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PERISTALTIC PUMPING OF JEFFREY FLUID WITH VARIABLE VISCOSITY IN A TUBE UNDER THE EFFECT OF MAGNETIC FIELD

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Abstract: In this paper, the peristaltic flow of a Jeffrey fluid with variable viscosity under the effect of magnetic field is investigated under the assumptions of long wavelength and low Reynolds number. The flow is examined in a wave frame of reference moving with velocity of the wave. The problem is formulated using perturbation expansion in terms of viscosity parameter α. The governing equations are developed upto first-order in the viscosity parameter α. The zero order system yields the classical Poiseuille flow when the Hartmann number M tends to zero and λ_1 tends to zero. We simplify a complicated group of products of Bessel functions by approximating the polynomial. The effects of Hartmann number M, viscosity parameter α and material parameter λ_1 on the pumping

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characteristics and friction force are discussed in detail through the graphs.

Keywords: Hartmann number; Jeffrey fluid; peristalsis; variable viscosity.

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1. Introduction

The analysis of flow dynamic of a fluid in a circular tube induced by a traveling wave on its wall has numerous applications in various branches of science. The peristaltic transport is a physical mechanism that occurs due to the action of a progressive wave which propagates along the length of a distensible tube containing fluid. The peristaltic mechanism is nature's way of moving the content within hollow structures by successive contraction of their muscular fibers. The mechanism is responsible for transport of biological fluids such as urine in the ureter, chyme in gastro-intestinal tract, semen in the vas deferens and ovum in the female fallopian tube. Considerable analysis of this principle has been carried out, primarily for a Newtonian fluid with a periodic train of sinusoidal peristaltic waves. The inertia free peristaltic flow with long wave length analysis is given by Shapiro et al. [13]. The early developments on mathematical modeling and experimental fluid mechanism of peristaltic flow are given in a comprehensive review by Jaffrin and Shapiro [10].

Moreover many of the physiological fluids of known to be non-Newtonian. Peristaltic transport of blood in small vessels was investigated using the visco-elastic, power-law, micropolar, Casson fluid models by (Bohme and Friedrich [4]; Radha-Krishnamacharya [11]; Srinivasacharya et al. [14]; Srivastava and Srivastava [15]) respectively. Biomagnitic fluid dynamics is a relatively new area that deals with the fluid dynamics of magneto hydrodynamic biological fluid. During the last decades extensive literature is available on the MHD flows of biological fluids. The magnetic hydrodynamic flow of blood in a channel having walls that execute peristaltic waves using long wave length approximation has been discussed by Agrawal and Anwaruddin [2]. Abd El Hakeem et al. [1] have studied the hydromagnetic flow of fluid with variable viscosity in uniform tube with peristalsis. Peristaltic flow of

Johnson-Segalman fluid under effect of a magnetic field is studied by Elshahed and Haroun [5]. Hayat et al. [7] have discussed peristaltic transport of a third order fluid under the effect of a magnetic field in a tube. Hayat et al. [6] have studied the effect of endoscope on the peristaltic flow of a Jeffrey fluid. Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube is discussed by Hayat and Ali [8]. Hayat et al. [9] have investigated the influence of an endoscope on the peristaltic flow of a Jeffrey fluid under the effective of magnetic filed in a tube. Ali et al. [3] have investigated peristaltic flow of MHD fluid in a channel with variable viscosity under the effect of slip condition. Reddappa et al. [12] have investigated the peristaltic transport of a Jeffrey fluid in an inclined planar channel with variable viscosity under the effect of a magnetic field. Recently, Subba Reddy et al. [16] have studied the effects of magnetic field and slip on the peristaltic flow of Jeffrey fluid through a porous medium in an asymmetric channel.

To the best of our knowledge, no investigation has been made yet to analyze the peristaltic flow of a Jeffrey fluid with variable viscosity under the effect of magnetic field. The flow is examined in a wave frame of reference moving with velocity of the wave. The problem is formulated using perturbation expansion in terms of viscosity parameter α. The governing equations are developed upto first-order in the viscosity parameter α. The zero order system yields the classical Poiseuille flow when the Hartmann number M tends to zero and λ_1 tends to zero. We simplify a complicated group of products of Bessel functions by approximating the polynomial. The effects of Hartmann number M, viscosity parameter α and material parameter λ_1 on the pumping characteristics and friction force are studied in detail.

2. Mathematical formulation

Consider the axisymmetric flow of a Jeffery fluid in a uniform circular tube with a sinusoidal peristaltic wave of small amplitude traveling down its wall. We further assume that wall is extensible and fluid is electrically conducting. A uniform magnetic field B_o is applied in the transverse direction to the flow. The magnetic Reynolds member is taken small so that the induced magnetic field is neglected. The geometry of wall surface is therefore described as

$$R = H(Z,t) = a + b \sin\left(\frac{2\pi}{\lambda}(Z - ct)\right) \tag{2.1}$$

in which a is the average radius of the undisturbed tube, b is the amplitude of the peristaltic wave, λ is the wavelength, c is the wave propagation speed, and t is the time, R and Z are the cylindrical coordinate with Z measures along the axis of the tube and R is in the radial direction. Let (U, W) be the velocity components in fixed frame of reference (R, Z). Fig. 1 shows the physical model of the tube.

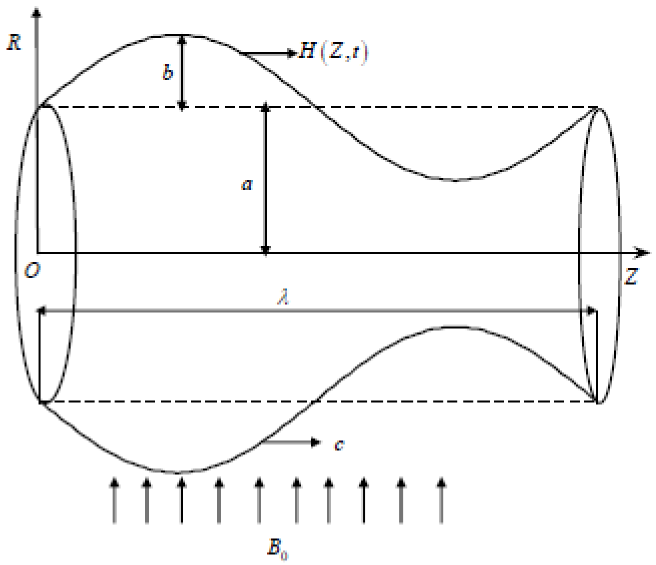

Fig. 1. Physical model.

The constitutive equations for an incompressible Jeffery fluid are

$$T = -pI + S \tag{2.2}$$

$$S = \frac{\mu(R)}{1+\lambda_1}(\dot{\gamma} + \lambda_2 \ddot{\gamma}) \tag{2.3}$$

where T and S are Cauchy stress tensor and extra stress tensor respectively, p is the pressure I is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, μ is the dynamic viscosity, $\dot{\gamma}$ is the shear rate and dots over the quantity indicate differentiation with respect to time.

In the fixed frame of reference (R, Z) the flow is unsteady. However, in a coordinate frame moving with the wave speed c (wave frame) (r, z) the boundary shape is stationary.

The transformation from fixed frame to wave frame is given below as

$$z = Z - ct,\ r = R,\ w(r,\ z) = W - c,\ u(r,\ z) = U. \tag{2.4}$$

where u and w being the velocity components in the wave frame.

The governing hydrodynamic equations are the equations of conservation of mass and momentum. The momentum equation here is modified to account for the interaction between magnetic field and fluid flow through the ponder motive force.

The governing equations in the wave frame are given as follows

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.5}$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) - \frac{S_{\theta\theta}}{r} \tag{2.6}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) - \sigma B_0^2(w+c) \tag{2.7}$$

where ρ is the density, σ is the electrical conductivity of the fluid and

$$S = \frac{\mu(r)}{1+\lambda_1}\left(1+\lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\right)\dot{\gamma}. \tag{2.8}$$

Introducing the following non-dimensional variables

$$\bar{r} = \frac{r}{a}, \bar{z} = \frac{z}{\lambda}, \bar{u} = \frac{u}{c\delta}, \bar{p} = \frac{pa^2}{\mu_0 c\lambda} \quad , \quad \bar{\mu}(r) = \frac{\mu(r)}{\mu_0}, \bar{w} = \frac{w}{c}, \delta = \frac{a}{\lambda}, S = \frac{aS}{\mu_0 c},$$

$$\phi = \frac{b}{a}, h = 1 + \phi\sin 2\pi z,$$

where ϕ is the amplitude ratio, μ_0 is the viscosity and δ is the wave number, in the equations (2.5) – (2.8), we get

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.9}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{\delta}{r}\frac{\partial}{\partial r}(rS_{rr}) + \delta^2\frac{\partial}{\partial z}(S_{rz}) - \frac{\partial}{r}(S_{\theta\theta}) \tag{2.10}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \delta\frac{\partial}{\partial z}(S_{zz}) - M^2(w+1) \tag{2.11}$$

where

$$S_{rr}=\frac{2\delta}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\frac{\partial u}{\partial r},$$

$$S_{rz}=\frac{1}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\left(\frac{\partial w}{\partial r}+\delta^2\frac{\partial u}{\partial z}\right),$$

$$S_{\theta\theta}=\frac{2\delta}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\frac{u}{r},$$

$$S_{zz}=\frac{2\delta}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\frac{\partial w}{\partial z},$$

$M=aB_0\sqrt{\frac{\sigma}{\mu_0}}$ is the Hartmann number and $\mathrm{Re}=\frac{\rho ac}{\mu_0}$ is the Reynolds member.

The corresponding non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r}=0, u=0 \quad \text{at} \quad r=0, \tag{2.12}$$

$$w=-1 \quad \text{at} \quad r=h=1+\phi\sin 2\pi z. \tag{2.13}$$

Using the long wavelength approximation ($\delta<<1$) and low Reynolds number ($\mathrm{Re}\to 0$) assumptions, the equations (2.10) and (2.11) becomes

$$\frac{\partial p}{\partial r}=0 \tag{2.14}$$

$$\frac{\partial p}{\partial z}=\frac{1}{(1+\lambda_1)r}\frac{\partial}{\partial r}\left(r\mu(r)\frac{\partial w}{\partial r}\right)-M^2(w+1). \tag{2.15}$$

From the equations (2.14) and (2.15), we have

$$\frac{dp}{dz}=\frac{1}{(1+\lambda_1)r}\frac{\partial}{\partial r}r\mu(r)\frac{\partial w}{\partial r}-M^2(w+1). \tag{2.16}$$

The effect of viscosity variation on peristaltic flow can be investigated for any given function $\mu(r)$. For the present investigation, we assume that viscosity variation in the dimensionless form

$$\mu(r)=e^{-\alpha r} \quad \text{or} \quad \mu(r)=1-\alpha r \quad \text{for} \quad \alpha<<1. \tag{2.17}$$

The dimensionless volume flow rate in the wave frame is given by

$$q=2\int_0^h wrdr. \tag{2.18}$$

The dimensionless instantaneous volume flow rate in the fixed frame of reference is given by

$$Q\left(x,t\right)=2\int_0^h wrdr=2\int_0^h (w+1)rdr=q+h^2\,. \tag{2.19}$$

The dimensionless time mean flow over a period $T\left(=\lambda/c\right)$ of the peristaltic wave is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Q(x,t)dt=q+1+\frac{\phi^2}{2}\,. \tag{2.20}$$

From Eq. (2.20), we have $q=\overline{Q}-1-\dfrac{\phi^2}{2}$.

The non-dimensional expressions for the pressure rise Δp per one wave length and friction force F (on the wall) are respectively given as

$$\Delta p=\int_0^1 \frac{dp}{dz}dz \tag{2.21}$$

and $$F=\int_0^1 h^2\left(-\frac{dp}{dz}\right)dz\,. \tag{2.22}$$

3. Perturbation solution

We look for a regular perturbation in terms of all small parameter α as follows

$$w=w_0+\alpha w_1+O(\alpha^2) \tag{3.1}$$

$$u=u_0+\alpha u_1+O(\alpha^2) \tag{3.2}$$

$$\frac{dp}{dz}=\frac{dp_0}{dz}+\alpha\frac{dp_1}{dz}+O(\alpha^2) \tag{3.3}$$

$$q=q_0+\alpha q_1+O(\alpha^2) \tag{3.4}$$

Substitutions from equations (3.1) – (3.3) in the equations (2.16), (2.12) and (2.13), we get

3.1 The system of order zero

$$\frac{1}{r}\frac{\partial(ru_0)}{\partial r}+\frac{\partial w_0}{\partial z}=0 \tag{3.5}$$

$$\frac{\partial p_0}{\partial r}=0 \tag{3.6}$$

$$\frac{\partial p_0}{\partial z}=\frac{1}{(1+\lambda_1)r}\frac{\partial}{\partial r}\left(r\frac{\partial w_0}{\partial r}\right)-M^2(w_0+1)\,. \tag{3.7}$$

With the dimensionless boundary conditions

$$\frac{\partial w_0}{\partial r}=0, u_0=0 \quad \text{at} \quad r=0\,, \tag{3.8}$$

$$w_0=-1, u_0=\frac{-dh}{dz}\,, \quad \text{at} \quad r=h=1+\phi\sin 2\pi z\,. \tag{3.9}$$

3.2 The system of order one

$$\frac{1}{r}\frac{\partial(ru_1)}{\partial r}+\frac{\partial w_1}{\partial z}=0 \tag{3.10}$$

$$\frac{\partial p_1}{\partial r}=0 \tag{3.11}$$

$$\frac{\partial p_1}{\partial z}=\frac{1}{(1+\lambda_1)}\frac{\partial}{\partial r}\left(-r^2\frac{\partial w_0}{\partial r}+r\frac{\partial w_1}{\partial r}\right)-M^2 w_1 \tag{3.12}$$

with the dimensionless boundary conditions

$$\frac{\partial w_1}{\partial r}=0, u_1=0 \quad \text{at} \quad r=0 \tag{3.13}$$

$$w_1=0, u_1=0 \quad \text{at} \quad r=h=1+\phi\sin 2\pi z\,. \tag{3.14}$$

3.3. Solution of order zero

Solving Eq. (3.7) using the boundary conditions (3.8) and (3.9), we get

$$w_0=\frac{dp_0}{dz}\frac{(1+\lambda_1)}{N^2 I_0(Nh)}\left(I_0(Nr)-I_0(Nh)\right)-1 \tag{3.15}$$

where $N=\sqrt{(1+\lambda_1)}M$.

The volume flow rate q_0 in the moving coordinate system is given by

$$q_0=\int_0^h rw_0 dr\,. \tag{3.16}$$

Substituting from (3.15) into (3.16) and solving the result for $\frac{dp_0}{dz}$ yields

$$\frac{dp_0}{dz}=\frac{N^4 I_0(Nh)(2q_0+h^2)}{(1+\lambda_1)\left[2NhI_1(Nh)-N^2h^2 I_0(Nh)\right]}\,. \tag{3.17}$$

3.4 Solution of order one

Substituting from Eq. (3.15) in Eq. (3.12) yields.

$$r^2\frac{\partial^2 w_1}{\partial r^2}+r\frac{\partial w_1}{\partial r}-N^2r^2w_1=(1+\lambda_1)\left[\frac{dp_1}{dz}r^2+\frac{G}{N^2I_0(Nh)}\frac{dp_0}{dz}\right], \tag{3.18}$$

where $G=N^2r^3I_0(Nr)+Nr^2I_1(Nr)$.

Differentiating equation (3.18) and with respect to r, yields

$$r^2\frac{\partial^2 S}{\partial r^2}+r\frac{\partial S}{\partial r}-(N^2r^2+1)S=\frac{dp_0}{dz}\frac{(1+\lambda_1)}{N^2I_0(Nh)}G_1, \tag{3.19}$$

where $G_1=2N^2r^2I_0(Nr)+(N^3r^3-Nr)I_1(Nr)$,

$$S=\frac{\partial w_1}{\partial r}. \tag{3.20}$$

The determination of a particular solution of Eq. (3.19) corresponding to this group of terms is complicated, and to avoid tedious manipulation we recall a similar group of terms. We represent the right hand side of Eq. (3.19) by a polynomial in the following form

$$r^2\frac{\partial^2 s}{\partial r^2}+r\frac{\partial s}{\partial r}-(N^2r^2+1)S=\frac{1+\lambda_1}{N^2I_0(Nh)}\frac{dp_0}{dz}\sum_{k=0}^{\infty}b_k(Nr)^{2k+2}, \tag{3.21}$$

where

$$b_k=\frac{(2k+1)(2k+3)}{2^{2k+1}\left(\Gamma\left(k+1\right)\right)^2(k+1)} \quad \text{for} \quad k=0,1,2,3,\ldots \tag{3.22}$$

The reason for the lower limit of the sum being zero and for the even power of *r* in the series is that, when the right-hand side of equation (3.19) is expanded in a power series in (*Nr*) using a series expansion of $I_0\left(Nr\right)$ and $I_1\left(Nr\right)$, we obtain only even power beginning with $\left(Nr\right)^2$ and then we can determine b_k.

Therefore, Eq. (3.18) can be rewritten as

$$r^2\frac{\partial^2 w_1}{\partial r^2}+r\frac{\partial w_1}{\partial r}-N^2r^2w_1=(1+\lambda_1)\frac{dp_1}{dz}r^2+\frac{(1+\lambda_1)}{N^3I_0(Nh)}\frac{dp_0}{dz}\sum_{k=0}^{\infty}\frac{b_k}{2k+1}(Nr)^{2k+2}. \tag{3.23}$$

Solving Eq. (3.23) using the boundary conditions (3.13) and (3.14), we get

$$w_1=\frac{(1+\lambda_1)}{N^2I_0(Nh)}\frac{dp_1}{dz}\left(I_0(Nr)-I_0(Nh)\right)+\frac{(1+\lambda_1)}{N^3I_0(Nh)}\frac{dp_0}{dz}\sum_{k=0}^{\infty}\frac{a_k(Nr)^{2k+3}}{2k+3}$$

$$-\frac{I_0(Nr)(1+\lambda_1)}{N^3\left(I_0(Nh)\right)^2}\frac{dp_0}{dz}\sum_{k=0}^{\infty}\frac{a_k(Nh)^{2k+3}}{2k+3}, \tag{3.24}$$

where

$$a_0=\frac{1}{2}, a_k=\frac{b_k+a_{k-1}}{(2k+1)(2k+3)} \quad \text{for} \quad k=0,1,2,3,\dots, \tag{3.25}$$

$I_0\left(Nr\right)$ and $I_1\left(Nr\right)$ are the modified Bessel functions of the first kind.

The volume flow rate q_1 in the moving coordinate system is given by

$$q_1=\int_0^h w_1 r dr. \tag{3.26}$$

Substituting from Eq. (3.24) into the Eq. (3.26) and solving the results for $\frac{dp_1}{dz}$, yields

$$\begin{aligned}\frac{dp_1}{dz}=&\frac{2q_1N^4I_0(Nh)}{(1+\lambda_1)\left(2NhI_1(Nh)-(Nh)^2I_0(Nh)\right)}+\frac{A_1}{(1+\lambda_1)}\sum_{k=0}^{\infty}\frac{a_k(Nh)^{2k+3}}{2k+3}\\&+\frac{A_2}{(1+\lambda_1)}\sum_{k=0}^{\infty}\frac{a_k(Nh)^{2k+5}}{2k+5}\end{aligned}, \tag{3.27}$$

where $A_1=\dfrac{N^3(2q_0+h^2)}{2NhI_1(Nh)-(Nh)^2I_0(Nh)}$,

$$A_2=\frac{N^3I_0(Nh)(2q_0+h^2)}{\left(2NhI_1(Nh)-(Nh)^2I_0(Nh)\right)^2}.$$

Substituting from equations (3.15), (3.24) into the Eq. (3.1) using the relation $\frac{dp_0}{dz}=\frac{dp}{dz}-\alpha\frac{dp_1}{dz}$ and negating terms greater than $O(\alpha)$ we get

$$\begin{aligned}w=&\frac{(1+\lambda_1)(I_0(Nr)-I_0(Nh))}{N^2I_0(Nh)}\frac{dp}{dz}+\alpha\frac{(1+\lambda_1)}{N^3I_0(Nh)}\frac{dp}{dz}\sum_{k=0}^{\infty}\frac{a_k(Nr)^{2k+3}}{2k+3}\\&-\alpha\frac{(1+\lambda_1)\frac{dp}{dz}\left(I_0(Nr)\right)}{N^3\left(I_0(Nh)\right)^2}\sum_{k=0}^{\infty}\frac{a_k(Nh)^{2k+3}}{2k+3}.\end{aligned} \tag{3.28}$$

Substituting from equations (3.17) and (3.27) into the Eq. (3.3) using the relation $q_0=q-\alpha q_1$, where q is defined through Eq. (2.18) and neglecting the terms greater than $O\left(\alpha\right)$, we have

$$\frac{dp}{dz}=\frac{N^4 I_0(Nh)(\overline{Q}-\frac{\phi^2}{2}-1+h^2)}{(1+\lambda_1)\left[2NhI_1(Nh)-(Nh)^2 I_0(Nh)\right]} + \frac{\alpha}{1+\lambda_1}\left\{B_1\sum_{k=0}^{\infty}\frac{a_k(Nh)^{2k+3}}{2k+3}+B_2\sum_{k=0}^{\infty}\frac{a_k(Nh)^{2k+5}}{2k+5}\right\}. \tag{3.29}$$

where B_1 and B_2 are given by

$$B_1=\frac{N^3(\overline{Q}-\frac{\phi^2}{2}-1+h^2)}{2NhI_1(Nh)-(Nh)^2 I_0(Nh)} \quad \text{and} \quad B_2=\frac{N^3 I_0(Nh)(\overline{Q}-\frac{\phi^2}{2}-1+h^2)}{\left(2NhI_1(Nh)-(Nh)^2 I_0(Nh)\right)^2}.$$

The pressure rise Δp and friction force F (on the wall) in the tube length λ in their non-dimensional forms are given by

$$\Delta p=\int_0^1\frac{dp}{dz}dz\,, \tag{3.30}$$

$$F=\int_0^1 h^2\left(-\frac{dp}{dz}\right)dz\,. \tag{3.31}$$

4. Main results

A regular perturbation series in terms of the viscosity parameter (α) is used to obtain solution to the field equations for peristaltic flow of a Jeffery fluid in an axisymmetric tube. Since the integrals in equations (3.30) and (3.31) are not integrable in closed form, we have evaluated it numerically using a **MATHEMATICA** package. The values of various parameters for the transport of mucus in the small intestine, as reported in Shukla et al. (1980) and Srivastava et al. (1983) are $c=2cm/\min$, $a=1.25cm$, $\lambda=8.01cm$. The values of viscosity parameter α as reported in Srivastava et al. (1983) are $\alpha=0$ and $\alpha=0.1$. It may be noted that the theory of long wave length and zero Reynolds number of the present investigation remains applicable here, since the radius of the small intestine is very small compared with the wave length.

Fig. 2 shows the variation of pressure rise Δp with time averaged volume flow rate $\overline{Q}$ for different values of Hartmann number M with $\alpha=0.1$, $\phi=0.5$, and $\lambda_1=0.2$. Any two pumping curves intersect at a point in the first quadrant. To the left of this point pumping increases and to the right pumping, $(\Delta p > 0)$ free pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ are all decreases with increasing the Hartmann number M.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter with $\phi = 0.5$, $\lambda_1 = 0.2$ and $M = 1$ as shown in Fig. 3. For $\Delta p > 0$, the time average flux $\overline{Q}$ increases with a decrease in α for $\Delta p \leq 0$, the $\overline{Q}$ increases with an increase in α.

Fig. 4 depicts the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of λ_1 with $\phi = 0.5$, $\alpha = 0.1$ and $M = 1$. In the pumping and co-pumping regions, $\overline{Q}$ decreases with an increase in λ_1. Whereas in the co-pumping region $\overline{Q}$ increases with an increase in λ_1.

The variation of pressure rise Δp with time mean flow rate $\overline{Q}$ for different values of amplitude ratio with $\alpha = 0.1$, $M = 1$ and $\lambda_1 = 0.2$ as presented in Fig. 5. Both pumping and free pumping increases with an increase in ϕ. But in the co-pumping region, $\overline{Q}$ increases with an increase in amplitude ratio ϕ $\left(0 \leq \phi < 1\right)$, for an appropriately chosen $\Delta p(< 0)$.

In order to see the effects of Hartmann number, viscosity parameter, material parameter and amplitude ratio on the friction force on the wall of the tube, we have plotted figures 6-9. From Fig. 6 it is noted that, the friction force increases with increase in the M. From Fig. 7, it is observed that, the friction force first increase and then decrease with an increase in viscosity parameter α. From Fig. 8, it is noted that, as the material parameter λ_1 increases the friction force first increases and then decreases. From Fig. 9, it is observed that the friction force decreases with an increase in ϕ. In general, figures 2-9 show that the friction force has opposite character in comparison to the pressure rise.

5. Conclusions

In this paper, we investigated the peristaltic transport of magnetohydro-dynamic (MHD) Jeffrey fluid with variable viscosity in a tube. The analytical expressions are constructed for the axial velocity, axial pressure gradient, pressure rise and frictional force. The effects of Hartmann number M, amplitude ratio ϕ, viscosity parameter α and λ_1 on pumping characteristics and frictional force. It is found that, an increase in M leads to an increase in the magnitudes of Δp and F. The magnitudes of Δp and F decreases with an increase in both viscosity parameter α and λ_1. As the amplitude ratio increases the magnitudes of Δp and F increases. Further, as $\lambda_1 \to 0$ our results coincides with the results obtained by Abd El Hakeem et al. [1].

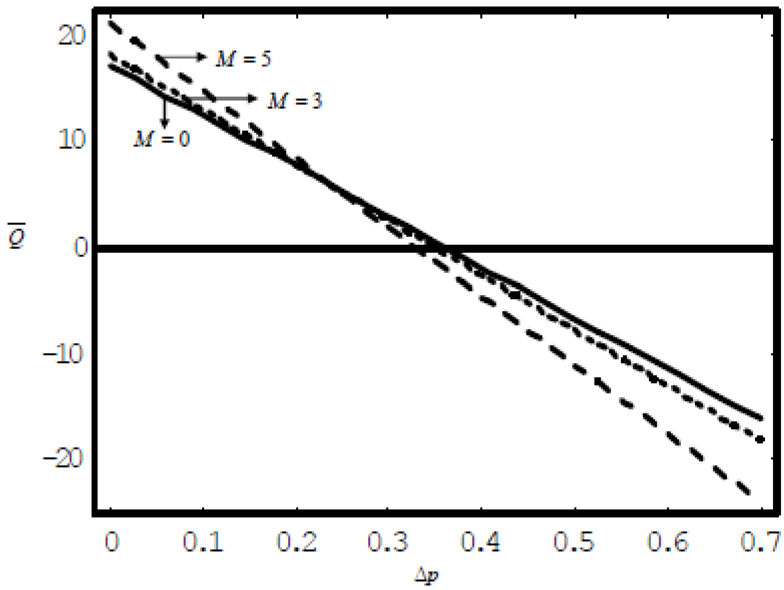

Fig. 2. The variation of pressure rise Δp with $\overline{Q}$ for different values of Hartmann number M with $\phi = 0.5, \alpha = 0.1$ and $\lambda_1 = 0.2$.

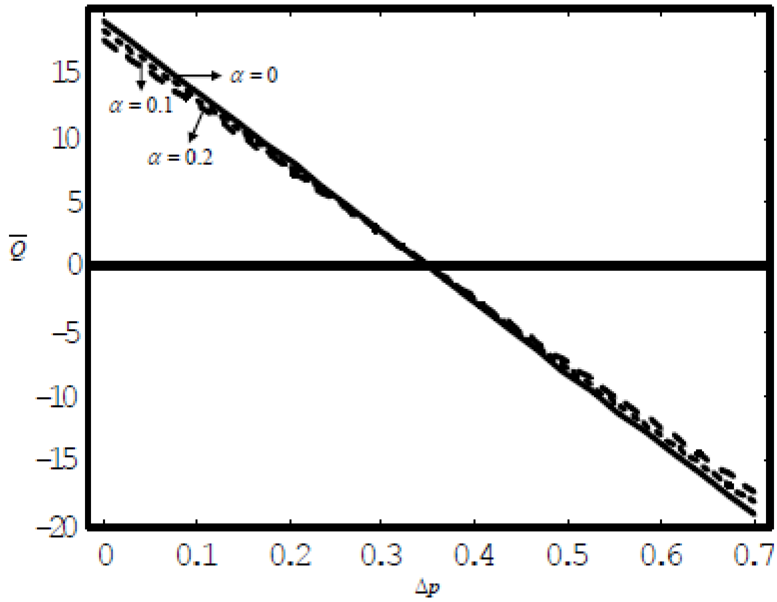

Fig. 3. The variation of pressure rise Δp with $\overline{Q}$ for different values of viscosity parameter α with $\phi = 0.5, M = 1$ and $\lambda_1 = 0.2$.

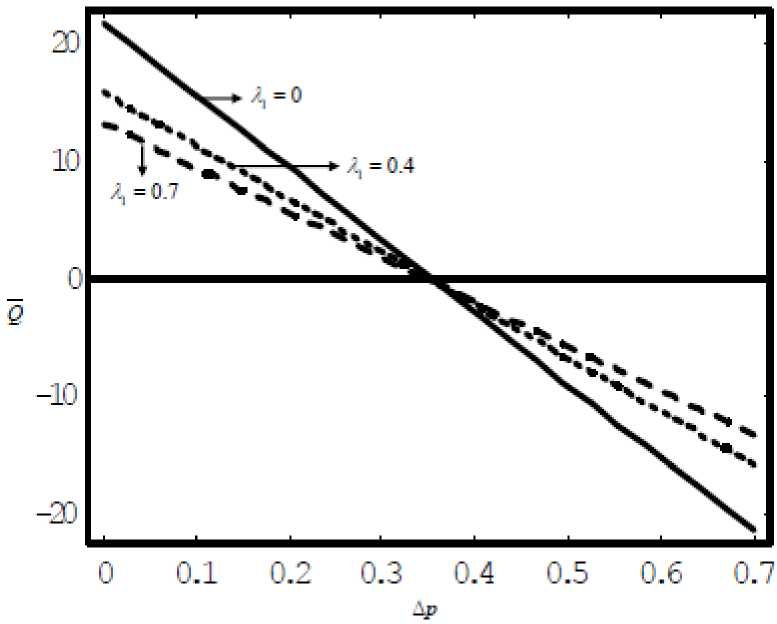

Fig. 4. The variation of pressure rise Δp with $\overline{Q}$ for different values of λ_1 with $\phi = 0.5, \alpha = 0.1$ and $M = 1$.

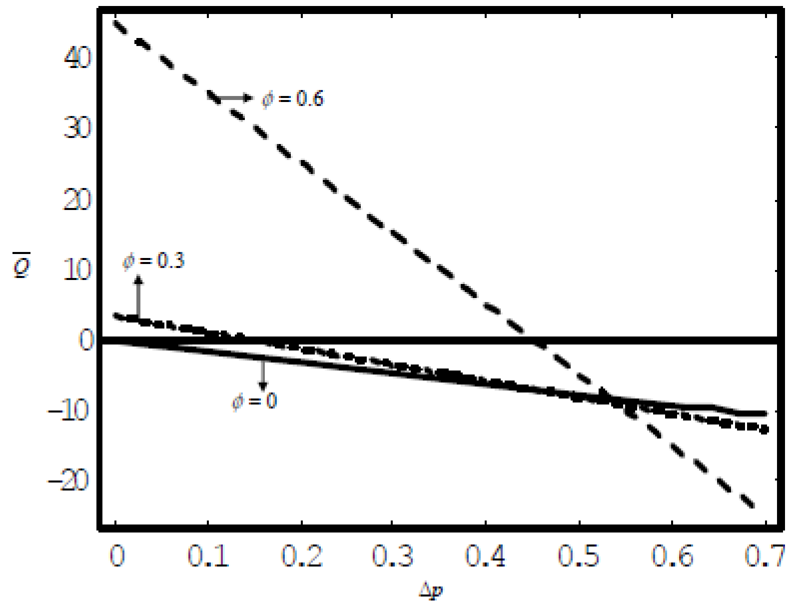

Fig. 5. The variation of pressure rise Δp with $\overline{Q}$ for different values of amplitude ratio ϕ with $\lambda_1 = 0.2, \alpha = 0.1$ and $M = 1$.

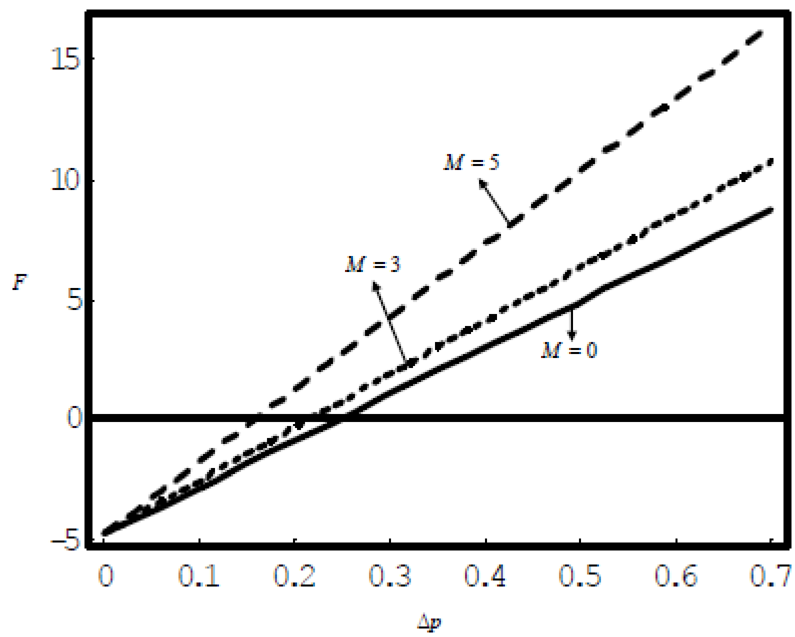

Fig. 6. The variation of Friction force F with $\overline{Q}$ for different values of Hartmann

number M with $\phi = 0.5, \alpha = 0.1$ and $\lambda_1 = 0.2$.

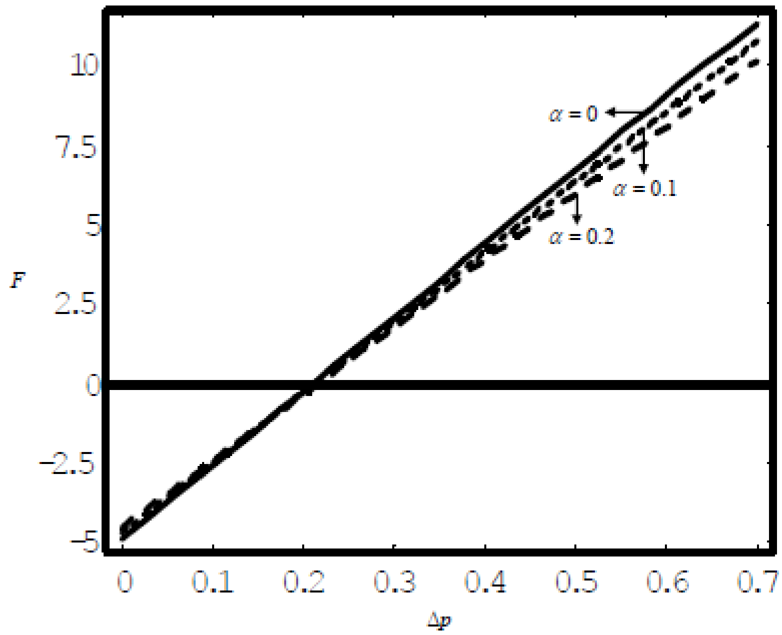

Fig. 7. The variation of Friction force F with $\overline{Q}$ for different values of viscosity parameter α with $\phi = 0.5, M = 1$ and $\lambda_1 = 0.2$.

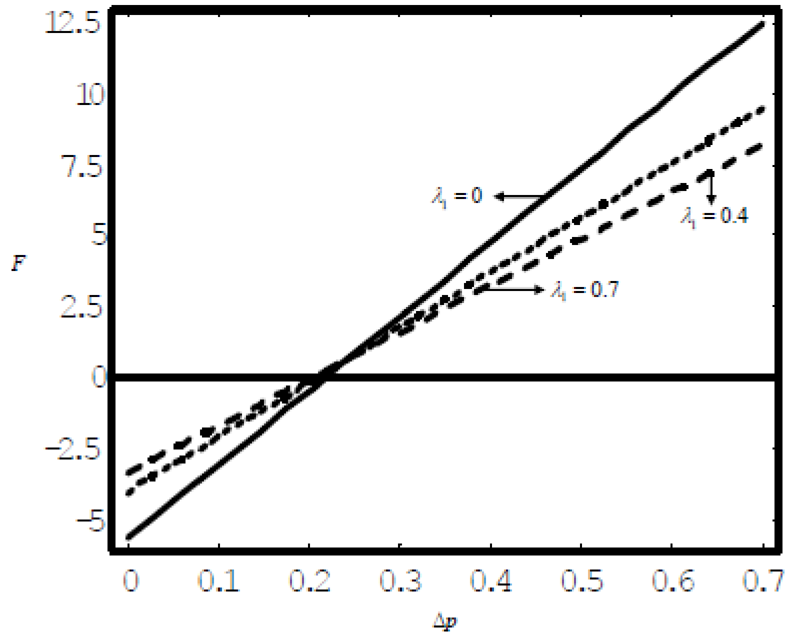

Fig. 8. The variation of Friction force F with $\overline{Q}$ for different values of λ_1 with $\phi = 0.5, \alpha = 0.1$ and $M = 1$.

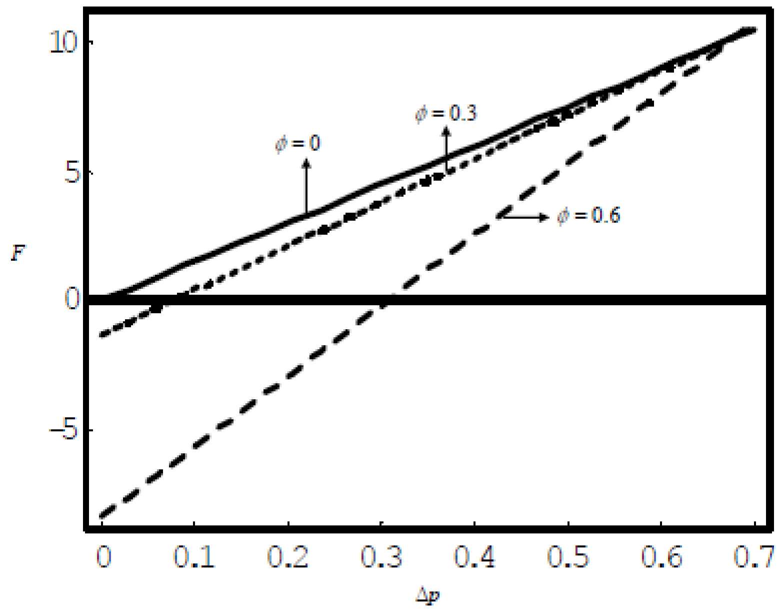

Fig. 9. The variation of Friction force F with $\overline{Q}$ for different values of amplitude ratio ϕ with $\lambda_1 = 0.2, \alpha = 0.1$ and $M = 1$.

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