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THE LIMIT OBJECT OF HAUSDORFF SPECTRUM IN THE CATEGORY TLC

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Abstract. The category $\mathscr{H}$ of Hausdorff spectra $\mathscr{X} = \{X_s, \mathfrak{F}, h_{s's}\}$ is introduced by E.I.Smirnov into the discussion by means of an appropriate factorization of the category of Hausdorff spectra Spect$\mathscr{G}$ over the category $\mathscr{G}$ [4]. If $\mathscr{G}$ is a semiabelian complete subcategory of the category TG, then $\mathscr{H}$ is a semiabelian category in the sense of V. P. Palamodov. The direct and inverse spectra of a family of objects are particular cases of Hausdorff spectra – it suffices to put $\mathfrak{F} = |\mathfrak{F}|$, $h_{s's} = q_{F'F}$ in the direct case and $\mathfrak{F} = \{|\mathfrak{F}|\}$, $h_{s's} : X_s \rightsquigarrow X_{s'}$ $(s' \to s)$, $q_{F'F} = i_{|F|} = i_{|\mathfrak{F}|}$ in the inverse case. In this case for each Hausdorff spectrum $\mathscr{X} = \{X_s, \mathfrak{F}, h_{s's}\}$ over $\mathscr{G}$ there exists a unique (up to isomorphism) object of the category $\mathscr{G}$, the H-limit of the Hausdorff spectrum $\mathscr{X}$, which we denote by $\underset{\overrightarrow{\mathfrak{F}}}{\overleftarrow{\lim}}\, h_{s's}X_s$. Thus the additive and covariant functor of the H-limit of a Hausdorff spectrum Haus : $\mathscr{H} \to \mathscr{G}$ is defined and we remark that it is natural in the categorical sense.

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1. Introduction

The study which was carried out in [1], [2] of the derivatives of the projective limit functor acting from the category of countable inverse spectra with values in the category of locally convex spaces made it possible to resolve universally homomorphism questions about a given mapping in terms of the exactness of a certain complex in the abelian category of vector spaces. Later in [3] a broad generalization of the concepts of direct and inverse spectra of objects of an additive semiabelian category $\mathscr{G}$ was introduced: the concept of a Hausdorff spectrum, analogous to the δs-operation in descriptive set theory. This idea is characteristic even for algebraic topology, general algebra, category theory and the theory of generalized functions. The construction of Hausdorff spectra $\mathscr{X} = \{X_s, \mathfrak{F}, h_{s's}\}$ is achieved by successive standard extension of a small category of indices Ω. The category $\mathscr{H}$ of Hausdorff spectra turns out to be additive and semiabelian under a suitable definition of mapping of spectra. In particular, $\mathscr{H}$ contains V. P. Palamodov's category of countable inverse spectra with values in the category TLC of locally convex spaces [1].

2. Main Results

Let $\mathscr{S}$ be the category TLC of locally convex spaces over $\mathbb{R}$ or $\mathbb{C}$ and let $H : |\mathfrak{F}| \to \mathscr{S}$ be a functor of the Hausdorff spectrum $\{X_s, \mathfrak{F}, h_{s's}\}$. We will construct an object of the category TLC, which is defined by means of the Hausdorff spectrum $\{X_s, \mathfrak{F}, h_{s's}\}$; this object will be unique up to isomorphism in the category TLC.

Specifically, the spaces X_s $(s \in |\mathfrak{F}|)$ are locally convex and the morphisms $h_{s's} : X_s \to X_{s'}$ are continuous linear operators, therefore each morphism $\omega_{FF'} : F' \to F$ $(F, F' \in \mathfrak{F})$ generates a continuous linear operator $q_{F'F} : \prod_F X_s \to \prod_{F'} X_{s'}$, defined by the collection of morphisms $(h_{s's})_{F'F}$ in such a way that if $\alpha = (x_s)_{s \in |F|} \in \prod_F X_s$, then $q_{F'F}(\alpha) = \alpha'$, where $\alpha' = (h_{s's} x_s)_{s' \in |F'|} \in \prod_{F'} X_{s'}$.

Theorem 2.1. *There exists a unique (up to isomorphism) object of the category TLC as a limit of Hausdorff spectrum* $\{X_s, \mathfrak{F}, h_{s's}\}$.

Proof. Let us consider the set $\widehat{S} = \bigcup_{\mathfrak{F}} \prod_F X_s$ and let us introduce on $\widehat{S}$ the equivalence relation R in the following manner. If $\alpha = (x_s)_{s \in |F|} \in \prod_F X_s$ and $\alpha' = (x_s)_{s' \in |F'|} \in \prod_{F'} X_{s'}$, where $F, F' \in \mathfrak{F}$, then we will say that $\alpha \sim \alpha' (\operatorname{mod} R)$ if there exist $F^* \in \mathfrak{F}$ and $T^* \in F^*$ such that

$$\omega_{FF^*} : F^* \to F, \quad \omega_{F'F^*} : F^* \to F' \quad \text{and} \quad h_{s^*s} x_s = h_{s^*s'} x_{s'} \quad (s^* \in T^*).$$

Let us show that R is in fact an equivalence relation: reflexivity and symmetry of the relation R are obvious; we will establish its transitivity. Suppose that $\alpha \sim \alpha' (\mathrm{mod}\ R)$ and $\alpha' \sim \alpha'' (\mathrm{mod}\ R)$, where $\alpha'' = (x_{s''})_{s'' \in |F''|} \in \prod_{F''} X_{s''}$, $F'' \in \mathfrak{F}$ and $F^{**} \in \mathfrak{F}$, $T^{**} \in F^{**}$ are such that

$$\omega_{F'F^{**}} : F^{**} \to F', \quad \omega_{F''F^{**}} : F^{**} \to F'' \quad \text{and} \quad h_{s^{**}s'} x_{s'} = h_{s^{**}s''} x_{s''}$$

for $s^{**} \in T^{**}$. Because of the admissibilty of the class $\mathfrak{F}$ there exists $\widehat{F} \in \mathfrak{F}$ such that

$$\omega_{F^*\widehat{F}} : \widehat{F} \to F^*, \quad \omega_{F^{**}\widehat{F}} : \widehat{F} \to F^{**},$$

and there also exists $\widehat{T} \in \widehat{F}$ such that

$$\widehat{T} \subset \left(\omega^{-1}_{F^*\widehat{F}} T^*\right) \cap \left(\omega^{-1}_{F^{**}\widehat{F}} T^{**}\right).$$

Let $\hat{s} \in \widehat{T}$. Then because of the admissibility of the class $\mathfrak{F}$ for $\alpha \sim \alpha' (\mathrm{mod}\ R)$ we obtain

$$h_{\hat{s}s^*}(h_{s^*s} x_s) = h_{\hat{s}s^*}(h_{s^*s'} x_{s'}) = h_{\hat{s}s'} x_{s'},$$

while for $\alpha' \sim \alpha'' (\mathrm{mod}\ R)$ we obtain correspondingly

$$h_{\hat{s}s^{**}}(h_{s^{**}s^*} x_{s^*}) = h_{\hat{s}s^{**}}(h_{s^{**}s'_1} x_{s'_1}) = h_{\hat{s}s'_1} x_{s'_1}.$$

However, because the class $\mathfrak{F}$ is directed in the category $\mathscr{S}^\circ$, we have $h_{\hat{s}s'} \equiv h_{\hat{s}s'_1}$ and, consequently, $x_{s'} = x_{s'_1}$. Thus the following relationship holds for $\hat{s} \in \widehat{T}$:

$$h_{\hat{s}s} x_s = h_{\hat{s}s^*}(h_{s^*s} x_s) = h_{\hat{s}s^{**}}(h_{s^{**}s''} x_{s''}) = h_{\hat{s}s''} x_{s''}.$$

The last assertion means that $\alpha \sim \alpha'' (\mathrm{mod}\ R)$, which it was required to show.

Now let $S = \widehat{S}/R$ and let $\psi : \widehat{S} \to S$ be the canonical mapping. If $\xi, \eta \in S$, then we define the sum $\xi + \eta$ as the class containing the element

$$(h_{s_{12}s_1} x_{s_1} + h_{s_{12}s_2} x_{s_2})_{s_{12} \in |F_{12}|} \in \prod_{F_{12}} X_{s_{12}},$$

where

$$\omega_{F_1F_{12}} : F_{12} \to F_1, \quad \omega_{F_2F_{12}} : F_{12} \to F_2,$$

and

$$q_{F_{12}F_1} = (h_{s_{12}s_1})_{F_{12}F_1}, \quad q_{F_{12}F_2} = (h_{s_{12}s_2})_{F_{12}F_2},$$

$$\alpha = (x_{s_1})_{s_1 \in |F_1|} \in \xi, \quad \beta = (x_{s_2})_{s_2 \in |F_2|} \in \eta.$$

Let us show that the class $\xi+\eta$ is defined in a unique manner, independent of the choice of representatives α and β in the respective classes ξ and η. Let $\alpha'=(x_{s'_1})_{s'_1\in|F'_1|}\in\xi$ and $\beta'=(x_{s'_2})_{s'_2\in|F'_2|}\in\eta$ so that

$$(h_{s'_{12}s'_1}x_{s'_1}+h_{s'_{12}s'_2}x_{s'_2})_{s'_{12}\in|F'_{12}|}\in\prod_{F'_{12}}X_{s'_{12}},$$

where

$$\omega_{F'_1F'_{12}}:F'_{12}\to F'_1,\quad \omega_{F'_2F'_{12}}:F'_{12}\to F'_2.$$

Since $\alpha\sim\alpha'(\operatorname{mod} R)$ and $\beta\sim\beta'(\operatorname{mod} R)$, there exist $F_1^*,F_2^*,F^*\in\mathfrak{F}$ such that the following diagrams are commutative $(i=1,2)$:

(1)
$$\begin{array}{ccc} F_i^* & \xrightarrow{\ \omega_{F_iF_i^*}\ } & F_i \\ {\scriptstyle \omega_{F_i^*F^*}}\uparrow & & \uparrow{\scriptstyle \omega_{F_iF_{12}}} \\ F^* & \xrightarrow{\ \omega_{F_{12}F^*}\ } & F_{12}, \end{array}$$

(2)
$$\begin{array}{ccc} F_i^* & \xrightarrow{\ \omega_{F'_iF_i^*}\ } & F'_i \\ {\scriptstyle \omega_{F_i^*F^*}}\uparrow & & \uparrow{\scriptstyle \omega_{F'_iF'_{12}}} \\ F^* & \xrightarrow{\ \omega_{F'_{12}F^*}\ } & F'_{12}. \end{array}$$

Thus there exist $T_1^*,T_2^*\in F^*$ such that

$$h_{s^*s_i}x_{s_i}=h_{s^*s'_i}x_{s'_i}\quad (s^*\in T_i^*)\quad\text{for}\quad (i=1,2).$$

Then for $T^*\in F^*,T^*\subset T_1^*\cap T_2^*$ the last relationship is satisfied simultaneously for $i=1,2$. Therefore

$$h_{s^*s_1}x_{s_1}+h_{s^*s_2}x_{s_2}=h_{s^*s'_1}x_{s'_1}+h_{s^*s'_2}x_{s'_2}\quad\text{for}\quad s^*\in T^*,$$

and, consequently,

$$\alpha+\beta\sim\alpha'+\beta'(\operatorname{mod} R).$$

Thus the class $\xi+\eta$ is defined in a unique manner. Correctness of the definition of the class $\lambda\xi$ ($\lambda\in\mathbb{R}$ or $\mathbb{C}$, $\xi\in S$) can be shown similarly. This means that the set S is a vector space over $\mathbb{R}$ or $\mathbb{C}$. We continue the construction. For each $F\in\mathfrak{F}$ and $T\in F$ we define the vector space

$$V_F^T=\{\alpha\in\prod_F X_s : x_s=h_{s\hat{s}}x_{\hat{s}},\ s,\hat{s}\in T\}$$

and we show that $\psi V_F^T=\psi V_{F'}^T$, whenever $T\in F$ and $T\in F'$. In fact, let $\xi\in\psi V_F^T$ and $\alpha\in V_F^T$ where $\alpha=(x_s)_{s\in|F|}\in\xi$. We choose an element $\alpha'\in V_{F'}^T$ such that $\alpha'=(x_{s'})_{s'\in|F'|}$ and $x_{s'}=x_s$ $(s\in T,\ s=s')$.

By property (2) of admissibility of the class $\mathfrak{F}$ and its directedness in the category $\mathscr{S}^\circ$ there exist $F^* \in \mathfrak{F}$ and $T^* \in F^*$ such that $\omega^F_{TT^*} \equiv \omega^{F'}_{TT^*}$, where

$$T^* = \omega^{-1}_{FF^*} T = \omega^{-1}_{F'F^*} T, \quad \omega_{FF^*} : F^* \to F, \quad \omega_{F'F^*} : F^* \to F' .$$

Thus $h_{s^*s} x_s = h_{s^*s'} x_{s'}$ $(s^* \in T^*)$ and, consequently, $\alpha \sim \alpha' (\operatorname{mod} R)$. The last assertion means that $\psi\alpha' = \xi$ and $\xi \in \psi V^T_{F'}$. The reverse inclusion is shown similarly, therefore

$$\psi V^T_F = \psi V^T_{F'} \quad (T \in F,\ T \in F') .$$

Put $X_T = \psi V^T_F$ $(F \in \mathfrak{F})$. We now show that the set

$$X = \bigcup_{F \in \mathfrak{F}} \bigcap_{T \in F} X_T$$

is a vector space. In fact, we will show that, if $\xi, \eta \in X$, then $\xi + \eta \in X$. Suppose that $\xi \in X_{T_1}$ for all $T_1 \in F_1$ where $F_1 \in \mathfrak{F}$ and $\eta \in X_{T_2}$ for all $T_2 \in F_2$ where $F_2 \in \mathfrak{F}$; moreover the class $\xi \in X_{T_1}$ and the class $\eta \in X_{T_2}$ if and only if ξ contains elements

$$\alpha_{T_1} = (x_{s_1}(T_1))_{s_1 \in |F_1|} \in \prod_{F_1} X_{s_1} \quad (T_1 \in F_1)$$

such that $x_{\hat{s}_1}(T_1) = \hat{h}_{\hat{s}_1 s_1} x_{s_1}(T_1)$ for $s_1, \hat{s}_1 \in T_1$, and η contains elements

$$\alpha_{T_2} = (x_{s_2}(T_2))_{s_2 \in |F_2|} \in \prod_{F_2} X_{s_2} \quad (T_2 \in F_2)$$

such that $x_{\hat{s}_2}(T_2) = \hat{h}_{\hat{s}_2 s_2} x_{s_2}(T_2)$ for $s_2, \hat{s}_2 \in T_2$. Let $F_{12} \in \mathfrak{F}$ and $\omega_{F_1F_{12}} : F_{12} \to F_1$, $\omega_{F_2F_{12}} : F_{12} \to F_2$. We will show that the class $\xi + \eta$, which, according to the definition, contains the elements

$$(h_{s_{12}s_1} x_{s_1}(T_1) + h_{s_{12}s_2} x_{s_2}(T_2))_{s_{12} \in |F_{12}|} \quad (T_1 \in F_1,\ T_2 \in F_2) ,$$

satisfies the relationship

$$\xi + \eta \in \bigcap_{T_{12} \in F_{12}} X_{T_{12}} .$$

In fact, suppose that $T_{12} \in F_{12}$ and $T_1 \in F_1$, $T_2 \in F_2$ are such that

$$\omega^{-1}_{F_1F_{12}} T_1 = \omega^{-1}_{F_2F_{12}} T_2 = T_{12}$$

and

$$(h_{s_{12}s_1} x_{s_1}(T_1))_{s_{12} \in |F_{12}|} \in \prod_{F_{12}} X_{s_{12}} , \quad (h_{s_{12}s_2} x_{s_2}(T_2))_{s_{12} \in |F_{12}|} \in \prod_{F_{12}} X_{s_{12}} .$$

Further, let $s_{12}, \hat{s}_{12} \in T_{12}$ and $\omega_{s_{12}\hat{s}_{12}} : \hat{s}_{12} \to s_{12}$ be such that

$$x_{s_1}(T_1) \in X_{s_1} , \quad x_{\hat{s}_1}(T_1) \in X_{\hat{s}_1}$$

and

$$h_{s_{12}s_1}x_{s_1}(T_1) \in X_{s_{12}}\,, \quad h_{\hat{s}_{12}\hat{s}_1}x_{\hat{s}_1}(T_1) \in X_{\hat{s}_{12}}\,,$$

where $s_1, \hat{s}_1 \in T_1$. Because the class T_1 is directed in the category Ω there exists an element $s_1^* \in T_1$ such that $\omega_{s_1^* s_1} : s_1 \to s_1^*$, $\omega_{s_1^* \hat{s}_1} : \hat{s}_1 \to s_1^*$, and moreover, by assumption, the relations

$$x_{s_1}(T_1) = \hat{h}_{s_1 s_1^*}x_{s_1^*}(T_1) \quad \text{and} \quad x_{\hat{s}_1}(T_1) = \hat{h}_{\hat{s}_1 s_1^*}x_{s_1^*}(T_1)$$

hold. Now it follows from the specification of a Hausdorff spectrum that the following diagram is commutative:

(3)
$$\begin{array}{ccccc}
X_{s_1^*} & \xrightarrow{\hat{h}_{s_1 s_1^*}} & X_{\hat{s}_1} & \xrightarrow{h_{\hat{s}_{12}\hat{s}_1}} & X_{\hat{s}_{12}} \\
 & \searrow^{\hat{h}_{s_1 s_1^*}} & & & \uparrow \hat{h}_{\hat{s}_{12}s_{12}} \\
 & & X_{s_1} & \xrightarrow{h_{s_{12}s_1}} & X_{\hat{s}_{12}}
\end{array}$$

Consequently, the relation

$$h_{\hat{s}_{12}\hat{s}_1}x_{\hat{s}_1}(T_1) = \hat{h}_{\hat{s}_{12}s_{12}}\left(h_{s_{12}s_1}x_{s_1}(T_1)\right)$$

is satisfied for $s_{12}, \hat{s}_{12} \in T_{12}$. This shows that $(h_{s_{12}s_1}x_{s_1}(T_1))_{s_{12}\in|F_{12}|} \in \xi \in X_{T_{12}}$. It can be shown similarly that $(h_{s_{12}s_2}x_{s_2}(T_2))_{s_{12}\in|F_{12}|} \in \eta \in X_{T_{12}}$, from which it follows that $\xi + \eta \in X_{T_{12}}$. But $T_{12} \in F_{12}$ was chosen arbitrarily, therefore

$$\xi + \eta \in \bigcap_{T_{12}\in F_{12}} X_{T_{12}}\,,$$

which it was required to establish.

It is clear that $\lambda\xi \in X$ ($\lambda \in \mathbb{R}$ or $\mathbb{C}$) and the appropriate axioms are satisfied, so that X is a vector space. Now we provide X with a locally convex topology τ_H in the following manner.

Suppose that the locally convex spaces X_s ($s \in |\mathfrak{F}|$) have locally convex topologies τ_s and let $F_1 \in \mathfrak{F}$, $F_2 \in \mathfrak{F}$ and $T \in F_1$, $T \in F_2$. Then the vector space $V_{F_i}^T$ ($i = 1,2$) is provided with the upper bound σ_{T,F_i} of the preimages of the topologies of the spaces X_s ($s \in T$) under the projections $\pi_s^{F_i} : \prod_{F_i} X_s \to X_s$ (the weakest locally convex topology on $\prod_{F_i} X_s$ which is continuously embedded in each of the topologies $(\pi_s^{F_i})^{-1}\tau_s$ ($s \in T$) respectively). We will show that the images $\psi\sigma_{T,F_1}$ and $\psi\sigma_{T,F_2}$ generate one and the same locally convex topology σ_T on the vector space X_T. In fact, if the W_{s_i} are absolutely convex neighbourhoods of zero in the spaces X_{s_i}, where $s_i \in T$, $i = 1,2,\ldots,m$, then according to the constructions of equivalence defined above we obtain

$$\psi\left(\bigcap_{i=1}^{m}(\pi_{s_i}^{F_1})^{-1}W_{s_i}\right) = \psi\left(\bigcap_{i=1}^{m}(\pi_{s_i}^{F_2})^{-1}W_{s_i}\right),$$

which by the linearity of the restriction of ψ to $V^T_{F_i}$ $(i = 1,2)$ implies the identity

$$\psi\sigma_{T,F_1} = \psi\sigma_{T,F_2} = \sigma_T .$$

However, $F_1, F_2 \in \mathfrak{F}$ were chosen arbitrarily for given T, therefore the locally convex topology σ_T on X_T is defined in a unique manner.

Now let us denote by $\widehat{\sigma}_T$ the strongest translation invariant topology on S which induces on X_T the topology σ_T $(T \in F)$. Next we define on the space S for each $F \in \mathfrak{F}$ the topology $\widehat{\sigma}_{(F)}$ which is the upper bound of the topologies $\widehat{\sigma}_T$ $(T \in F)$. The space S with the topology $\widehat{\sigma}_{(F)}$ $(F \in \mathfrak{F})$ will be a topological vector group [8].

Now for each $F \in \mathfrak{F}$ we denote by $X_{(F)}$ the space X with the topology $\sigma_{(F)}$ induced by the topology $\widehat{\sigma}_{(F)}$. Finally, we provide X with the locally convex topology τ_H which is the strongest locally convex topology on X for which all the embeddings of the spaces $X_{(F)}$ $(F \in \mathfrak{F})$ in the space (X, τ_H) are continuous. The theorem is proved.

Definition 2.2. We call the vector space X provided with the topology τ_H the *H-limit of the Hausdorff spectrum* $\{X_s, \mathfrak{F}, h_{s's}\}$ *over the category TLC* and we write

$$X = \underset{\mathfrak{F}}{\underrightarrow{\overleftarrow{\lim}}}\, h_{s's} X_s .$$

If $\{X_s, \mathfrak{F}, i_{s's}\}$ is a simple Hausdorff spectrum corresponding to a Suslin limit (Y, τ^*) [10], so that

$$Y = \bigcup_{F\in\mathfrak{F}} \bigcap_{s\in F} X_s ,$$

then it is not difficult to show that (Y, τ^*) is isomorphic to the H-limit of the simple Hausdorff spectrum

$$(X, \tau_H) = \underset{\mathfrak{F}}{\underrightarrow{\overleftarrow{\lim}}}\, i_{s's} X_s .$$

Along with the topology τ_H we will consider on the space X the locally convex topology $\widehat{\tau}_H$ which is the strongest locally convex topology on X for which all the embeddings into the space $(X, \widehat{\tau}_H)$ of the spaces $X_F = \bigcap_{T\in F} X_T$ $(F \in \mathfrak{F})$ provided with the projective topology are continuous. We will call the vector space X provided with the topology $\widehat{\tau}_H$ the *strong H-limit of the Hausdorff spectrum* $\{X_s, \mathfrak{F}, h_{s's}\}$ *over the category TLC*. It is clear that $\tau_H \le \widehat{\tau}_H$. Conditions for the coincidence of the topologies τ_H and $\widehat{\tau}_H$ will be established below.

Let $\mathscr{X} = \{X_s, \mathfrak{F}, h_{s's}\}$ be a Hausdorff spectrum over the category $\mathscr{G}$.

Definition 2.3. We will call an object Z of the category $\mathscr{G}$ a *categorical H-limit of the Hausdorff spectrum $\mathscr{X}$ over $\mathscr{G}$* if for any objects $A, B \in \mathscr{G}$ and mappings of spectra

$$A \xrightarrow{a} \mathscr{X} \xrightarrow{b} B$$

there exists a unique sequence in $\mathscr{G}$

$$A \xrightarrow{\alpha} Z \xrightarrow{\beta} B$$

such that the diagram

(4)
$$\begin{array}{ccc} A & \xrightarrow{a} & \mathscr{X} \\ \downarrow{\scriptstyle \alpha} & & \downarrow{\scriptstyle b} \\ Z & \xrightarrow{\beta} & B \end{array}$$

is commutative in the category Spect$\mathscr{G}$.

The concepts of projective and inductive limits over the category $\mathscr{G}$ are special cases of categorical H-limits. For example, let $\mathscr{X}$ be an inverse spectrum of objects from $\mathscr{G}$. Then (Lim) holds and moreover any object X_s from $\mathscr{X}$ can be taken for $B \in \mathscr{G}$ with the identity morphism $b_s : X_s \to X_s$ forming the mapping of spectra $b^s : \mathscr{X} \to X_s$ $(s \in |F|)$. Thus the diagram

(5)
$$\begin{array}{ccc} A & \xrightarrow{a} & \mathscr{X} \\ \downarrow{\scriptstyle \alpha} & & \downarrow{\scriptstyle b} \\ Z & \xrightarrow{\beta} & \mathscr{X} \end{array}$$

is commutative, where $b = (b^s)$, $\beta = (\beta^s)$, $\beta^s : Z \to X_s$ $(s \in |F|)$ and b is the identity morphism of the category Spect$\mathscr{G}$. Therefore the diagram

(6)
$$\begin{array}{ccc} A & & \\ \downarrow{\scriptstyle a} & \overset{a}{\searrow} & \\ Z & \xrightarrow{\beta} & \mathscr{X} \end{array}$$

is commutative for any object $A \in \mathscr{G}$.

The categorical H-limit of a Hausdorff spectrum (the functor Haus) exists in any semiabelian category $\mathscr{G}$ with direct sums and products (for example, the category of vector spaces L, the category TLG of topological vector groups, the category TLC of locally convex spaces).

We provide an example where the categorical H-limit is defined in a unique manner (up to categorical isomorphism).

Example 2.4. Let A be some s-set contained in a separated topological space T so that

$$\widetilde{A} = \bigcup_{F \in \mathfrak{F}} \bigcap_{s \in |F|} R_s$$

and the family $\{\bigcap_{s\in|F|} R_s\}_{F\in\mathfrak{F}}$ forms a fundamental system of nonempty compact subsets of $\widetilde{A}$ and moreover $H(\widetilde{A}) : |\mathfrak{F}| \to \mathscr{G}$ is a contravariant functor of the simple Hausdorff spectrum and $\mathscr{G} = \mathrm{Ord}_\beta(T)$. If $P \subset T$ we will denote by $\mathscr{K}(P)$ the vector space of functions f whose supports are contained in P and by L the family of vector spaces $\mathscr{K}(P)$ $(P \subset T)$ which is partially ordered by inclusion; let $\mathscr{L} = \mathrm{Ord}\, L$. We put

$$\mathscr{K}(\widetilde{A}) = \bigcup_{F \in \mathfrak{F}} \bigcap_{s \in |F|} \mathscr{K}(R_s)$$

and show that $\mathscr{K}(\widetilde{A})$ is the categorical H-limit of the Hausdorff spectrum

$$\mathscr{K} = \{\mathscr{K}(R_s), \mathfrak{F}, i_{s's}\}$$

over the category $\mathscr{L}$. Let $A, B \in \mathscr{L}$ and

$$A \xrightarrow{\ a\ } \mathscr{K} \xrightarrow{\ b\ } B\,.$$

The morphism $A \xrightarrow{\ a\ } \mathscr{K}$ signifies that there exists $F \in \mathfrak{F}$ such that $A \xrightarrow{\ a_s\ } \mathscr{K}(R_s)$ $(s \in |F|)$, and moreover a_s is the identity embedding of A in $\mathscr{K}(R_s)$. Therefore $A \subset \bigcap_{s\in|F|} \mathscr{K}(R_s)$ and, consequently, there exists a unique identity embedding

$$\alpha : A \to \mathscr{K}(\widetilde{A})\,.$$

Similarly, if $\mathscr{K} \xrightarrow{\ b\ } B$, then for each $F \in \mathfrak{F}$ there exist morphisms $b_{s_F^*} : \mathscr{K}(R_{s_F^*}) \to B$ (for individual $s_F^* \in F$ and for each $F \in \mathfrak{F}$), where $b_{s_F^*}$ is the identity embedding of $\mathscr{K}(R_{s_F^*})$ in B. Thus, $\bigcup_{F\in\mathfrak{F}} \mathscr{K}(R_{s_F^*}) \subset B$ and, consequently, the unique identity embedding

$$\mathscr{K}(\widetilde{A}) = \bigcap_{(s_F)} \bigcup_{F \in \mathfrak{F}} \mathscr{K}(R_{s_F}) \xrightarrow{\ \beta\ } B$$

exists. Commutativity is obvious.

At the same time, if $Z \in \mathscr{L}$ and satisfies (Lim), then $A \subset Z$: in particular, this holds for $A = \bigcap_{s\in F} \mathscr{K}(R_s)$ and moreover for any $F \in \mathfrak{F}$, therefore $\mathscr{K}(\widetilde{A}) \subset Z$. Now if we set $E = \bigcup_{F\in\mathfrak{F}} \mathscr{K}(R_{s_F})$ for some sequence $(s_F)_{\mathfrak{F}}$ (recall that $|\mathfrak{F}|$ is an at most countable set, therefore among the s_F $(F \in \mathfrak{F})$ the set of distinct elements is at most countable), then $E \subset \mathscr{K}(\bigcup_{\mathfrak{F}} R_{s_F})$. Thus, for $B = \mathscr{K}(\bigcup_{\mathfrak{F}} R_{s_F})$, choosing any sequence $(s_F)_{\mathfrak{F}}$, we obtain $Z \subset \bigcap_{(s_F)} \mathscr{K}(\bigcup_{\mathfrak{F}} R_{s_F})$. However $\bigcap_{(s_F)} \mathscr{K}(\bigcup_{\mathfrak{F}} R_{s_F}) = \mathscr{K}(A)$, since if

$\varphi \in \mathscr{K}(\bigcup_{\mathfrak{F}} R_{s_F})$ for each $(s_F)_{\mathfrak{F}}$, then $\operatorname{supp}\varphi \subset \bigcap_{(s_F)} \bigcup_{\mathfrak{F}} R_{s_F} = \widetilde{A}$ or $\varphi \in \mathscr{K}(\widetilde{A})$. Therefore, $Z = \mathscr{K}(\widetilde{A})$, which also implies the uniqueness of the categorical H-limit over Z.

On the other hand, an s-set is itself the unique categorical H-limit

$$\widetilde{A} = \bigcup_{F\in\mathfrak{F}} \bigcap_{s\in F} R_s$$

over the category $\mathscr{G} = \operatorname{Ord}\beta(T)$.

Example 2.5. In the category $\mathscr{E}ns$ of sets the H-limit of a Hausdorff spectrum exists and is the unique (up to categorical isomorphism) categorical H-limit.

Example 2.6. Let $\eta = \{x_n, \mathfrak{F}, \leq\}$, $\mathbb{N} = |\mathfrak{F}|$ be a Hausdorff spectrum over the category $\mathscr{G} = \operatorname{Ord}\mathbb{R}$ ($\mathbb{R}$ is the set of real numbers). We put $x^* = \sup\limits_{F\in\mathfrak{F}} \inf\limits_{n\in F} x_n$ and show that x^* is the categorical H-limit. Let $a, b \in \mathbb{R}$ and

$$a \to \eta \to b\,,$$

where the morphism $a \to \eta$ means that there exists $F^* \in \mathfrak{F}$ such that $a \leq x_n$ $(n \in F^*)$, while the morphism $\eta \to b$ means that for each $F \in \mathfrak{F}$ there exists x_{n_F} (for individul $n_F \in F$ and for each $F \in \mathfrak{F}$) such that $x_{n_F} \leq b$ $(F \in \mathfrak{F})$. Thus, $a \leq \inf\limits_{n\in F^*} x_n$ and $a \leq x^*$; at the same time, for $F \in \mathfrak{F}$ we have $\inf\limits_{n\in F} x_n \leq b$ and, consequently, $x^* \leq b$ – this implies that x^* is the categorical H-limit of the Hausdorff spectrum η. We now establish the uniqueness of the categorical H-limit. Let $a \leq z \leq b$ $(a, b \in \mathbb{R})$. Since we can put $a = \inf\limits_{F} x_n$ for any $F \in \mathfrak{F}$, then $x^* \leq z$; if in addition $b = \sup\limits_{\mathfrak{F}} x_{n_F}$ for all possible choices of $(n_F)_{\mathfrak{F}}$, then $z \leq \inf\limits_{(n_F)} \sup\limits_{\mathfrak{F}} x_{n_F}$. However,

$$\sup_{F\in\mathfrak{F}} \inf_{n\in F} x_n = \inf_{(n_F)} \sup_{\mathfrak{F}} x_{n_F}\,, \tag{j}$$

therefore $x^* = z$. The identity (j) is established as follows: clearly,

$$\sup_{F\in\mathfrak{F}} \inf_{n\in F} x_n \leq \inf_{(n_F)} \sup_{\mathfrak{F}} x_{n_F}\,;$$

let us suppose that equality does not hold, so that for some $\delta > 0$

$$\inf_{n\in F} x_n < \inf_{(n_F)} \sup_{\mathfrak{F}} x_{n_F} - \delta \quad \forall F \in \mathfrak{F}\,.$$

Thus, for each $F \in \mathfrak{F}$ there exists $n_F^* \in F$ such that

$$x_{n_F^*} < \inf_{(n_F)} \sup_{\mathfrak{F}} x_{n_F} - \delta$$

or

$$\sup_{\mathfrak{F}} x_{n_F^*} \leq \inf_{(n_F)} \sup_{\mathfrak{F}} x_{n_F} - \delta\,;$$

the last inequality implies that $\delta \le 0$, which is impossible.

Example 2.7. Let L be any complete lattice, $\mathscr{L} = \operatorname{Ord} L$ and let $\xi = \{x_\alpha, \mathfrak{F}, \le\}$ be a Hausdorff spectrum over the category $\mathscr{L}$. The following proposition gives a sufficient condition for the uniqueness of the categorical H-limit.

Proposition 2.8. Let L be a complete totally distributive lattice and let $\xi = \{x_\alpha, \mathfrak{F}, \le\}$ be a Hausdorff spectrum over the category $\mathscr{L}$. Then there exists a unique categorical H-limit

$$x^* = \sup_{F \in \mathfrak{F}} \inf_{\alpha \in F} x_\alpha .$$

In particular, the categorical H-limit is unique in any complete chain or closed sublattice of a direct product of complete chains [5].

Definition 2.9. Let $X = \overset{\longleftarrow}{\underset{\mathfrak{F}}{\underrightarrow{\lim}}} h_{s's} X_s$ where $\{X_s, \mathfrak{F}, h_{s's}\}$ is a Hausdorff spectrum. We will say that the H-limit X is *regular* if the following conditions are satisfied: for each $F \in \mathfrak{F}$ the space X is closed in $(S, \widehat{\sigma}_{(F)})$ and convergence of the net $a_\gamma \in X$ $(\gamma \in P)$ in the spaces $(S, \widehat{\sigma}_T)$ $(T \in F)$ implies its convergence in $(S, \widehat{\sigma}_{(F)})$.

We also note that the constructions of the H-limit in the category TLC which were introduced above can be repeated with no substantial changes for the case of a Hausdorff spectrum over some semiabelian subcategory of the category TG of topological groups.

The projective and inductive limits of separated spaces X_s are special cases of the regular H-limit. Moreover, if for X each projective topology $\sigma_{(F)}$ $(F \in \mathfrak{F})$ is complete, then the H-limit is regular. Sometimes we will speak of a regular Hausdorff spectrum $\mathscr{X}$ rather than a regular H-limit.

3. Applications

Let $\{\mathscr{S}_U, \rho_{UV}\}$ be a presheaf of abelian groups over a topological space $\mathscr{D}$, Ω a nonempty partially ordered set and $\mathfrak{F}$ an admissible class for Ω (we may assume without loss of generality that $\Omega = |\mathfrak{F}|$). Let us denote by $\hat{H}(\mathscr{S})$ a covariant functor from $\operatorname{Ord}\Omega$ to $\operatorname{Ord}\mathscr{U}$, where $\mathscr{U}$ is a base of open sets in $\mathscr{D}$, and by $\check{H}(\mathscr{S})$ a contravariant functor from $\operatorname{Ord}\mathscr{U}$ to the category of abelian groups so that an abelian group $\mathscr{S}_U$ is defined for each $U \in \mathscr{U}$ and a homomorphism $\rho_{UV} : \mathscr{S}_U \to \mathscr{S}_V$ is defined for each pair $U \subset V$. Then $H = \check{H}(\mathscr{S}) \circ \hat{H}(\mathscr{S})$ is a contravariant functor of the Hausdorff spectrum $\mathscr{X}(\mathscr{S}) = \{\mathscr{S}_{U_s}, \mathfrak{F}, \rho_{U_{s'} U_s}\}$, which we will call the *Hausdorff spectrum associated with the presheaf* $\{\mathscr{S}_U, \rho_{UV}\}$. Let X be the H-limit of the Hausdorff spectrum $\mathscr{X}(\mathscr{S})$ in the category of abelian groups and let

$$A = \bigcap_{F \in \mathfrak{F}} \bigcup_{s \in |F|} U_s \,.$$

Proposition 3.1. Let $\mathscr{S}$ be the sheaf of germs of holomorphic functions on an open set $\mathscr{D} \subset \mathbb{C}^n$, associated with the presheaf $\{\mathscr{S}_U, \rho_{UV}\}$, and let $\mathscr{X}(\mathscr{S}) = \{\mathscr{S}_{U_s}, \mathfrak{F}, \rho_{U_{s'}U_s}\}$ be the associated true Hausdorff spectrum. Then the H-limit of the Hausdorff spectrum $\mathscr{X}(\mathscr{S})$ is isomorphic to the vector space of sections $\Gamma(A, \mathscr{S})$ of the sheaf $\mathscr{S}$ over the set A.

Proof. By the conditions relating to $\{\mathscr{S}_U, \rho_{UV}\}$, we may put $\mathscr{S}_U = \Gamma(U, \mathscr{S})$ $(U \in \mathscr{U})$. Further, let

$$X = \underset{\mathfrak{F}}{\overleftarrow{\underrightarrow{\lim}}} \rho_{U_{s'}U_s} \Gamma(U_s, \mathscr{S}) \,,$$

so that

$$X = \bigcup_{F \in \mathfrak{F}} \bigcap_{T \in F} \psi(V_F^T) \,.$$

If $x \in X$, there exists $F \in \mathfrak{F}$ such that $x \in \psi(V_F^T)$ $(T \in F)$, that is to say, there exists a selection

$$\xi(T) = (f_s^T)_{s \in |F|}$$

such that $\psi(f_s^T) = x$ for each $T \in F$. For any $U \in \mathscr{U}_z$ $(z \in \mathscr{D})$ the homomorphism $\rho_{zU} : \Gamma(U, \mathscr{S}) \to \mathscr{S}_z$ generates for $f \in \Gamma(U, \mathscr{S})$ the set of points

$$\rho_U(f) = \bigcup_{z \in U} \rho_{zU}(f) \subset \mathscr{S} \,,$$

therefore let us put

$$\rho_x^T = \bigcup_{s \in T} \rho_{U_s}(f_s^T) \,;$$

it is clear that ρ_x^T generates the section f^T on the open set $U_T = \bigcup_{s \in T} U_s$, since the correspondence

$$z \in U_T \overset{f^T}{\longmapsto} \rho_x^T \cap \mathscr{S}_z \subset \mathscr{S}$$

is single-valued and continuous. Moreover, if $\rho_{UV} : \rho_V(g) \mapsto \rho_U(f)$, then $\rho_U(f) \subset \rho_V(g)$, so let us put

$$\rho_x^\xi = \bigcup_{F^* \succ F} \bigcup_{\substack{s^* \in |F^*| \\ s \in T}} \rho_{U_{s^*}U_s}(\rho_{U_s}(f_s^T)) \,,$$

where necessarily

$$\rho_{U_{s^*}U_s}(\rho_{U_s}(f_s^T)) = \rho_{U_{s^*}U_s}(\rho_{U_s}(f_s^{T'})) \quad (T, T' \in F) \,.$$

Let us put

$$U_{\rho_x} = \bigcap_{\xi} U_{\rho_x^\xi}, \quad \text{where} \quad U_{\rho_x^\xi} \subset \bigcup_{s\in|F|} U_s;$$

in this connection we have in particular,

$$\rho_{U_s}(f_s^T) \cap \rho_{U_s}(f_s^{T'}) \supset \rho_{U_{s^*}U_s}(\rho_{U_s}(f_s^T)).$$

It is also clear that for each ξ the correspondence

$$z \in U_{\rho_x^\xi} \mapsto \rho_x^\xi \cap \mathscr{S}_z$$

is single-valued and continuous. Although, in general, it is not guaranteed that $U_{\rho_x} \neq \emptyset$, we will show nevertheless that $U_{\rho_x} \supset A$ under the conditions of the proposition, specifically because the H-limit of the Hausdorff spectrum $\mathscr{X}(\mathscr{S})$ is true. Let the selection $\xi(T) = (f_s^T)_{s\in|F|}$ $(T \in F)$ generating the element $x \in X$ be fixed. Then because the Hausdorff spectrum $\mathscr{X}(\mathscr{S})$ is true we may assume that $f_s^{T_1} = f_s^{T_2}$ $(s \in T_1 \cap T_2)$ and, consequently, there exists $\xi = (f_s)_{s\in|F|} \in \bigcap_{T\in F} V_F^T$ such that

$$x \in \psi((f_s)_{|F|}) \quad \text{and} \quad f_{s'} = \rho_{U_{s'}U_s}(f_s) \quad (s, s' \in |F|).$$

It is clear that $\rho_x^\xi = \bigcup_{s\in|F|} \rho_{U_{s'}U_s}(f_s)$. Now let $z \in A$. Then $z \in U_{\rho_x^\xi}$ for any $\xi(F)$ $(F \in \mathfrak{F})$ and, moreover,

$$\rho_x^\xi(z) = \rho_x^\xi \cap \mathscr{S}_z = \rho_{zU_s}(f_s) \quad \text{for} \quad z \in U_s \quad (s \in |F|).$$

Let us show that $\rho_x^\xi(z) = \rho_x^{\xi'}(z)$ for any ξ, ξ'. In fact, let $\xi = (f_s)_{|F|}$, $\xi' = (f'_{s'})_{|F'|}$ and $x = \psi(\xi)$, $x' = \psi(\xi')$. Since $\xi \sim \xi'$, there exists $F^* \in \mathfrak{F}$, where $F^* \succ F'$ and $F^* \succ F'$, such that for each $T^* \in F^*$ we can find $T \in F$ and $T' \in F'$ such that

$$\omega_{TT^*} : T^* \to T, \quad \omega_{T'T^*} : T^* \to T' \quad \text{and} \quad \rho_{U_{s^*}U_s}(f_s) = \rho_{U_{s^*}U_{s'}}(f'_{s'}),$$

where $s^* \in T^*$. However, $z \in \bigcup_{s^*\in|F^*|} U_{s^*}$, and so it remains to choose $s_0^* \in |F^*|$, such that

$$z \in U_{s_0^*} \quad \text{and} \quad \rho_{zU_s}(f_s) = \rho_{zU_{s'}}(f'_{s'}) \quad (s^* \to s, s^* \to s').$$

Thus $z \in U_{\rho_x}$. Furthermore, let us put $x(z) = \rho_x^\xi(z)|_A$, so that $x(z)$ is a section of $\mathscr{S}$ on A, $x(z) \in \Gamma(A, \mathscr{S})$. In this way we have constructed a morphism $\mathscr{H} : X \to \Gamma(A, \mathscr{S})$. Given $f_A = \mathscr{H}(x)$, $f_A = \mathscr{H}(y)$, let us prove that $x = y$. In fact, at each point $z \in A$ there exists an open ball $B(z, \varepsilon)$ of the local homeomorphism $\pi : \mathscr{S} \to \mathscr{D}$ at the point $f_A(z)$. Let us put $U = \bigcup_{z\in A} B(z, \varepsilon/2)$ and determine the section $f_z \in \Gamma(B(z, \varepsilon/2), \mathscr{S})$ passing through the point $s = f_A(z) \in \mathscr{S}$ such that

$$f_z|_A = f_A|_{B(z,\varepsilon/2)}$$

(we note that $\varepsilon = \varepsilon(z)$). Let

$$B_{ij} = B(z_i, \varepsilon_i/2) \cap B(z_j, \varepsilon_j/2), \quad B_{ij} \cap A \neq \emptyset, \quad z_0 \in B_{ij} \cap A$$

for some $z_i, z_j \in A$. Then $f_{z_i}(z_0) = f_{z_j}(z_0)$, and, consequently, there is an open ball $B_0 \subset B(z_0, \varepsilon_0/2)$ of the local homeomorphism at the point $s_0 = f_{z_i}(z_0)$ such that $B_0 \subset B_{ij}$ and $f_{z_i}|_{B_0} = f_{z_j}|_{B_0}$. However, because of the isomorphism $\Gamma(B_{ij}, \mathscr{S}) \to \mathscr{S}_{B_{ij}}$ the holomorphic functions f_{z_i} and f_{z_j} coincide on the connected open set B_{ij} [9, Theorem A6]. The last observation means that $f_{z_i}|_{B_{ij}} = f_{z_j}|_{B_{ij}}$. Now suppose that

$$B_{ij} \cap A = \emptyset, \quad \text{but} \quad B'_{ij}(\varepsilon_i, \varepsilon_j) \cap A \neq \emptyset, \quad z' \in B'_{ij} \cap A.$$

Clearly, we will obtain by similar reasoning $f'_{z_i}|_{B'_{ij}} = f'_{z_j}|_{B'_{ij}}$. But we have $f'_{z_i}|_{B(z_i,\varepsilon_i/2)} = f_{z_i}$ and $f'_{z_j}|_{B(z_j,\varepsilon_j/2)} = f_{z_j}$, so that $f_{z_i}|_{B_{ij}} = f_{z_j}|_{B_{ij}}$ (in the case where $B_{ij} \neq \emptyset$). Now there remains the third possibility for $B_{ij} \neq \emptyset$, namely when $B'_{ij} \cap A = \emptyset$. In this case the sections f_{z_i}, f_{z_j} on B_{ij} do not necessarily coincide, therefore let us put $M = \overline{\bigcup B_{ij}}$, where the bar denotes closure in $\mathbb{C}^n$ and the union is taken over all B_{ij} of this third type. It is clear that $M \cap A = \emptyset$, since in the contrary case for $z^* \in M \cap A$ there would exist B^*_{ij} of the third type such that $z^* \in B^{*\prime}_{ij}$, which is impossible by construction. Let us put $U(f_A) = U \backslash M$, so that $U(f_A) \supset A$ and $U(f_A)$ is an open subset of $\mathbb{C}^n$. Then there exists $f \in \Gamma(U(f_A), \mathscr{S})$ such that $f|_A = f_A$ and, moreover,

$$f|_{U(f_A) \cap B(z,\varepsilon/2)} = f_z|_{U(f_A) \cap B(z,\varepsilon/2)} \quad (z \in A);$$

also the section on $U(f_A)$ of f with the property $f|_A = f_A$ is uniquely determined (nevertheless, ϕ_A, the corresponding holomorphic function on A, is extended holomorphically to $U(f_A)$ in a manner which, in general, is not unique).

Now if $\psi(\xi) = x$, $\psi(\eta) = y$, it follows from the fact that the family of open sets $\{\bigcup_{s \in |F|} U_s\}_{F \in \mathfrak{F}}$ is fundamental for A that there exists $F^* \in \mathfrak{F}$ such that $U(f_A) \supset \bigcup_{F^*} U_{s^*}$, and moreover by construction

$$\rho_x^{\xi}|_{\bigcup_{F^*} U_{s^*}} = \rho_y^{\eta}|_{\bigcup_{F^*} U_{s^*}}.$$

The last assertion means that $\xi \sim \eta$ and, consequently, $x = y$. Moreover, the fact that $\{\bigcup_F U_s\}_{F \in \mathfrak{F}}$ is fundamental for A and the constructions carried out above allow us to conclude that $\mathscr{H} : X \to \Gamma(A, \mathscr{S})$ is an isomorphism. The proposition is proved.

4. Conclusions

Numerical spectra are widely used in the solution of the actual problems concerning sheaves, their variations - vector bundles and the moduli spaces of this objects [7]. On the other hand, in the previous section we established connection between Hausdorff spectra and sheaves, which play a key role in modern algebraic geometry and related areas. Thus, a new activity for development of the theory of sheaves (vector bundles) on algebraic varieties by means of ideology of the Hausdorff spectra (the spectra of the non-numerical nature) is of serious interest.

Conflict of Interests

The authors declare that there is no conflict of interests.

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