and $s_A = \{u/A \,:\, u \in s\}$, $t_A = \{v/A \,:\, v \in t\}$. Then

(a) (X, s, t) is $FPT_2(i) \Rightarrow (A, s_A, t_A)$ is $FPT_2(i)$.

(b) (X, s, t) is $FPT_2(ii) \Rightarrow (A, s_A, t_A)$ is $FPT_2(ii)$.

(c) (X, s, t) is $FPT_2(iii) \Rightarrow (A, s_A, t_A)$ is $FPT_2(iii)$.

(d) (X, s, t) is $FPT_2(iv) \Rightarrow (A, s_A, t_A)$ is $FPT_2(iv)$.

(e) (X, s, t) is $FPT_2(v) \Rightarrow (A, s_A, t_A)$ is $FPT_2(v)$.

Proof: (a) Suppose (X, s, t) is $FPT_2(i)$. We have to show that (A, s_A, t_A) is $FPT_2(i)$. Let x_r, $y_s \in S(A)$ with x$\neq y$. Then $x_r,\ y_s \in S(X)$ with x$\neq y$. Since (X, s, t) is $FPT_2(i)$, then there exist fuzzy sets $\mu \in s,\ \lambda \in t$ such that $x_r q\mu,\ y_s q\lambda$ and $\mu \cap \lambda = 0$. Now it is clear that $\mu/A \in s_A$, $\lambda/A \in t_A$ for every $\mu \in s,\ \lambda \in t$ respectively.

Now, $x_r q\mu$, $y_s q\lambda$ implies that $\mu(x) + r > 1$ and $\lambda(y) + s > 1$.

But, $(\mu/A)(x) = \mu(x)$ and $(\lambda/A)(y) = \lambda(y)$. Then $(\mu/A)(x) + r > 1$ and $(\lambda/A)(y) + s > 1$. So, $x_r q(\mu/A)$, $y_s q(\lambda/A)$.

Also, $(\mu/A) \cap (\lambda/A) = (\mu \cap \lambda)/A = 0$, since $\mu \cap \lambda = 0$. Hence (A, s_A, t_A) is $FPT_2(i)$.

Proofs of (b), (c), (d) and (e) are similar.

Theorem 3.8. Let $(X, T_1,\ T_2)$ be a bitopological space. Then

(a) $(X,\ T_1,\ T_2)$ is $PT_2 \Leftrightarrow (X,\ \omega(T_1),\ \omega(T_2))$ is $FPT_2(i)$.

(b) $(X,\ T_1,\ T_2)$ is $PT_2 \Leftrightarrow (X,\ \omega(T_1),\ \omega(T_2))$ is $FPT_2(ii)$.

(c) $(X,\ T_1,\ T_2)$ is $PT_2 \Leftrightarrow (X,\ \omega(T_1),\ \omega(T_2))$ is $FPT_2(iii)$.

(d) $(X,\ T_1,\ T_2)$ is $PT_2 \Leftrightarrow (X,\ \omega(T_1),\ \omega(T_2))$ is $FPT_2(iv)$.

(e) $(X,\ T_1,\ T_2)$ is $PT_2 \Leftrightarrow (X,\ \omega(T_1),\ \omega(T_2))$ is $FPT_2(v)$.

Proof: (a) Suppose that (X, T_1, T_2) is PT_2. We have to show that $(X, \omega(T_1), \omega(T_2))$ is $FPT_2(i)$. Let $x_p,\ y_r \in S(X)$ with $x \neq y$. Since (X, T_1, T_2) is PT_2, then there exist $U \in T_1$, $V \in T_2$ such that

$x \in U$, $y \in V$ and $U \cap V = \emptyset$. This implies $\left(1_U \in N_Q\left(x_p,\ \omega(T_1)\right)\right)$, $(1_V \in N_Q(y_r,\ \omega(T_2))$ and $1_V \cap 1_U\ = 0$. Hence $(X, \omega(T_1), \omega(T_2))$ is $FPT_2(i)$.

Conversely, suppose that $(X, \omega(T_1), \omega(T_2))$ is $FPT_2(i)$. We have to show that (X, T_1, T_2) is PT_2. Let $x,\ y \in X$ such that $x \neq y$. Since $(X, \omega(T_1),\ \omega(T_2))$ is $FPT_2(i)$, then $(\exists\, \mu \in N_Q(x_1,\ \omega(T_1))$, $(\exists \eta \in N_Q(y_1,\ \omega(T_2))$ and $\mu \cap\ \eta = 0$.

Now, $\mu \in N_Q(x_1,\ \omega(T_1))$, $\eta \in N_Q(y_1,\ \omega(T_2)$ implies that $\mu(x) + 1 > 1$ and $\eta(y) + 1 > 1$. That is, $\mu(x) > 0$ and $\eta(y) > 0$. Hence $x \in \mu^{-1}(0,1] \in T_1$, $y \in \eta^{-1}(0,1] \in T_2$.

To show that $\mu^{-1}(0,1] \cap \eta^{-1}(0,1] = 0$, suppose that $\mu^{-1}(0,1] \cap \eta^{-1}(0,1] \neq 0$. Then there exists $z \in \mu^{-1}(0,1] \cap \eta^{-1}(0,1]$ such that $\mu(z) > 0$ and $\eta(z) > 0$. Consequently $(\mu \cap\ \eta)(z) \neq 0$ which contradicts the fact that $\mu \cap\ \eta = 0$.

Proofs of (c) and (d) are similar and for the proof of (b), cf. [9].

Theorem 3.9. Given $\{(X_i, s_i, t_i) : i \in \Lambda\}$ be a family of fuzzy bijtopological spaces. Then the product fbts $(\prod X_i,\ \prod s_i,\ \prod t_i)$ is $FPT_2(j)$ if each coordinate space (X_i, s_i, t_i) is $FPT_2(j)$, where $j = i, ii, iii, iv, v$.

Proof: Suppose each coordinate space (X_i, s_i, t_i) is $FPT_2(i)$. We shall show that the product space is $FPT_2(i)$. Let $x_r,\ \ y_s \in S(\prod X_i)$ with $x \neq y$. Again suppose that $x = \prod x_i$, $y = \prod y_i$. Then $x_i \neq y_i$ for some $i \in \Lambda$, since $x \neq y$. Now consider $(x_i)_r, (y_i)_s \in S(X_i)$. Since (X_i, s_i, t_i) is $FPT_2(i)$, then there exist $\mu_i \in s_i$, $\lambda_i \in t_i$ such that $(x_i)_r\, q \mu_i$, $(y_i)_s\, q \lambda_i$ and $\mu_i \cap \lambda_i = 0$. Now consider $\mu = \prod \mu_j$ and $\lambda = \prod \lambda_j$, where $\mu_i = \lambda_i = 1$ for $i \neq j$ and $\mu_j = \mu_j$, $\lambda_j = \lambda_j$. Then $\mu \in \prod s_i$, $\lambda \in \prod t_i$ and we can easily show that $x_r q \mu$, $y_s q \lambda$ and $\mu \cap \lambda = 0$. Hence the product space is $FPT_2(i)$.

Other proofs are similar.

Theorem 3.10. A bijective mapping from an fts (X, t) to an fts (Y, s) preserves the value of a fuzzy singleton (fuzzy point).

Proof: Let c_r be a fuzzy singleton in X. So, there exist a point $a \in Y$ such that $f(c) = a$. Now $f(c_r)(a) = f(c_r)(f(c)) = \sup c_r(c) = c_r(c) = r$, since f is bijective. Hence a_r has same value as c_r.

Note: Preimage of any fuzzy singleton (fuzzy point) under bijective mapping preserves its value.

The ideas of the following theorems 3.11, 3.12 are taken from [11].

Theorem 3.11. Let (X,s,t) and (Y,s_1,t_1) be two fuzzy bitopological spaces and let $f:X\to Y$ be bijective and FP-open. Then

(a) (X,s,t) is $FPT_2(i)$ $\Rightarrow (Y,s_1,t_1)$ is $FPT_2(i)$.

(b) (X,s,t) is $FPT_2(ii)$ $\Rightarrow (Y,s_1,t_1)$ is $FPT_2(ii)$.

(c) (X,s,t) is $FPT_2(iii)$ $\Rightarrow (Y,s_1,t_1)$ is $FPT_2(iii)$.

(d) (X,s,t) is $FPT_2(iv)$ $\Rightarrow (Y,s_1,t_1)$ is $FPT_2(iv)$.

(e) (X,s,t) is $FPT_2(v)$ $\Rightarrow (Y,s_1,t_1)$ is $FPT_2(v)$.

Proof: (a) Suppose (X,s,t) is $FPT_2(i)$. We shall show that (Y,s_1,t_1) is $FPT_2(i)$. Let $a_r,\ b_q\in S(Y)$ with $a\neq b$. Since f is bijective, then there exist distinct fuzzy singletons $c_r,\ d_q$ in X such that $f(c)=a,\ f(d)=b$ and $c\neq d$. Again since (X,s,t) is $FPT_2(i)$, then there exist fuzzy sets $\mu,\in s,\lambda\in t$ such that $c_rq\ \mu,\ d_qq\lambda$ and $\mu\cap\lambda=0$.

Now, $c_rq\ \mu,\ d_qq\lambda$ implies that $\mu(c)+r>1$ and $\lambda(d)+q>1$.

But $f(\mu)(a)=f(\mu)\big(f(c)\big)=\sup\mu(c)=\mu(c)$, since f is bijective. So $f(\mu)(a)+r>1$, since $\mu(c)+r>1$. Hence $a_rqf(\mu)$. Similarly, $b_qqf(\lambda)$.

Also, $f(\mu\cap\lambda)(a)=\sup(\mu\cap\lambda)(c)\ :f(c)=a$

$f(\mu\cap\lambda)(b)=\sup(\mu\cap\lambda)(d)\ :f(d)=b$.

Hence $(\mu\cap\lambda)=0\quad\Rightarrow f(\mu)\cap f(\lambda)=0$.

Since f is FP-open, then $f(\mu)\in s_1,\ f(\eta)\in t_1$. Now, it is clear that there exist $f(\mu)\in s_1$, $f(\eta)\in t_1$ such that $a_rqf(\mu)$, $b_qqf(\lambda)$ and $f(\mu)\cap f(\lambda)=0$. Hence (Y,s_1,t_1) is $FPT_2(i)$.

Similarly, (b), (c), (d) and (e) can be proved.

Theorem 3.12. Let (X,s,t) and (Y,s_1,t_1) be two fuzzy bitopological spaces and $f:X\to Y$ be FP-continuous and bijective. Then

(a) (Y,s_1,t_1) is $FPT_2(i)$ $\Rightarrow$ (X,s,t) is $FPT_2(i)$.

(b) (Y,s_1,t_1) is $FPT_2(ii)$ $\Rightarrow$ (X,s,t) is $FPT_2(ii)$.

(c) (Y,s_1,t_1) is $FPT_2(iii)$ $\Rightarrow$ (X,s,t) is $FPT_2(iii)$.

(d) (Y,s_1,t_1) is $FPT_2(iv)$ $\Rightarrow$ (X,s,t) is $FPT_2(iv)$.

Proof: We shall prove (a) only.

Suppose (Y,s_1,t_1) is $FPT_2(i)$. We claim that (X,s,t) is $FPT_2(i)$. For this, let $c_r,d_q\in S(X)$ with $c\neq d$. Then there exist distinct fuzzy singletons $a_r,\ b_q$ in Y such that $f(c)=a,f(d)=b$

and $a \neq b$, since f is one-one. Again since (Y, s_1, t_1) is $FPT_2(i)$, then there exist fuzzy sets $\mu, \in s$, $\lambda \in t$ such that $a_r q\mu$, $b_q q\lambda$ and $\lambda \cap \mu = 0$.

This implies that $\mu(a) + r > 1, \lambda(b) + q > 1$ and $\lambda \cap \mu = 0$.

That is, $\mu(f(c)) + r > 1, \lambda(f(d)) + q > 1$ and $f^{-1}(\lambda \cap \mu) = 0$.

That is, $f^{-1}(\mu)(c) + r > 1, f^{-1}(\lambda)(d) + q > 1$ and $f^{-1}(\lambda) \cap f^{-1}(\mu) = 0$.

That is, $c_r q f^{-1}(\mu)$, $d_q q f^{-1}(\lambda)$ and $f^{-1}(\lambda) \cap f^{-1}(\mu) = 0$.

Since f is FP-continuous, then $f^{-1}(\mu) \in s$, $f^{-1}(\eta) \in t$. Now, we see that there exist $f^{-1}(\mu) \in s$, $f^{-1}(\eta) \in t$ such that $c_r q f^{-1}(\mu)$, $d_q q f^{-1}(\lambda)$ and $f^{-1}(\lambda) \cap f^{-1}(\mu) = 0$. Hence (X, s, t) is $FPT_2(i)$.

Conflict of Interests

The author declares that there is no conflict of interests.

REFERENCES

[1] A.S. Abu Sufiya, A.A. Fora, M. W. Warner, Fuzzy separation axioms and fuzzy continuity in fuzzy bitopological spaces, Fuzzy sets and Systems, 62 (1994), 367-373.

[2] D. M. Ali, On Certain Separation and Connectedness Concepts in Fuzzy Topology, Ph. D. Thesis, Banaras Hindu University, India, 1990.

[3] C. L. Chang, Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, 24 (1968), 182 – 190.

[4] M. S. Hossain and D. M. Ali, On T_1 Fuzzy Bitopological Spaces, Journal of Bangladesh Academy of Sciences, 31 (2007), 129 –135.

[5] A. Kandil and M. El-Shafee, Separation axioms for fuzzy bitopological spaces, Journal of Institute of Mathematics and Computer Sciences, 4 (3) (1991), 373-383.

[6] A. Kandil, A.A. Nouh and S.A. El-Sheikh, Strong and ultra separation axioms on fuzzy bitopological spaces, Fuzzy Sets and Systems, 105 (1999), 459-467.

[7] J. C. Kelly, Bitopological spaces, Proceeding of the London Mathematical Society, 13 (1963), 71-89.

[8] R. Lowen, Fuzzy topological spaces and fuzzy compactness, Journal of Mathematical Analysis and Applications, 56 (1976), 621-623.

[9] A. A. Nouh, On separation axioms in fuzzy bitopological spaces, Fuzzy Sets and Systems, 80 (1996), 225-236.

[10] W. Rudin. Real and complex analysis. Copyright © 1966, 1974, by McGraw –Hill Inc., 33-59.

[11] M. A. M. Talukder and D. M. Ali, Certain Properties of Countably Q –Compact Fuzzy sets, Journal of Mathematical and Computational Science, 4 (2014), 446-462.

[12] C. K. Wong, Fuzzy Points and Local Properties of Fuzzy Topology, Journal of Mathematical Analysis and Applications, 46 (1974), 316-328.

[13] L.A. Zadeh, Fuzy sets, Information and control, 8 (1965), 338-353.