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CERTAIN TRANSFORMATIONS AND SUMMATIONS OF BASIC HYPERGEOMETRIC SERIES

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Abstract. In the present work we have established some new transformations and summations of basic hypergeometric series by making the use of WP-Bailey pairs.

Keywords: Bailey's lemma; Basic hypergeometric series; Transformation; Summation.

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1. Introduction

For $|q| < 1, \quad (a;q)_n = (1-a)(1-aq)..........(1-aq^{n-1}); n = 1, 2....$

$$(a;q)_0 = 1; \qquad (a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n)$$

$$(a;q)_n = \frac{(a;q)_\infty}{(aq^n;q)_\infty}.$$

where a is real or complex.

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A Basic Hypergeometric Series is defined as

$$
{}_r\phi_s(a_1,a_2,a_3....a_r;b_1,b_2,b_3......b_s;q,z) = \sum_{n=0}^{\infty}\frac{(a_1;q)_n \quad (a_2;q)_n \quad ... \quad (a_r;q)_n}{(q;q)_n(b_1;q)_n \quad (b_2;q)_n....(b_s;q)_n}[(-1)^n q^{\frac{n(n-1)}{2}}]^{1+s-r}z^n.
$$

For $0<|q|<1$, the series converges absolutely for all z if $r \le s$ and for $|z|<1$ if $r=s+1$. This series also converges absolutely if $|q|>1$ and $|z|<|b_1b_2...b_s|/|a_1a_2...a_r|$.

In 1944, Bailey [1] introduced a very useful and simple identity known as Bailey's lemma. The Bailey's lemma states that, if

$$
\beta_n = \sum_{r=0}^{n}\alpha_r u_{n-r}\upsilon_{n+r} \tag{1.1}
$$

and

$$
\gamma_n = \sum_{r=n}^{\infty}\delta_r u_{r-n}\upsilon_{n+r}, \tag{1.2}
$$

then under the suitable convergence conditions and if change in the order of summations is allowed

$$
\sum_{n=0}^{\infty}\alpha_n\gamma_n = \sum_{n=0}^{\infty}\beta_n\delta_n, \tag{1.3}
$$

where α_r, δ_r, u_r and υ_r are functions of r such that β_n and γ_n exist. The proof of the lemma is trivial.

Taking $u_r = 1/(q;q)_r$ and $v_r = 1/(aq;q)_r$ in (1.1), we have

$$
\beta_n = \sum_{r=0}^{n}\frac{\alpha_r}{(q;q)_{n-r}(aq;q)_{n+r}}. \tag{1.4}
$$

The pair of sequence (α_n, β_n) that satisfies (1.4) is called a Bailey pair relative to the parameter a.

The Bailey lemma has been a simple and effective tool in proving Rogers-Ramanujan type of identities and also a verity of transformations of basic hypergeometric series [2]. Slater [3, 4] used Bailey's lemma and gave the long list of 130 identities of Roger-Ramanujan type. After Slater the Bailey lemma have been extensively used to prove Rogers-Ramanujan type of identities and its generalizations [5-8]. Very recently, Warnaar [9] has written a very elegant survey of Bailey lemma. Andrews et al [10-13] exploited very effective the mechanism of Bailey's transform in the form of Bailey pair and Bailey chain. In particular, WP-Bailey pair (α_n, β_n) [14] satisfying

$$\beta_n = \sum_{r=0}^{n} \frac{(k/a;q)_{n-r}(k;q)_{n+r}}{(q;q)_{n-r}(aq;q)_{n+r}} \alpha_r. \tag{1.5}$$

For $k = 0$ in (1.5), we get the standard Bailey pair (1.4). The relation (1.5) follows by setting $u_r = \frac{(k/a;q)_r}{(q;q)_r}$ and $v_r = \frac{(k;q)_r}{(aq;q)_r}$ in (1.1). The same substitutions in (1.2), gives

$$\gamma_n = \frac{(k;q)_{2n}}{(aq;q)_{2n}} \sum_{r=0}^{\infty} \frac{(k/a;q)_r (kq^{2n};q)_r}{(q;q)_r (aq^{2n+1};q)_r} \delta_{r+n}. \tag{1.6}$$

In the present paper, we have established a number of transformations and summations of basic hypergeometric series by making use of (1.5) and (1.6). Some interesting special cases have also been deduced.

We define a WP-Bailey Unit Bailey pair as

$$\alpha_n = \frac{(a, q\sqrt{a}, -q\sqrt{a}, a/k;q)_n}{(q, \sqrt{a}, -\sqrt{a}, kq;q)_n} (k/a)^n, \tag{1.7}$$

$$\beta_n = \begin{cases} 1, & n = 0, \\ 0, & n > 0. \end{cases}$$

The trivial WP-Bailey pair is defined as

$$\beta_n = \frac{(k, k/a;q)_n}{(q, aq;q)_n}, \tag{1.8}$$

$$\alpha_n = \begin{cases} 1, & n = 0, \\ 0, & n > 0. \end{cases}$$

A WP-Bailey pair due to Singh [15] is

$$\alpha_n = \frac{(a, q\sqrt{a}, -q\sqrt{a}, y, z, a^2q/kyz;q)_n}{(q, \sqrt{a}, -\sqrt{a}, aq/y, aq/z, kyz/a;q)_n} (k/a)^n,$$

$$\beta_n = \frac{(ky/a, kz/a, k, aq/yz;q)_n}{(q, aq/y, aq/z, kyz/a;q)_n}. \tag{1.9}$$

In our analysis we shall also require the following known results,

$$ {}_4\phi_3(a,-q\sqrt{a},b,c;-\sqrt{a},aq/b,aq/c;q,q\sqrt{a}/bc) = \frac{(aq,q\sqrt{a}/b,q\sqrt{a}/c,aq/bc;q)_\infty}{(aq/b,aq/c,q\sqrt{a},q\sqrt{a}/bc;q)_\infty}. \quad (1.10) $$

$(see[16], II.13)$

$$ {}_3\phi_2(a,\lambda q,b;\lambda,q\lambda^2/b;q,\lambda^2/ab^2) = \frac{1-\lambda+\lambda/b(1-\lambda/a)}{(1-\lambda)(1+\lambda/b)} \frac{(\lambda^2/b^2,q\lambda^2/ab;q)_\infty}{(q\lambda^2/b,\lambda^2/ab^2;q)_\infty}, \quad (1.11) $$

$|\lambda^2/ab^2| < 1.$

$(see[16]pp.103)$

$$ {}_2\phi_1(a,b;aq/b;q,-q/b) = \frac{(-q;q)_\infty(aq,aq^2/b^2;q^2)_\infty}{(-q/b,aq/b;q)_\infty}. \quad (1.12) $$

$(see[16]II.9)$

$$ {}_4\phi_3(a,q\sqrt{a},-q\sqrt{a},b;\sqrt{a},-\sqrt{a},aq/b;q,1/b^2q) = \frac{(a/b^2,1/bq;q)_\infty}{(aq/b,1/b^2q;q)_\infty}. \quad (1.13) $$

$(see[17])$

$$ {}_8\phi_7(a,q\sqrt{a},-q\sqrt{a},\sqrt{(a/b)},-\sqrt{(a/b)},\sqrt{(aq/b)},-\sqrt{(aq/b)},b;\sqrt{a},-\sqrt{a},q\sqrt{(ab)},-q\sqrt{(ab)} $$
$$,\sqrt{(abq)},-\sqrt{(abq)},aq/b;q;bq) = \frac{(aq,b^2q;q)_\infty}{(bq,abq;q)_\infty}, \quad (1.14) $$

$|bq| < 1.$

$(see[18])$

2. Main results

If (α_n,β_n) is a WP-Bailey pair, then under suitable convergence conditions, the following relations are true

$$ \sum_{n=0}^{\infty} \frac{(-q\sqrt{k},c;q)_n}{(-\sqrt{k},kq/c;q)_n} (\frac{aq}{c\sqrt{k}})^n \beta_n = $$

$$\frac{(kq,aq/\sqrt{k},q\sqrt{k}/c,aq/c;q)_\infty}{(aq/c\sqrt{k},aq,kq/c,q\sqrt{k};q)_\infty}\sum_{n=0}^{\infty}\frac{(k;q)_{2n}}{(kq;q)_{2n}}\frac{(-q\sqrt{k},q\sqrt{k},c;q)_n}{(-\sqrt{k},aq/\sqrt{k},aq/c;q)_n}(\frac{aq}{c\sqrt{k}})^n\alpha_n. \tag{2.1}$$

$$\sum_{n=0}^{\infty}\frac{(q^{n+1}\sqrt{ak};q)_n}{(q^n\sqrt{ak};q)_n}(\frac{a^2}{k^2})^n\beta_n =$$

$$\frac{(a/k,a^2q/k;q)_\infty}{(a^2/k^2,aq;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,kq^n,q^{n+1}\sqrt{ak};q)_n}{(q^n\sqrt{ak},a^2q/k,a^2q^{n+1}/k;q)_n}\frac{(1-\sqrt{ak}q^{2n}+\sqrt{a}/(\sqrt{k}-a^{3/2}q^{2n}))}{(1-q^{2n}\sqrt{(ak)})(1+\sqrt{a}/\sqrt{k})}(\frac{a^2}{k^2})^n\alpha_n. \tag{2.2}$$

$$\sum_{n=0}^{\infty}\frac{(q\sqrt{k},-q\sqrt{k};q)_n}{(\sqrt{k},-\sqrt{k};q)_n}(\frac{a^2}{k^2q})^n\beta_n =$$

$$\frac{(a^2/k,a/kq;q)_\infty}{(aq,a^2/k^2q;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,kq^n,q\sqrt{k},-q\sqrt{k};q)_n}{(a^2/k,a^2q^n/k,\sqrt{k},-\sqrt{k};q)_n}(\frac{a^2}{qk^2})^n\alpha_n. \tag{2.3}$$

$$\sum_{n=0}^{\infty}\frac{(q\sqrt{k},-q\sqrt{k},\sqrt{a},-\sqrt{a},\sqrt{aq},-\sqrt{aq};q)_n}{(\sqrt{k},-\sqrt{k},-kq\sqrt{(1/a)},kq\sqrt{(1/a)},k\sqrt{(q/a)},-k\sqrt{(q/a)};q)_n}(\frac{kq}{a})^n\beta_n =$$

$$\frac{(kq,k^2q/a^2;q)_\infty}{(kq/a,k^2q/a;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,kq^n,q\sqrt{k},-q\sqrt{k},\sqrt{a},-\sqrt{a},\sqrt{(aq)},-\sqrt{(aq)};q)_n}{(\sqrt{k},-\sqrt{k},kq\sqrt{(1/a)},-kq\sqrt{(1/a)},k\sqrt{(q/a)},-k\sqrt{(q/a)};q)_n}$$

$$\frac{(k^2q/a,k^2q^{(n+1)}/a;q)_n}{(aq,aq^{n+1},kq,kq^{n+1};q)_n}(\frac{kq}{a})^n\alpha_n. \tag{2.4}$$

$$\sum_{n=0}^{\infty}(\frac{-aq}{k})^n\beta_n = \frac{(kq,a^2q^2/k;q^2)_\infty(-q;q)_\infty}{(-aq/k,aq;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,kq^n;q)_n}{(kq,a^2q^2/k;q^2)_{2n}}(\frac{-aq}{k})^n\alpha_n. \tag{2.5}$$

Proof 2.1. Substituting $a=kq^{2n}, b=k/a$ and $c=cq^n$ in (1.10), we have

$$\begin{aligned}&{}_4\phi_3(kq^{2n},-q^{n+1}\sqrt{k},k/a,cq^n;-q^n\sqrt{k},aq^{2n+1},kq^{n+1}/c;q,aq/c\sqrt{k})\\&=\frac{(kq,aq/\sqrt{k},q\sqrt{k}/c,aq/c;q)_\infty(aq;q)_{2n}(kq/c,q\sqrt{k};q)_n}{(aq/c\sqrt{k},aq,kq/c,q\sqrt{k};q)_\infty(kq;q)_{2n}(aq/\sqrt{k},aq/c;q)_n}.\end{aligned} \tag{2.6}$$

Putting $\delta_r = \frac{(c,-q\sqrt{k};q)_r}{(-\sqrt{k},kq/c;q)_r}(\frac{aq}{c\sqrt{k}})^r$ in (1.6) and making the use of (2.6), we get

$$\gamma_n = \frac{(k;q)_{2n}(q\sqrt{k},-q\sqrt{k},c;q)_n(kq,aq/\sqrt{k},q\sqrt{k}/c,aq/c;q)_\infty}{(kq;q)_{2n}(-\sqrt{k},aq/\sqrt{k},aq/c;q)_n(aq,aq/c\sqrt{k},kq/c,q\sqrt{k};q)_\infty}(\frac{aq}{c\sqrt{k}})^n.$$

Substituting δ_n and γ_n as above in (1.3), we get (2.1).

Proof 2.2 Setting $a=k/a, b=kq^{2n}$ and $\lambda=q^{2n}\sqrt{ak}$ in (1.11), we get

$$ {}_3\phi_2(k/a, q^{2n+1}\sqrt{ak}, kq^{2n}; q^{2n}\sqrt{ak}, aq^{2n+1}; q, a^2/k^2) = \frac{(a/k, a^2q/k;q)_\infty (aq;q)_{2n}}{(a^2/k^2, aq;q)_\infty (a^2q/k;q)_{2n}} $$

$$ \times \frac{(1-q^{2n}\sqrt{ak}+\sqrt{a}/(\sqrt{k}-a^{3/2}q^{2n}))}{(1-q^{2n}\sqrt{ak})(1+\sqrt{a}/\sqrt{k})}, \tag{2.7} $$

$|a^2/k^2| < 1$.

Choosing $\delta_r = \frac{(q^{n+1}\sqrt{(ak)};q)_r}{(q^n\sqrt{(ak)};q)_r}(\frac{a^2}{k^2})^r$ in (1.6) and substituting in (2.7), we have

$$ \gamma_n = \frac{(1-q^{2n}\sqrt{ak}+\sqrt{a}/(\sqrt{k}-a^{3/2}q^{2n}))}{(1-q^{2n}\sqrt{(ak)})(1+\sqrt{a}/\sqrt{k})} \frac{(a/k, a^2q/k;q)_\infty (k, kq^n, q^{n+1}\sqrt{(ak)};q)_n}{(aq, a^2/k^2;q)_\infty (q^n\sqrt{(ak)}, a^2q/k, a^2q^{n+1}/k;q)_n} (\frac{a^2}{k^2})^n. $$

using δ_n and γ_n in (1.3), we obtain (2.2).

Proof 2.3. Choosing $a = kq^{2n}$ and $b = k/a$ in (1.13), we get

$$ {}_4\phi_3(kq^{2n}, q^{n+1}\sqrt{k}, -q^{n+1}\sqrt{k}, k/a; q^n\sqrt{k}, -q^n\sqrt{k}, aq^{2n+1}; q, a^2/k^2q) = \frac{(a^2q^{2n}/k, a/kq;q)_\infty}{(aq^{2n+1}, a^2/k^2q;q)_\infty}. \tag{2.8} $$

Now taking $\delta_r = \frac{(q\sqrt{k}, -q\sqrt{k};q)_r}{(\sqrt{k}, -\sqrt{k};q)_r}(\frac{a^2}{k^2q})^r$ in (1.6) and using (2.8), we get

$$ \gamma_n = \frac{(a^2/k, a/(kq);q)_\infty (k;q)_{2n} (q\sqrt{k}, -q\sqrt{k};q)_n}{(aq, a^2/(qk^2);q)_\infty (a^2/k;q)_{2n} (\sqrt{k}, -\sqrt{k};q)_n} (\frac{a^2}{qk^2})^n. $$

substituting δ_n and γ_n in (1.3), we obtained (2.3).

Proof 2.4. Taking $a = kq^{2n}$ and $b = k/a$ in (1.14), we get

$$ {}_8\phi_7(kq^{2n}, q^{n+1}\sqrt{k}, -q^{n+1}\sqrt{k}, q^n\sqrt{a}, -q^n\sqrt{a}, q^n\sqrt{aq}, -q^n\sqrt{aq}, k/a; q^n\sqrt{k}, -q^n\sqrt{k}, $$

$$ kq^{n+1}/\sqrt{a}, -kq^{n+1}/\sqrt{a}, kq^n\sqrt{(q/a)}, -kq^n\sqrt{(q/a)}, aq^{2n+1}; q, kq/a) $$

$$ = \frac{(kq^{2n+1}, k^2q/a^2;q)_\infty}{(kq/a, k^2q^{2n+1}/a;q)_\infty}, \tag{2.9} $$

$|kq/a| < 1$.

Putting $\delta_r = \frac{(q\sqrt{k}, -q\sqrt{k}, \sqrt{a}, -\sqrt{a}, \sqrt{(aq)}, -\sqrt{(aq)};q)_r}{(\sqrt{k}, -\sqrt{k}, kq\sqrt{(1/a)}, -kq\sqrt{(1/a)}, k\sqrt{(q/a)}, -k\sqrt{(q/a)};q)_r}(\frac{kq}{a})^r$ in (1.6) and applying (2.9), we have

$$ \gamma_n = \frac{(kq, qk^2/a^2;q)_\infty (qk^2/a, k;q)_{2n} (q\sqrt{k}, -q\sqrt{k}, \sqrt{a}, -\sqrt{a}, \sqrt{(aq)};q)_n}{(kq/a, k^2q/a;q)_\infty (kq, aq;q)_{2n} (\sqrt{k}, -\sqrt{k}, kq\sqrt{(1/a)}, -kq\sqrt{(1/a)};q)_n} $$

$$\times \frac{(-\sqrt{(aq)};q)_n}{(k\sqrt{(q/a)},-k\sqrt{(q/a)};q)_n}(\frac{kq}{a})^n.$$

substituting δ_n and γ_n in (1.3), we get (2.4).

Proof 2.5. Setting $a = kq^{2n}$ and $b = k/a$ in (1.12), we get

$$_2\phi_1(kq^{2n},k/a;aq^{2n+1};q,-aq/k) = \frac{(-q;q)_\infty(kq^{2n+1},a^2q^{2n+2}/k;q^2)_\infty}{(-aq/k,aq^{(2n+1)};q)_\infty}. \tag{2.10}$$

Taking $\delta_r = (\frac{-aq}{k})^r$ in (1.6) and using (2.10), we get

$$\gamma_n = \frac{(-q;q)_\infty(kq,a^2q^2/k;q^2)_\infty(k,kq^n;q)_n}{(aq,-aq/k;q)_\infty(kq,a^2q^2/k;q^2)_{2n}}(\frac{-aq}{k})^n.$$

substituting δ_n and γ_n in (1.3), we get (2.5).

3. APPLICATIONS

By using (1.7) in (2.1) and taking $n \to \infty$, we get

$$_8\phi_7(k,q\sqrt{k},-q\sqrt{k},c,a,q\sqrt{a},-q\sqrt{a},a/k;kq,-\sqrt{k},aq/\sqrt{k}\sqrt{a},-\sqrt{a},kq,aq/c;q,q\sqrt{k}/c)$$
$$= \frac{(q\sqrt{k},kq/c,aq,aq/c\sqrt{k};q)_\infty}{(kq,aq/\sqrt{k},q\sqrt{k}/c,aq/c;q)_\infty}. \tag{3.1}$$

Again by making the use of (1.9) in (2.1) and taking $n \to \infty$, we obtain

$$_{10}\phi_9(k,-q\sqrt{k},c,q\sqrt{k},a,q\sqrt{a},-q\sqrt{a},y,z,a^2q/kyz;kq,-\sqrt{k},aq/\sqrt{k},$$
$$aq/c,\sqrt{a},-\sqrt{a},aq/y,aq/z,kyz/a;q,q\sqrt{k}/c)$$
$$= \frac{(aq/c\sqrt{k},aq,kq/c,q\sqrt{k};q)_\infty}{(kq,aq/\sqrt{k},q\sqrt{k}/c,aq/c;q)_\infty}\ {}_6\phi_5(-q\sqrt{k},c,ky/a,kz/a,k,aq/yz;-\sqrt{k},$$
$$kq/c,aq/y,aq/z,kyz/a;q,aq/c\sqrt{k}). \tag{3.2}$$

On using (1.8) in (2.2) and taking $n \to \infty$, we get

$$_2\phi_1(k,k/a;aq;q,a^2/k^2) = \frac{(a/k,a^2q/k;q)_\infty}{(aq,a^2/k^2;q)_\infty}\frac{(1-\sqrt{ak}+\sqrt{a}/(\sqrt{k}-a^{3/2})}{(1-\sqrt{ak})(1+\sqrt{a}/\sqrt{k})}. \tag{3.3}$$

By making the use of (1.7) in (2.3) and then taking $n \to \infty$, we obtain

$$_7\phi_6(a,q\sqrt{a},-q\sqrt{a},a/k,k,q\sqrt{k},-q\sqrt{k};\sqrt{a},-\sqrt{a},kq,a^2/k,\sqrt{k},-\sqrt{k};q,a/kq)$$
$$= \frac{(aq,a^2/k^2q;q)_\infty}{(a^2/k,a/kq;q)_\infty}. \tag{3.4}$$

In (2.3) using (1.9) and then taking $n \to \infty$, we get the following transformation

$$ {}_9\phi_8(k, q\sqrt{k}, -q\sqrt{k}, a, q\sqrt{a}, -q\sqrt{a}, y, z, a^2q/kyz; a^2/k, \sqrt{k}, -\sqrt{k}, \sqrt{a}, -\sqrt{a}, aq/y, $$

$$ aq/z, kyz/a; q, a/kq) $$

$$ = \frac{(aq, a^2/k^2q; q)_\infty}{(a^2/k, a/kq; q)_\infty} \quad {}_6\phi_5(q\sqrt{k}, -q\sqrt{k}, ky/a, kz/a, k, aq/yz; \sqrt{k}, -\sqrt{k}, aq/y, aq/z, kyz/a; q, a^2/k^2q). \tag{3.5} $$

Again in (2.4) making the use of (1.7) and taking $n \to \infty$, we obtain the following summation

$$ {}_{12}\phi_{11}(k, q\sqrt{k}, -q\sqrt{k}, \sqrt{a}, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}, k^2q/a, a, q\sqrt{a}, -q\sqrt{a}, a/k; \sqrt{k}, -\sqrt{k} $$

$$ kq\sqrt{1/a}, -kq\sqrt{1/a}, k\sqrt{q/a}, -k\sqrt{q/a}, \sqrt{a}, -\sqrt{a}, kq, aq, kq; q; k^2q/a^2) $$

$$ = \frac{(kq/a, k^2q/a; q)_\infty}{(kq, k^2q/a^2; q)_\infty}. \tag{3.6} $$

Now use (1.9) in (2.4) and taking $n \to \infty$,we have

$$ {}_{14}\phi_{13}(k, q\sqrt{k}, -q\sqrt{k}, \sqrt{a}, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}, k^2q/a, a, q\sqrt{a}, -q\sqrt{a}, y, z, a^2q/kyz; \sqrt{k}, $$

$$ -\sqrt{k}, kq\sqrt{1/a}, -kq\sqrt{1/a}, k\sqrt{q/a}, -k\sqrt{q/a}, aq, kq, \sqrt{a}, -\sqrt{a}, aq/y, aq/z, kyz/a; q; k^2q/a^2) $$

$$ = \frac{(kq/a, k^2q/a; q)_\infty}{(kq, k^2q/a^2; q)_\infty} \quad {}_{10}\phi_9(q\sqrt{k}, -q\sqrt{k}, \sqrt{a}, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}, ky/a, kz/a, k, aq/yz; $$

$$ \sqrt{k}, -\sqrt{k}, kq\sqrt{1/a}, -kq\sqrt{1/a}, k\sqrt{q/a}, -k\sqrt{q/a}, aq/y, aq/z, kyz/a; q; kq/a). \tag{3.7} $$

By using (1.7) in (2.5), we have

$$ \sum_{n=0}^{\infty} \frac{(k, kq^n, a, q\sqrt{a}, -q\sqrt{a}, a/k; q)_n}{(q, \sqrt{a}, -\sqrt{a}, kq; q)_n (kq, a^2q^2/k; q^2)_{2n}} (-q)^n = \frac{(-aq/k, aq; q)_\infty}{(-q; q)_\infty (kq, a^2q^2/k; q^2)_\infty}. \tag{3.8} $$

and again in (2.5) using (1.9), we get

$$ {}_4\phi_3(ky/a, kz/a, k, aq/yz; aq/y, aq/z, kyz/a; q, -aq/k) $$

$$ = \frac{(-q; q)_\infty (kq, a^2q^2/k; q^2)_\infty}{(-aq/k, aq; q)_\infty} \sum_{n=0}^{\infty} \frac{(k, kq^n, a, q\sqrt{a}, -q\sqrt{a}, y, z, a^2q/kyz; q)_n}{(kq, a^2q^2/k; q^2)_{2n} (q, \sqrt{a}, -\sqrt{a}, aq/y, aq/z, kyz/a; q)_n} (-q)^n. \tag{3.9} $$

Conflict of Interests

The authors declare that there is no conflict of interests.

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