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INTERVAL VALUE FUZZY n-FOLD KU-IDEALS OF KU-ALGEBRAS

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Abstract. In this paper, we will introduce the concept of interval value fuzzy n-fold KU-ideal in KU-algebras, which is a generalization of interval value fuzzy KU-ideal of KU-algebras and we will obtain few properties that is similar to the properties of interval value fuzzy KU-ideal in KU-algebras, see [8]. Also, we construct some algorithms for folding theory applied to KU-ideals in KU-algebras.

Keywords: KU-algebra; n-fold KU-ideal; interval value fuzzy n-fold KU-ideal; image and the pre image of interval value fuzzy n-fold KU-ideal; product of interval value fuzzy n-fold KU-ideals.

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1. Introduction

Prabpayak and Leerawat [12, 13] constructed a new algebraic structure which is called KU-algebras and introduced the concept of homomorphisms for such algebras. Akram et al and Yaqoob et al [1, 14] introduced the notion of cubic sub-algebras and ideals in KU-algebras. They discussed relationship between a cubic subalgebra and a cubic KU-ideal. Zadeh [15] presented the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as groups, functional analysis, probability theory and topology. Muhiuddin [11] introduced the notions of bipolar fuzzy KU-subalgebras and bipolar fuzzy KU-ideals in KU-algebras. He considered the specifications of a bipolar fuzzy KU-subalgebra, a bipolar fuzzy KU-ideal in KU-algebras and discussed the relations between a bipolar fuzzy KU-subalgebra and a bipolar fuzzy KU-ideal. Gulistan et al [3] studied (α, β)-fuzzy KU-ideals in KU-algebras and discussed some

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special properties. Mostafa et al [8] introduced the notion of interval value fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to interval value fuzzy KU-ideals. Akram et al [2] introduced the notion of interval-valued $(\tilde{\theta}, \tilde{\delta})$-fuzzy KU-ideals of KU-algebras and obtained some related properties. Jun and Dudek [5] introduced the notion of n-fold BCC-ideals and obtained some related results. In [4] Jun, introduced n-fold fuzzy BCC-ideals and gave a relation between n-fold fuzzy BCC-ideal and a fuzzy BCK-ideal. Mostafa and Kareem [9, 10] introduced n-fold KU-ideals and fuzzy n-fold KU-ideals of KU-algebra. They obtained some related properties. In this paper, we will introduce a generalization of interval value fuzzy KU-ideal of KU-algebras. Therefore, few properties similar to the properties of interval value fuzzy KU-ideal in KU-algebras can be obtained. Also, few results of interval value fuzzy n-fold KU-ideals of KU-algebra under homomorphism have been discussed. Also, some algorithms for folding theory have been constructed.

2. Preliminaries

In this section, we will recall some known concepts related to KU-algebra from the literature which will be helpful in further study of this article.

Definition2.1. [12, 13] an algebra(X, ∗, 0) of type (2, 0) is said to be a KU -algebra, if for all $x, y, z \in X$, the following axioms are satisfied:

(ku_1) $(x * y) * [(y * z) * (x * z)] = 0$,

(ku_2) $x * 0 = 0$,

(ku_3) $0 * x = x$,

(ku_4) $x * y = 0$ and $y * x = 0$ implies $x = y$,

(ku_5) $x * x = 0$,

On a KU-algebra $(X, *, 0)$ we can define a binary relation $\leq$ on X by putting:

$x \leq y \Leftrightarrow y * x = 0$.

Thus a KU-algebra X satisfies the conditions:

$(ku_{1'})$ $(y * z) * (x * z) \leq (x * y)$

$(ku_{2'})$ $0 \leq x$

$(ku_{3'})$ $x \leq y, y \leq x$ implies $x = y$,

$(ku_{4'})$ $y * x \leq x$.

Theorem 2.2. [7]. In a KU-algebra X , the following axioms are satisfied:
For all $x, y, z \in X$,

(1) $x \leq y$ imply $y * z \leq x * z$,

(2) $x * (y * z) = y * (x * z)$,

(3) $((y * x) * x) \leq y$.

Definition 2.3[13]. A non-empty subset S of a KU-algebra (X,$*$, 0) is called a KU-sub algebra of X if $x * y \in S$ whenever $x, y \in S$.

Definition2.4 [12]. A non-empty subset I of a KU -algebra (X,$*$, 0) is called an ideal of X if for any $x, y \in X$,

(i) $\mathbf{0} \in I$,

(ii) $x * y, x \in I$ imply $y \in I$.

Definition2.5 [13]. Let I be a non empty subset of a KU-algebra X . Then I is said to be an KU-ideal of X , if

(I_1) $0 \in I$

(I_2) $\forall x, y, z \in X$, if $x * (y * z) \in I$ and $y \in I$, imply $x * z \in I$.

For any elements x and y of a KU-algebra X, $x^n * y$ denotes $x * (x * ...(x * y))$, where x occurs n times.

Definition2.6[10]. A nonempty subset I of a KU-algebra X is called n-fold KU-ideal of X if

(I) $0 \in I$

(II) $\forall x, y, z \in X$ there exists a natural number n such that $x^n * z \in I$ whenever $x^n * (y * z) \in I$ and $y \in I$.

For a KU-algebra X, obviously $\{0\}$ and X itself are n-fold KU-ideal of X for every positive integer n.

Example 2.7. Let X = {0, 1, 2, 3, 4} be a set with $*$ defined by the following table:

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	4
2	0	1	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

By using the algorithms in Appendix A, we can prove that (X, *, 0) is a KU-algebra and it is easy to check that $I = \{0,1,2,3\}$ is n-fold KU-ideal of X for every positive integer n.

Definition 2.8 [15]. Let X be a set, a fuzzy set μ in X is a function μ: X → [0, 1]. For a fuzzy set μ in X and $t \in [0,1]$. Define $U(\mu,t)$ to be the set $U(\mu,t) = \{x \in X : \mu(x) \geq t\}$, which is called a level set of μ.

Definition2.9[10]. A fuzzy set μ in a KU-algebra X is called n-fold fuzzy KU-ideal of X if

(N_1) $\mu(0) \geq \mu(x)$ for all $x \in X$.

(N_2) $\forall x, y, z \in X$, there exists a natural number n such that

$\mu(x^n * z) \geq \min\{\mu(x^n * (y * z)), \mu(y)\}$.

Example 2.10. Let X = {0, 1, 2, 3, 4} be a set with $*$ defined as in Example2.7, define a fuzzy set μ in X by $\mu(4) = 0.2$ and $\mu(x) = 0.7$ for all $x \neq 4$. Then μ is n-fold fuzzy KU-ideal of X.

Definition 2.11 [13]. Let $(X,*,0)$ and $(X',*',0')$ be KU-algebras, a homomorphism is a map $f: X \to X'$ satisfying $f(x*y) = f(x)*' f(y)$ for all $x, y \in X$.

Theorem 2.12[13]. Let f be a homomorphism of KU-algebra X into KU-algebra Y, then

(i) If 0 is the identity in X, then $f(0)$ is the identity in Y.

(ii) If S is a KU- subalgebra of X, then $f(S)$ is a KU-subalgrbra of Y.

(iii) If I is n-fold KU- ideal of X, then $f(I)$ is n-fold KU- ideal in Y.

(iv) If S is a KU- subalgebra of Y, then $f^{-1}(S)$ is a KU- algebra of X.

(v) If B is n-fold KU- ideal in $f(X)$, then $f^{-1}(B)$ is n-fold KU- ideal in X.

3. Interval value fuzzy n-fold KU-ideals of KU-algebras

In this section, we begin with the concepts of interval-valued fuzzy sets.

An interval number is $\tilde{a} = [a^L, a^U]$, where $0 \le a^L \le a^U \le 1$. Let D [0, 1] be denote the family of all closed sub-intervals of [0, 1], i.e.,

$$D[0,1] = \{\tilde{a} = [a^L, a^U] : a^L \le a^U \quad for\ a^L, a^U \in [0,1]\}.$$

We define the operations $\le$, $\ge$, $=$, $r\min$ and $r\max$ in case of two elements in D [0, 1]. We consider two elements $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$ in D [0, 1].

Then

1- $\tilde{a} \le \tilde{b}$ iff $a^L \le b^L, a^U \le b^U$;

2- $\tilde{a} \ge \tilde{b}$ iff $a^L \ge b^L, a^U \ge b^U$;

3- $\tilde{a} = \tilde{b}$ iff $a^L = b^L, a^U = b^U$;

4- $r\min\{\tilde{a}, \tilde{b}\} = [\min\{a^L, b^L\}, \min\{a^U, b^U\}]$;

5- $r\max\{\tilde{a}, \tilde{b}\} = [\max\{a^L, b^L\}, \max\{a^U, b^U\}]$

Here we consider that $\tilde{0} = [0,0]$ as least element and $\tilde{1} = [1,1]$ as greatest element.

Let $\tilde{a}_i \in D[0,1]$, where $i \in \Lambda$. We define

$$r\inf_{i\in\Lambda} \tilde{a}_i = \left[\inf_{i\in\Lambda} a^L{}_i, \inf_{i\in\Lambda} a^U{}_i\right] \text{ and } r\sup_{i\in\Lambda} \tilde{a}_i = \left[\sup_{i\in\Lambda} a^L{}_i, \sup_{i\in\Lambda} a^U{}_i\right]$$

An interval valued fuzzy set $\tilde{\mu}$ in X is defined as $\tilde{\mu} = \{\langle x\ ,\ [\mu^L(x), \mu^U(x)],\ x \in X \rangle\}$, where $\tilde{\mu} : X \to D[0,1]$ and $\mu^L(x) \le \mu^U(x)$, for all $x \in X$. Then the ordinary fuzzy sets $\mu^L : X \to [0,1]$ and $\mu^U : X \to [0,1]$ are called a lower fuzzy set and an upper fuzzy set of $\tilde{\mu}$ respectively.

Definition 3.1. Let X be a KU-algebra. An interval valued fuzzy set $\tilde{\mu}$ in X is called an interval valued fuzzy KU-subalgebra of X if $\tilde{\mu}(x * y) \ge r\min\{\tilde{\mu}(x), \tilde{\mu}(y)\}$, for all $x, y \in X$.

Definition3.2. An interval valued fuzzy set $\tilde{\mu}$ in a KU-algebra X is called an interval valued fuzzy ideal of X if

(i_1) $\tilde{\mu}(0) \ge \tilde{\mu}(x)$ for all $x \in X$.

(ii_2) $\forall x, y \in X$, $\tilde{\mu}(y) \ge r\min\{\tilde{\mu}(x * y), \tilde{\mu}(x)\}$.

Definition3.3. An interval valued fuzzy set $\tilde{\mu}$ in a KU-algebra X is called an interval valued fuzzy KU-ideal of X if

(f_1) $\tilde{\mu}(0) \ge \tilde{\mu}(x)$ for all $x \in X$.

(f_2) $\forall x, y, z \in X$, $\tilde{\mu}(x * z) \ge r\min\{\tilde{\mu}(x * (y * z)), \tilde{\mu}(y)\}$.

Lemma 3.4. If $\tilde{\mu}$ is an interval valued fuzzy ideal of KU-algebra X and if x ≤ y, then $\tilde{\mu}(x) \ge \tilde{\mu}(y)$.

Proof. If x ≤ y, then $y * x = 0$, by ku_3 $0 * x = x$ and for all $x \in X$, $\tilde{\mu}(0) \ge \tilde{\mu}(x)$. We get

$$\tilde{\mu}(0 * x) = \tilde{\mu}(x) \ge r\min\{\tilde{\mu}(0 * (y * x)), \tilde{\mu}(y)\} = r\min\{\tilde{\mu}(0 * 0), \tilde{\mu}(y)\}$$
$$= r\min\{\tilde{\mu}(0), \tilde{\mu}(y)\} = \tilde{\mu}(y).$$

Definition 3.5. An interval valued fuzzy $\tilde{\mu}$ is called an interval valued fuzzy relation on any set X, if $\tilde{\mu}$ is an interval valued fuzzy subset $\tilde{\mu} : X \times X \to D[0,1]$.

Definition 3.6. If $\tilde{\mu}$ is interval valued fuzzy relation on a set X and $\tilde{\beta}$ is an interval-valued fuzzy subset of X, then $\tilde{\mu}$ is an interval valued fuzzy relation on $\tilde{\beta}$ if

$\tilde{\mu}(x, y) \le r\min\{\tilde{\beta}(x), \tilde{\beta}(y)\}, \forall x, y \in X$.

Definition 3.7. Let $\tilde{\mu}$ and $\tilde{\beta}$ be two interval valued fuzzy subsets of a set X , the product of $\tilde{\mu}$ and $\tilde{\beta}$ is define by $(\tilde{\mu} \times \tilde{\beta})(x, y) = r\min\{\tilde{\mu}(x), \tilde{\beta}(y)\}, \forall x, y \in X$.

Definition 3.8. If $\tilde{\beta}$ is an interval valued fuzzy subset of a set X , the strongest interval valued fuzzy relation on X , that is, an interval valued fuzzy relation on $\tilde{\beta}$ is $\tilde{\mu}_{\tilde{\beta}}$ given by $\tilde{\mu}_{\tilde{\beta}}(x, y) = r\min\{\tilde{\beta}(x), \tilde{\beta}(y)\}, \forall x, y \in X$.

Definition3.9. An interval valued fuzzy set $\tilde{\mu}$ in a KU-algebra X is called an interval valued fuzzy n-fold KU-ideal of X if

(L_1) $\tilde{\mu}(0) \geq \tilde{\mu}(x)$ for all $x \in X$.

(L_2) $\forall x, y, z \in X$, there exists a natural number n such that

$\tilde{\mu}(x^n * z) \geq r\min\{\tilde{\mu}(x^n * (y * z)), \tilde{\mu}(y)\}$.

Remark 3.10. An interval valued 1-fold fuzzy KU-ideal is precisely an interval valued fuzzy KU-ideal.

Example 3.11. Let $X = \{0,1,2,3,4\}$ be a set with $*$ defined as in Example 2.7, define $\tilde{\mu}(x)$ as follows:

$$\tilde{\mu}(x) = \begin{cases} [0.3,0.9] & \text{if} \quad x = \{0,1,2,3\} \\ [0.1,0.6] & \text{if} \quad x = 4 \end{cases} .$$

It is easy to check that $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of *X*.

Lemma 3.12. In a KU-algebra X , every interval valued fuzzy n-fold KU-ideal is an interval valued fuzzy ideal.

Proof. Let $\tilde{\mu}$ be an interval valued n-fold fuzzy KU-ideal of a KU-algebra X . By taking $x = \mathbf{0}$ in (L_2) and using (ku_3), we get

$$\tilde{\mu}(z) = \tilde{\mu}(0^n * z) \geq r\min\{\tilde{\mu}(0^n * (y * z)), \tilde{\mu}(y)\} = r\min\{\tilde{\mu}(y * z), \tilde{\mu}(y)\} \text{ for all } y, z \in X .$$

Hence $\tilde{\mu}$ is an interval valued fuzzy ideal of X .

Lemma 3.13. Let $\tilde{\mu}$ be an interval valued fuzzy n-fold KU-ideal of a KU-algebra X, if the inequality $x^n * y \le z$ holds in X, Then $\tilde{\mu}(y) \ge r\min\{\tilde{\mu}(x^n), \tilde{\mu}(z)\}$.

Proof: Assume that the inequality $x^n * y \le z$ holds in X, then $z * (x^n * y) = 0$ and by (L_2)

$$\tilde{\mu}((x^n * y)) \ge r\min\{\tilde{\mu}(x^n * (z * y)), \mu(z)\}$$
$$= r\min\{\tilde{\mu}(z * (x^n * y)), \tilde{\mu}(z)\} = r\min\{\tilde{\mu}(0), \tilde{\mu}(z)\} = \tilde{\mu}(z).....(I)$$

but

$$\tilde{\mu}(0 * y) = \tilde{\mu}(y) \ge r\min\{\tilde{\mu}(0 * (x^n * y)), \tilde{\mu}(x^n)\} = r\min\{\tilde{\mu}(x^n * y), \tilde{\mu}(x^n)\}$$
$$\ge r\min\{\tilde{\mu}(z), \tilde{\mu}(x^n)\} \quad (by \ \ (I))$$

i.e. $\tilde{\mu}(y) \ge r\min\{\tilde{\mu}(x^n), \tilde{\mu}(z)\}$.

Proposition 3.14. If $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X , then

$$\tilde{\mu}(x^n * (x^n * y) \ge \tilde{\mu}(y)$$

Proof: By taking $z = x^n * y$ in (L_2) and using (ku_2) , we get

$$\tilde{\mu}(x^n * (x^n * y)) \ge r\min\{\tilde{\mu}(x^n * (y * (x^n * y)), \tilde{\mu}(y)\} = r\min\{\tilde{\mu}(x^n * (x^n * (y * y)), \tilde{\mu}(y)\}$$
$$= r\min\{\tilde{\mu}(x^n * (x^n * 0), \tilde{\mu}(y)\}$$
$$= r\min\{\tilde{\mu}(x^n * 0), \tilde{\mu}(y)\}$$
$$= r\min\{\tilde{\mu}(0), \tilde{\mu}(y)\} = \tilde{\mu}(y).$$

The proof is completed.

Proposition3.15. *If* $\tilde{\mu}$ *is* an interval valued *fuzzy n-fold KU-ideal, then*

$$\tilde{\mu}(x^n * (y * z)) \ge \tilde{\mu}(x^n * z)$$

Proof. Since
$$\begin{cases} \overbrace{(x^n * z) * (x^n * (y * z)) = x^n * ((x^n * z) * (y * z))) = x^n * (y * ((x^n * z) * z)) =}^{by(2), Th.2.2,\ ku_3} \\ \qquad = y * (x^n * ((x^n * z) * z) = y * ((x^n * z) * (x^n * z)) = y * 0 = 0 \end{cases}$$

, then we have $x^n * (y * z)) \le (x^n * z)$, by Lemma 3.4, we get

$\tilde{\mu}(x^n *(y*z)) \geq \tilde{\mu}(x^n * z)$. The proof is completed.

Proposition3.16. Let A be a nonempty subset of a KU-algebra X and $\tilde{\mu}$ be an interval valued fuzzy set in X define by $\tilde{\mu}(x) = \begin{cases} [t_1, t_2] & x \in A \\ [\alpha_1, \alpha_2] & otherwise \end{cases}$, where $t_1 > \alpha_1$, $t_2 > \alpha_2$ and $t_1, t_2, \alpha_1, \alpha_2 \in [0,1]$. Then $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X if and only if A is an interval valued fuzzy n-fold KU-ideal of X. Moreover $X_{\tilde{\mu}} = A$, where $X_{\tilde{\mu}} = \{x \in X : \tilde{\mu}(x) = \tilde{\mu}(0)\}$.

Proof: Assume that $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X. Since $\tilde{\mu}(0) \geq \tilde{\mu}(x)$ for all $x \in X$, we have $\mu(0) = [t_1, t_2]$ and so $\mathbf{0} \in A$. For any $x, y, z \in X$ such that $x^n * (y * z) \in A$ and $y \in A$. Using (L_2), we know that $\tilde{\mu}(x^n * z) \geq r\min\{\tilde{\mu}(x^n *(y*z)), \tilde{\mu}(y)\} = [t_1, t_2]$ and thus $\tilde{\mu}(x^n * z) = [t_1, t_2]$. Hence $x^n * z \in A$, and A is n-fold KU-ideal of X.

Conversely, suppose that A is n-fold KU-ideal of X. Since $\mathbf{0} \in A$, it follows that $\tilde{\mu}(0) = [t_1, t_2] \geq \tilde{\mu}(x)$ for all $x \in X$. Let $x, y, z \in X$. If $y \notin A$ and $x^n * z \in A$, then clearly $\tilde{\mu}(x^n * z) \geq r\min\{\tilde{\mu}(x^n *(y*z)), \tilde{\mu}(y)\}$.

Assume that $y \in A$ and $x^n * z \notin A$. Then by (II), we have $x^n * (y * z) \notin A$. Therefore $\tilde{\mu}(x^n * z) = [t_2, t_2] = r\min\{\tilde{\mu}(x^n *(y*z)), \tilde{\mu}(y)\}$. Finally we have that $X_{\tilde{\mu}} = \{x \in X : \tilde{\mu}(x) = \tilde{\mu}(0)\} = \{x \in X : \tilde{\mu}(x) = [t_1, t_2]\} = A$.

Theorem 3.17. Let $\tilde{\mu}$ be an interval valued fuzzy set in a KU-algebra X and n a positive integer. Then $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X if and only if the nonempty level set $U(\tilde{\mu}, t)$ of $\tilde{\mu}$ is n-fold KU-ideal of X. Then call $U(\tilde{\mu}, t)$ the level n-fold KU-ideal of $\tilde{\mu}$.

Proof: Suppose that $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X and $U(\tilde{\mu}, t) \neq \phi$ for any $\tilde{t} = [t_1, t_2] \in D[0,1]$, there exists $x \in U(\tilde{\mu}, \tilde{t})$ and so $\tilde{\mu}(x) \geq \tilde{t}$. It follows from (L_1) that $\tilde{\mu}(0) \geq \tilde{\mu}(x) \geq \tilde{t}$ so that $0 \in U(\tilde{\mu}, \tilde{t})$. Let $x, y, z \in X$ be such that $x^n * (y * z) \in U(\tilde{\mu}, \tilde{t})$ and $y \in U(\tilde{\mu}, \tilde{t})$. Using (L_2), we know that

$\tilde{\mu}(x^n * z) \geq r\min\{\tilde{\mu}(x^n * (y * z)), \tilde{\mu}(y)\} \geq r\min\{\tilde{t}, \tilde{t}\} = \tilde{t}$, thus $x^n * z \in U(\tilde{\mu}, \tilde{t})$. Hence $U(\tilde{\mu}, \tilde{t})$ is n-fold KU-ideal of X.

Conversely, suppose that $U(\tilde{\mu}, \tilde{t}) \neq \phi$ is n-fold KU-ideal of X for every $\tilde{t} \in D[0,1]$. For any $x \in X$, let $\tilde{\mu}(x) = \tilde{t}$, then $x \in U(\tilde{\mu}, \tilde{t})$. Since $0 \in U(\tilde{\mu}, \tilde{t})$, it follows that $\tilde{\mu}(0) \geq \tilde{t} = \tilde{\mu}(x)$ so that $\tilde{\mu}(0) \geq \tilde{\mu}(x)$ for all $x \in X$. Now, we need to show that $\tilde{\mu}$ satisfies (L_2). If not, then there exist $a, b, c \in X$ such that $\tilde{\mu}(a^n * c) \geq r\min\{\tilde{\mu}(a^n * (b * c)), \tilde{\mu}(b)\}$. By taking

$\tilde{t}_0 = \frac{1}{2}(\tilde{\mu}(a^n * c) + r\min\{\tilde{\mu}(a^n * (b * c)), \tilde{\mu}(b)\})$ then we have

$\tilde{\mu}(a^n * c) < \tilde{t}_0 < r\min\{\tilde{\mu}(a^n * (b * c)), \tilde{\mu}(b)\}$. Hence $(a^n * (b * c)) \in U(\tilde{\mu}, \tilde{t}_0)$ and $b \in U(\tilde{\mu}, \tilde{t}_0)$, but $a^n * c \notin U(\tilde{\mu}, \tilde{t}_0)$, which means that $U(\tilde{\mu}, \tilde{t}_0)$ is not n-fold KU-ideal of X. This is contradiction. Hence $\tilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X.

Lemma 3.18. Let $\tilde{\mu}$ be an interval valued fuzzy n-fold KU-ideal of a KU-algebra X and $\tilde{t}_1, \tilde{t}_2 \in D[0,1]$ with $\tilde{t}_1 > \tilde{t}_2$. Then

(i) $U(\tilde{\mu}, \tilde{t}_1) \subseteq U(\tilde{\mu}, \tilde{t}_2)$,

(ii) Whenever $\tilde{t}_1, \tilde{t}_2 \in \operatorname{Im}(\tilde{\mu})$, where $\operatorname{Im}(\tilde{\mu}) = \{\tilde{t}_i : i \in \Lambda\}$ then $U(\tilde{\mu}, \tilde{t}_1) \neq U(\tilde{\mu}, \tilde{t}_2)$,

(iii) $U(\tilde{\mu}, \tilde{t}_1) = U(\tilde{\mu}, \tilde{t}_2)$ if and only if there does not exist $x \in X$ such that $\tilde{t}_1 \leq \tilde{\mu}(x) < \tilde{t}_2$.

Proof: clear.

Theorem 3.19. Let $\tilde{\mu}$ be an interval valued fuzzy n-fold KU-ideal of a KU-algebra X with $\operatorname{Im}(\tilde{\mu}) = \{\tilde{t}_i : i \in \Lambda\}$ and $\Omega = \{U(\tilde{\mu}, \tilde{t}_i) : i \in \Lambda\}$ where Λ is an arbitrary index set. Then

(i) There exists a unique $i_0 \in \Lambda$ such that $\tilde{t}_{i_0} \geq \tilde{t}_i$ for all $i \in \Lambda$.

(ii) $X_{\tilde{\mu}} = \bigcap_{i \in \Lambda} U(\tilde{\mu}, \tilde{t}_i) = U(\tilde{\mu}, \tilde{t}_{i_0})$,

(iii) $X = \bigcup_{i \in \Lambda} U(\tilde{\mu}, \tilde{t}_i)$,

Proof: (i) since $\tilde{\mu}(0) \in \operatorname{Im}(\tilde{\mu})$, there exists a unique $i_0 \in \Lambda$ such that $\tilde{\mu}(0) = \tilde{t}_{i_0}$. Hence by (L_1), we get $\tilde{\mu}(x) \leq \tilde{\mu}(0) = \tilde{t}_{i_0}$ for all $x \in X$, and so $\tilde{t}_{i_0} \geq \tilde{t}_i$ for all $i \in \Lambda$.

(ii) We have that

$U(\tilde{\mu},\tilde{t}_{i_0}) = \{x \in X : \tilde{\mu}(x) \geq \tilde{t}_{i_0}\} = \{x \in X : \tilde{\mu}(x) = \tilde{t}_{i_0}\} = \{x \in X : \tilde{\mu}(x) = \tilde{\mu}(0)\} = X$.

Note that $U(\tilde{\mu},\tilde{t}_{i_0}) \subseteq U(\tilde{\mu},\tilde{t}_i)$ for all $i \in \Lambda$, so that $U(\tilde{\mu},\tilde{t}_{i_0}) \subseteq \bigcap_{i\in\Lambda} U(\tilde{\mu},\tilde{t}_i)$. Since $i_0 \in \Lambda$, it follows that $X_{\tilde{\mu}} = U(\tilde{\mu},\tilde{t}_{i_0}) = \bigcap_{i\in\Lambda} U(\tilde{\mu},\tilde{t}_i)$.

(iii) For any $x \in X$ we have $\tilde{\mu}(x) \in \operatorname{Im}(\tilde{\mu})$ and so there exists $i(x) \in \Lambda$ such that $\tilde{\mu}(x) = \tilde{t}_{i(x)}$.

This implies $x \in U(\tilde{\mu},\tilde{t}_{i(x)}) \subseteq \bigcup_{i\in\Lambda} U(\tilde{\mu},\tilde{t}_i)$. Hence $X = \bigcup_{i\in\Lambda} U(\tilde{\mu},\tilde{t}_i)$.

4. Image (Pre-image) of interval valued fuzzy n-fold KU-ideals under homomorphism

Definition4.1.

Let f be a mapping from the set X to the set Y . If $\tilde{\mu}$ is an interval valued fuzzy subset of X , then the fuzzy subset $\tilde{B}$ of Y defined by

$$f(\tilde{\mu})(y) = \tilde{B}(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \tilde{\mu}(x), & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

Is said to be the image of $\tilde{\mu}$ under f . Similarly if $\tilde{\beta}$ is a fuzzy subset of Y , then the fuzzy subset $\tilde{\mu} = \tilde{\beta} \circ f$ in X (i.e. the interval valued fuzzy subset defined by $\tilde{\mu}(x) = \tilde{\beta}(f(x))$ for all $x \in X$) is called the pre-image of $\tilde{\beta}$ under f .

Theorem 4.2. An onto homomorphic pre-image of an interval valued fuzzy n-fold KU-ideal is also an interval valued fuzzy n-fold KU-ideal.

Proof: Let $f : X \to X'$ be an onto homomorphism of KU-algebras, $\tilde{\beta}$ be an interval valued fuzzy n-fold KU-ideal of X' and $\tilde{\mu}$ be the pre-image of $\tilde{\beta}$ under f , then $\tilde{\mu}(x) = \tilde{\beta}(f(x))$, for all $x \in X$. Let $x \in X$, then $\tilde{\mu}(0) = \tilde{\beta}(f(0)) \geq \tilde{\beta}(f(x)) = \tilde{\mu}(x)$. Now let $x, y, z \in X$ then

$$\widetilde{\mu}(x^n * z) = \widetilde{\beta}(f(x^n * z) = \widetilde{\beta}(f(x^n) *' f(z)) \geq r\min\{\widetilde{\beta}(f(x^n) *' (f(y) *' f(z)), \widetilde{\beta}(f(y)\}$$
$$= r\min\{\widetilde{\beta}(f(x^n * (y * z))), \widetilde{\beta}(f(y)\}$$
$$= r\min\{\widetilde{\mu}(x^n * (y * z)), \widetilde{\mu}(y)\}$$

, the proof is completed.

Definition4.3. An interval valued fuzzy subset $\widetilde{\mu}$ of X has sup property if for any subset T of X, there exist $t_0 \in T$ such that $\widetilde{\mu}(t_0) = \underset{t \in T}{SUP} \ \widetilde{\mu}(t)$.

Theorem 4.4. Let $f : X \to X'$ be a homomorphism between two KU-algebras X and X'. For every interval valued fuzzy n-fold KU-ideal $\widetilde{\mu}$ in X, $f(\widetilde{\mu})$ is an interval valued fuzzy n-fold KU-ideal of X'.

Proof : By definition $\widetilde{B}(y') = f(\widetilde{\mu})(y') := \sup\limits_{x \in f^{-1}(y')} \widetilde{\mu}(x)$ for all $y' \in X'$ and $\sup\phi := \widetilde{0}$.

We have to prove that $\widetilde{B}((x')^n * z') \geq r\min\{\widetilde{B}((x')^n * (y' * z')), \widetilde{B}(y')\}, \forall x', y', z' \in X'$.

Let $f : X \to X'$ be an onto homomorphism of a KU-algebra, $\widetilde{\mu}$ be an interval valued fuzzy n-fold KU-ideal of X with sup property and $\widetilde{\beta}$ be the image of $\widetilde{\mu}$ under f, since $\widetilde{\mu}$ is an interval valued fuzzy n-fold KU-ideal of X, we have $\widetilde{\mu}(0) \geq \widetilde{\mu}(x)$ for all $x \in X$. Note that $\mathbf{0} \in f^{-1}(\mathbf{0'})$, where 0, $0'$ are the zero of X and X' respectively, Thus, $\widetilde{B}(0') = \sup\limits_{t \in f^{-1}(0')} \widetilde{\mu}(t) = \widetilde{\mu}(0) \geq \widetilde{\mu}(x)$, for all $x \in X$, which implies that $\widetilde{B}(0') \geq \sup\limits_{t \in f^{-1}(x')} \widetilde{\mu}(t) = \widetilde{B}(x')$, for any $x' \in X'$. Now, for any $x', y', z' \in X'$, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, $z_0 \in f^{-1}(z')$ be such that

$\widetilde{\mu}((x_0)^n * z_0) = \sup\limits_{t \in f^{-1}((x')^n * z')} \widetilde{\mu}(t)$, $\widetilde{\mu}(y_0) = \sup\limits_{t \in f^{-1}(y')} \widetilde{\mu}(t)$ and

$$\widetilde{\mu}((x_0)^n * (y_0 * z_0)) = \widetilde{B}\{f((x_0)^n * (y_0 * z_0))\} = \widetilde{B}((x')^n * (y' * z'))$$

$$= \sup_{((x_0)^n * (y_0 * z_0) \in f^{-1}((x')^n * (y' * z')}} \widetilde{\mu}((x_0)^n * (y_0 * z_0)\} = \sup_{t \in f^{-1}((x')^n * (y' * x'))} \widetilde{\mu}(t).$$

Then $\widetilde{B}((x')^n * z') = \sup\limits_{t \in f^{-1}((x')^n * z')} \widetilde{\mu}(t) = \widetilde{\mu}((x_0)^n * z_0) \geq r\min\{\widetilde{\mu}((x_0)^n * (y_0 * z_0), \widetilde{\mu}(y_0)\} =$

$$r\min\left\{\sup_{t\in f^{-1}((x')^n*(y'*z'))}\tilde{\mu}(t),\ \sup_{t\in f^{-1}(y')}\tilde{\mu}(t)\right\} = r\min\{\tilde{B}((x')^n*(y'*z')),\ \tilde{B}(y')\}.$$

Hence $\tilde{B}$ is an interval valued fuzzy n-fold KU-ideal of Y.

Proposition4.5. For a given interval valued fuzzy subset $\tilde{\beta}$ of a KU-algebra X, let $\tilde{\mu}_{\tilde{\beta}}$ be the strongest fuzzy relation on X. If $\tilde{\mu}_{\tilde{\beta}}$ is interval valued fuzzy n-fold KU-ideal of $X\times X$, then $\tilde{\beta}(x)\le\tilde{\beta}(0)$ for all $x\in X$.

Proof: Since $\tilde{\mu}_{\tilde{\beta}}$ is an interval valued fuzzy n-fold KU-ideal of $X\times X$, it follows from (L_1) that $\tilde{\mu}_{\tilde{\beta}}(x,x)=r\min\{\tilde{\beta}(x),\tilde{\beta}(x)\}\le r\min\{\tilde{\beta}(0),\tilde{\beta}(0)\}$, then $\tilde{\beta}(x)\le\tilde{\beta}(0)$.

Theorem 4.6. Let $\tilde{\mu}$ and $\tilde{\beta}$ be two interval valued fuzzy n-fold KU-ideals of a KU-algebra X, then $\tilde{\mu}\times\tilde{\beta}$ is an interval valued fuzzy n-fold KU-ideal of $X\times X$.

Proof: for any $(x,y)\in X\times X$, we have

$(\tilde{\mu}\times\tilde{\beta})(0,0)=r\min\{\tilde{\mu}(0),\tilde{\beta}(0)\}\ge r\min\{\tilde{\mu}(x),\tilde{\beta}(y)\}=(\tilde{\mu}\times\tilde{\beta})(x,y)$.

Now let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$, then

$$(\tilde{\mu}\times\tilde{\beta})(x_1^n*z_1,\ x_2^n*z_2)=r\min\left\{\tilde{\mu}(x_1^n,z_1),\ \tilde{\beta}(x_2^n,z_2)\right\}$$

$$\ge r\min\left\{r\min\left\{\tilde{\mu}(x_1^n*(y_1*z_1)),\tilde{\mu}(y_1)\right\},r\min\left\{\tilde{\beta}(x_2^n*(y_2*z_2)),\tilde{\beta}(y_2)\right\}\right\}$$

$$=r\min\left\{r\min\left\{\tilde{\mu}(x_1^n*(y_1*z_1)),\tilde{\beta}(x_2^n*(y_2*z_2))\right\},r\min\left\{\tilde{\mu}(y_1),\tilde{\beta}(y_2)\right\}\right\}$$

$$=r\min\left\{(\tilde{\mu}\times\tilde{\beta})(x_1^n*(y_1*z_1),x_2^n*(y_2*z_2)),(\tilde{\mu}\times\tilde{\beta})(y_1,y_2)\right\}$$

Hence $\tilde{\mu}\times\tilde{\beta}$ is an interval valued fuzzy n-fold KU-ideal of $X\times X$.

Analogous to theorem 3.2 [6] , we have a similar results for interval-valued n-fold KU- ideal , which can be proved in similar manner , we state the results without proof .

Theorem 4.7. Let $\tilde{\mu}$ and $\tilde{\beta}$ be two interval valued fuzzy subsets of a KU-algebra X, such that $\tilde{\mu}\times\tilde{\beta}$ is an interval valued fuzzy n-fold KU-ideal of $X\times X$, then

(i) either $\tilde{\mu}(x)\le\tilde{\mu}(0)$ or $\tilde{\beta}(x)\le\tilde{\beta}(0)$ for all $x\in X$,

(ii) if $\tilde{\mu}(x)\le\tilde{\mu}(0)$ for all $x\in X$, then either $\tilde{\mu}(x)\le\tilde{\beta}(0)$ or $\tilde{\beta}(x)\le\tilde{\beta}(0)$,

(iii) if $\tilde{\beta}(x)\le\tilde{\beta}(0)$ for all $x\in X$, then either $\tilde{\mu}(x)\le\tilde{\mu}(0)$ or $\tilde{\beta}(x)\le\tilde{\mu}(0)$,

(iv) either $\tilde{\mu}$ or $\tilde{\beta}$ is an interval valued fuzzy n-fold KU-ideal of X.

Theorem 4.8. Let $\tilde{\beta}$ be an interval valued fuzzy subset of a KU-algebra X and $\tilde{\mu}_{\tilde{\beta}}$ be the strongest fuzzy relation on X, then $\tilde{\beta}$ is an interval valued fuzzy n-fold KU-ideal of X if and only if $\tilde{\mu}_{\tilde{\beta}}$ is an interval valued fuzzy n-fold KU-ideal of $X\times X$.

Proof: Assume that $\tilde{\beta}$ is an interval-valued fuzzy KU-ideal of X, we note from (L_1) that:

$\tilde{\mu}_{\tilde{\beta}}(0,0)=r\min\{\tilde{\beta}(0),\tilde{\beta}(0)\}\ge r\min\{\tilde{\beta}(x),\tilde{\beta}(y)\}=\tilde{\mu}_{\tilde{\beta}}(x,y)$.

Now, for any $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$, we have from (L_2):

$$\tilde{\mu}_{\tilde{\beta}}(x_1{}^n*z_1\ ,x_2{}^n*z_2)=r\min\left\{\tilde{\beta}(x_1{}^n*z_1)\ ,\tilde{\beta}(x_2{}^n*z_2)\right\}$$

$$\ge r\min\left\{r\min\left\{\tilde{\beta}(x_1{}^n*(y_1*z_1)),\tilde{\beta}(y_1)\right\},r\min\left\{\tilde{\beta}(x_2{}^n*(y_2*z_2)),\tilde{\beta}(y_2)\right\}\right\}$$

$$=r\min\left\{r\min\left\{\tilde{\beta}(x_1{}^n*(y_1*z_1)),\tilde{\beta}(x_2{}^n*(y_2*z_2))\right\},r\min\left\{\tilde{\beta}(y_1),\tilde{\beta}(y_2)\right\}\right\}$$

$$=r\min\left\{(\tilde{\mu}_{\tilde{\beta}}(x_1{}^n*(y_1*z_1),x_2{}^n*(y_2*z_2)),\ \tilde{\mu}_{\tilde{\beta}}(y_1,y_2)\right\}$$

Hence $\tilde{\mu}_{\tilde{\beta}}$ is an interval valued fuzzy KU-ideal of $X\times X$.

Conversely: For all $\boldsymbol{(x,y)}\in X\times X$, we have

$\tilde{\mu}_{\tilde{\beta}}(0,0)=r\min\{\tilde{\beta}(0),\tilde{\beta}(0)\}\ge r\min\{\tilde{\beta}(x),\tilde{\beta}(y)\}=\tilde{\mu}_{\tilde{\beta}}(x,y)$ It follows that $\boldsymbol{\beta(0)\ge\beta(x)}$ for all $x\in X$, which prove (L_1).

Now, let $\boldsymbol{(x_1,x_2),(y_1,y_2),(z_1,z_2),}\in X\times X$, then

$$r\min\{\tilde{\beta}(x_1{}^n * z_1), \tilde{\beta}(x_2{}^n * z_2)\} = \tilde{\mu}_{\tilde{\beta}}(x_1{}^n * z_1, x_2{}^n * z_2)$$

$$\geq r\min\{\tilde{\mu}_{\tilde{\beta}}((x_1{}^n, x_2{}^n) * ((y_1, y_2) * (z_1, z_2)), \tilde{\mu}_{\tilde{\beta}}(y_1, y_2)\}$$

$$= r\min\{r\min\{\tilde{\mu}_{\tilde{\beta}}(x_1{}^n * (y_1 * z_1), x_2{}^n * (y_2 * z_2))\}, \tilde{\mu}_{\tilde{\beta}}(y_1, y_2)\}$$

$$= r\min\{r\min\{\tilde{\mu}_{\tilde{\beta}}(x_1{}^n * (y_1 * z_1), (x_2{}^n * (y_2 * z_2)\}, r\min\{\tilde{\mu}_{\tilde{\beta}}(y_1), \tilde{\mu}_{\tilde{\beta}}(y_2)\}\}$$

$$= r\min\{r\min\{\tilde{\beta}(x_1{}^n * (y_1 * z_1), \tilde{\beta}(x_2{}^n * (y_2 * z_2)\}, r\min\{\tilde{\beta}(y_1), \tilde{\beta}(y_2)\}\}$$

$$= r\min\{r\min\{\tilde{\beta}(x_1{}^n * (y_1 * z_1), \tilde{\beta}(y_1)\}, r\min\{\tilde{\beta}(x_2{}^n * (y_2 * z_2), \tilde{\beta}(y_2)\}\}$$

In particular, if we take $x_2 = y_2 = z_2 = \mathbf{0}$, then

$\tilde{\beta}(x_1{}^n * z_1) \geq r\min\{\tilde{\beta}(x_1{}^n * (y_1 * z_1)), \tilde{\beta}(y_1)\}$. This proves (L_2) and completes the proof.

Conclusion: we have studied the interval valued fuzzy foldedness of a KU-ideal in a KU-algebra. Also we discussed few results of interval valued fuzzy n-fold KU-ideal of KU-algebras under homomorphism, the image and the pre-image of interval valued fuzzy n-fold KU-ideals in KU-algebras are defined. How the image and the pre-image of interval valued fuzzy n-fold KU-ideals are studied. Moreover, the product of interval valued fuzzy n-fold KU-ideals to product KU-algebras is established. Furthermore, we construct some algorithms for folding theory applied to KU-ideals in a KU-algebra.

The main purpose of our future work is to investigate the foldedness of other types of interval valued fuzzy ideals such as an implicative (commutative, positive implicative) and $\tilde{\tau}$ -cubic n-fold KU-ideals of a KU-algebra.

Appendix A. Algorithms

This appendix contains all necessary algorithms

Algorithm for KU-algebras

Input (X : set, $*$: binary operation)

Output (" X is a KU-algebra or not")

Begin

If $X = \phi$ then go to (1.);

EndIf

If $0 \notin X$ then go to (1.);

EndIf

Stop: =false;

$i := 1$;

While $i \le |X|$ and not (Stop) do

If $x_i * x_i \neq 0$ then

Stop: = true;

EndIf

$j := 1$

While $j \le |X|$ and not (Stop) do

If $((y_j * x_i) * x_i) \neq 0$ then

Stop: = true;

EndIf

EndIf

$k := 1$

While $k \le |X|$ and not (Stop) do

If $(x_i * y_j) * ((y_j * z_k) * (x_i * z_k)) \neq 0$ then

Stop: = true;

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

(1.) Output (“ X is not a KU-algebra”)

Else

Output (“ X is a KU-algebra”)

EndIf

End

Algorithm for fuzzy subsets

Input (X : KU-algebra, $A: X \to [0,1]$);

Output (“ A is a fuzzy subset of X or not”)

Begin

Stop: =false;

$i := 1$;

While $i \le |X|$ and not (Stop) do

If ($A(x_i) < 0$) or ($A(x_i) > 1$) then

Stop: = true;

EndIf

EndIf While

If Stop then

Output (" A is a fuzzy subset of X ")

Else

Output (" A is not a fuzzy subset of X ")

EndIf

End.

Algorithm for n-fold KU-ideals

Input (X : KU-algebra, I : subset of $X, n \in N$);

Output (" I is an n-fold KU-ideal of X or not");

Begin

If $I = \phi$ then go to (1.);

EndIf

If $0 \notin I$ then go to (1.);

EndIf

Stop: =false;

$i := 1$;

While $i \le |X|$ and not (Stop) do

$j := 1$

While $j \le |X|$ and not (Stop) do

$k := 1$

While $k \le |X|$ and not (Stop) do

If $(x^n{}_i * (y_j * z_k)) \in I$ and $y_j \in I$ then

If $(x^n{}_i * z_k) \notin I$ then

Stop: = true;

EndIf

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

Output (“ I is an n-fold KU-ideal of X ”)

Else

(1.) Output (“ I is not an n-fold KU-ideal of X ”)

EndIf

End.

Algorithm for fuzzy n-fold KU-ideals

Input (X : KU-algebra, $*$: binary operation, A : fuzzy subset of X);

Output (“ A is a fuzzy n-fold KU-ideal of X or not”)

Begin

Stop: =false;

$i := 1$;

While $i \le |X|$ and not (Stop) do

If $A(0) < A(x_i)$ then

Stop: = true;

EndIf

$j := 1$

While $j \le |X|$ and not (Stop) do

$k := 1$

While $k \le |X|$ and not (Stop) do

If $A(x_i^n * z_k) < \min(A(x_i^n * (y_j * z_k)), A(y_j))$ then

Stop: = true;

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

Output (“ A is not a fuzzy n-fold KU-ideal of X ”)

Else

Output (“ A is a fuzzy n-fold KU-ideal of X ”)

EndIf

End

Conflict of Interests

The author declares that there is no conflict of interests.

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