Available online at http://scik.org
J. Math. Comput. Sci. 5 (2015), No. 3, 297-319
ISSN: 1927-5307

DOUBT INTUITIONISTIC FUZZY MAGNIFIED TRANSLATION MEDIAL IDEALS IN BCI-ALGEBRAS

SAMY M. MOSTAFA[1,*], ABDELAZIZ E. RADWAN[2], AMANY M. MENSHAWY[1], REHAM GHANEM[1]
[1]Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Cairo, Egypt
[2]Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt

Copyright © 2015 Mostafa, Radwan, Menshawy and Ghanem. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. In this paper, we introduce the notion of doubt intuitionistic fuzzy medial ideals, doubt intuitionistic fuzzy magnified translation medial ideals in BCI-algebras and investigate some interesting results. Moreover, some algorithms for medial ideals, fuzzy set and doubt intuitionistic fuzzy medial ideals have been constructed.

Keywords: medial BCI-algebras; fuzzy medial ideals in BCI-algebras; intuitionistic fuzzy medial ideal.
2010 AMS Classification: 03G25, 06F35, 08A72.

1. Introduction

The notion of BCK-algebras was proposed by Iami and Iseki [6,7,9]in 1966. In the same year, Iseki [8] introduced the notion of a BCI-algebra which is a generalization of BCK-algebra. Since then numerous mathematical papers have been written investigating the algebraic properties of the BCK / BCI-algebras and their relationship with other structures including lattices and Boolean algebras. There is a great deal of literature which has been produced on the theory of BCK/BCI-algebras, in particular, emphasis seems to have been put on the ideal theory of BCK/BCI-algebras [15,16,17,19, 21]. For the general development of BCK/BCI-algebras the ideal theory plays an important role. The concept of fuzzy sets was first introduced by Zadeh [28]. From that time, the theory of fuzzy sets which has been developed in many directions and found applications in a wide variety of fields [5,11 ,13,14,18,24,25,26]. In 1991, Xi [27] applied the concept of fuzzy sets to BCI, BCK, MV-algebras. The ideal theory and its fuzzification play

*Corresponding author
Received December 30, 2014

an important role .In [20] J.Meng and Y.B.Jun studied medial BCI-algebras. In [23] S.M.Mostafa, Y.B.Jun and A.El-menshawy introduce the notion of medial ideals in BCI-algebras, they state the fuzzification of medial ideals and investigate its properties. Biswas in [4] gave the idea of anti fuzzy subgroups. Jun [12] defined a doubt fuzzy sub-algebra, doubt fuzzy ideal, doubt fuzzy implicative ideal, and doubt fuzzy prime ideal in BCI-algebras and got some results about it. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1,2] as a generalization of the notion of fuzzy sets. After that many researchers consider the Fuzzifications of ideals and sub-algebras in BCK/BCI-algebras. Menshawy [21] introduced the notion of intuitionistic fuzzy medial ideals and investigated some simple but elegant results. Kyoung, Jun and Doh [10] discussed fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications of fuzzy subalgebras in BCK/BCI-algebras and introduced the relations among fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications .In [3], the authors have studied doubt intuitionistic fuzzy sub-algebras , doubt intuitionistic fuzzy ideals in BCK=BCI-algebras and introduced the relations among doubt intuitionistic fuzzy ideals and doubt intuinistic fuzzy H-ideals .Here in this paper, we modify the ideas of Atanassov [1,2] , Jun [10,12] to introduce the notion of doubt intuitionistic fuzzy magnified translation medial ideals in BCI-algebras and obtain some interesting results. Moreover, some algorithms for medial ideals, fuzzy set and doubt intuitionistic fuzzy medial ideals have been constructed.

2. Preliminaries

We review some definitions and properties that will be useful in our results.

Definition 2.1 [8]. An algebraic system $(X,*,0)$ of type (2, 0) is called a BCI-algebra if it satisfying the following conditions:

(BCI-1) $((x*y)*(x*z))*(z*y)=0$,

(BCI-2) $(x*(x*y))*y=0$,

(BCI-3) $x*x=0$,

(BCI-4) $x*y=0$ and $y*x=0$ imply $x=y$.

For all x, y and $z \in X$. In a BCI-algebra *X*, we can define a partial ordering" $\leq$ " by $x \leq y$ if and only if $x * y = 0$.

In what follows, *X* will denote a BCI-algebra unless otherwise specified.

Definition 2.2 [20]. A BCI-algebra $(X, *, 0)$ of type (2, 0) is called a medial BCI-algebra if it satisfying the following condition: $(x * y) * (z * u) = (x * z) * (y * u)$,for all x, y, z and $u \in X$.

Lemma 2.3[20]. An algebra (X, $*$, 0) of type (2, 0) is a medial BCI-algebra if and only if it satisfies the following conditions:

(i) $x * (y * z) = z * (y * x)$

(ii) $x * 0 = x$

(iii) $x * x = 0$

Lemma 2.4[20]. In a medial BCI-algebra X, the following holds:

$x * (x * y) = y$, for all $x, y \in X$.

Lemma 2.5.Let *X* be a medial BCI-algebra, then $0 * (y * x) = x * y$, for all $x, y \in X$.

Proof. Clear.

Definition 2.6. A non empty subset *S* of a medial BCI-algebra *X* is said to be medial sub-algebra of *X*, if $x * y \in S$, for all $x, y \in S$.

Definition 2.7 [8]. A non-empty subset *I* of a BCI-algebra *X* is said to be a BCI-ideal of *X* if it satisfies:

(I_1) $0 \in I$,

(I_2) $x * y \in I$ and $y \in I$ implies $x \in I$ for all $x, y \in X$.

Definition 2.8[23]. A non empty subset *M* of a medial BCI-algebra *X* is said to be a medial ideal of *X* if it satisfies:

(M_1) $0 \in M$,

(M_2) $z*(y*x)\in M$ and $y*z\in M$ imply $x\in M$ for all x, y and $z\in X$.

Proposition 2.9[23]. Any medial ideal of a BCI-algebra must be a BCI- ideal but the converse is not true.

Proposition 2.10. Any BCI- ideal of a medial BCI-algebra is a medial ideal.
Proof. Let *M* be a BCI- ideal in a medial BCI-algebra *X*, such that $z*(y*x)\in M, y*z\in M$, for all $x, y, z\in X$, by lemma 2.3(i), we have $x*(y*z)\in M$, $y*z\in M$. But M is a BCI-ideal, therefore $x\in M$ .Then *M* is a medial ideal.

Example 2.11. Let *X* = {0,1,2,3,4,5} be a set with a binary operation $*$ defined by the following table:

*	0	1	2	3	4	5
0	0	0	0	0	4	4
1	1	0	1	0	4	4
2	2	2	0	0	4	4
3	3	2	1	0	4	4
4	4	4	4	4	0	0
5	5	4	5	4	1	0

Using the algorithms in Appendix B , we can prove that $(X,*,0)$ is a BCI-algebra and A = {0, 1, 2, 3} is a medial-ideal of X.

3. Doubt fuzzy medial ideal

Definition 3.1**.[12].** Let *X* be a BCI-algebra. a fuzzy set μ in *X* is called doubt fuzzy BCI-ideal of *X* if it satisfies:

(FI_1) $\mu(0)\le\mu(x)$,

(FI_2) $\mu(x)\le\max\{\mu(x*y),\mu(y)\}$, for all x, y and $z\in X$.

Definition 3.2. Let *X* be a BCI-algebra. A fuzzy set μ in *X* is called doubt fuzzy medial ideal of *X* if it satisfies:

(FM1) $\mu(0) \leq \mu(x)$,

(FM2) $\mu(x) \leq \max\{\mu(z*(y*x)), \mu(y*z)\}$, for all x, y and $z \in X$.

Lemma 3.3. Any doubt fuzzy medial-ideal of a BCI-algebra is doubt fuzzy subalgebra of *X*.

Proof. In definition 3.2, put $z = 0$ in (FM2) and using lemma 2.4, we have

$$\mu(x) \leq \max\{\mu(0*(y*x)), \mu(y*0)\} = \max\{\mu(x*y), \mu(y)\}.$$

4. Doubt intuitionistic fuzzy medial ideals in BCI-algebras

Definition 4.1 [1]. An Intuitionistic fuzzy set (briefly IFS) A in a nonempty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership and degree of non membership, respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$, $\forall x \in X$ An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, in X can be identified to an order pair (μ_A, λ_A) in $I^X \times I^X$. We shall use the symbol $A = (\mu_A, \lambda_A)$ for IFS $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

Definition 4.2.[3]. An IFS $A = (\mu_A, \lambda_A)$ in a BCI-algebra *X* is called doubt intuitionistic fuzzy subalgebra of *X* if it satisfies the following :

(IFMS1) $\mu_A(x*y) \leq \max\{\mu_A(x), \mu_A(y)\}$,

(IFMS2) $\lambda_A(x*y) \geq \min\{\lambda_A(x), \lambda_A(y)\}$, for all $x, y \in X$.

Example 4.3. Let $X = \{0,1,2,3,4,5\}$ as in example 2.11, and $A = (\mu_A, \lambda_A)$ be an I F S in *X* defined by $\mu_A(1) = \mu_A(2) = \mu_A(3) = \mu_A(4) = \mu_A(5) = 0.5 > 0.2 = \mu_A(0)$,

and $\lambda_A(1) = \lambda_A(2) = \lambda_A(3) = \lambda_A(4) = \lambda_A(5) = 0.3 < 0.7 = \lambda_A(0)$.

Then $A = (\mu_A, \lambda_A)$ is a doubt intuitionistic fuzzy subalgebra of *X*.

Lemma 4.4. Every doubt intuitionistic fuzzy subalgebra $A=(\mu_A,\lambda_A)$ of X satisfies the inequalities $\mu_A(0)\le\mu_A(x)$, and $\lambda_A(0)\ge\lambda_A(x)$ for all $x\in X$.

Proof. Clear.

Definition 4.5[3]. An IFS $A=(\mu_A,\lambda_A)$ in X is called doubt intuitionistic fuzzy BCI-ideal of X if it satisfies the following inequalities:

(IFI1) $\mu_A(0)\le\mu_A(x)$ and $\lambda_A(0)\ge\lambda_A(x)$

(IFI2) $\mu_A(x)\le\max\{\mu_A(x*y),\mu_A(y)\}$,

(IFI3) $\lambda_A(x)\ge\min\{\lambda_A(x*y),\lambda_A(y)\}$, for all $x,y\in X$.

Definition 4.6. An IFS $A=(\mu_A,\lambda_A)$ in X is called doubt intuitionistic fuzzy medial ideal of X if it satisfies the following inequalities.

(IFM1) $\mu_A(0)\le\mu_A(x)$ and $\lambda_A(0)\ge\lambda_A(x)$

(IFM2) $\mu_A(x)\le\max\{\mu_A(z*(y*x)),\mu_A(y*z)\}$,

(IFM3) $\lambda_A(x)\ge\min\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}$, for all $x,y,z\in X$.

Example 4.7. Let $X=\{0,1,2,3\}$ be a set with a binary operation $*$ define by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define

X	0	1	2	3
μ_A	0.1	0.4	0.5	0.8
λ_B	0.9	0.6	0.2	0.2

Using the algorithms in Appendix B ,we can prove that, $A=(\mu_A,\lambda_A)$ is doubt intuitionistic fuzzy medial ideal (sub-algebra) of *X*.

Lemma 4.8. Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy medial ideal of *X*. If $x\le y$ in *X*, then $\mu_A(x)\le\mu_A(y)$, $\lambda_A(x)\ge\lambda_A(y)$, for all $x,y\in X$.

Proof. Let $x,y\in X$ be such that $x\le y$, then $x*y=0$. From (IFM2), , lemma2.5), we have

$\mu_A(x)\le\max\{\mu_A(0*(y*x)),\mu_A(y*0)\}=\max\{\mu_A((x*y),\mu_A(y)\}$

$=\max\{\mu_A(0),\mu_A(y)\}=\mu_A(y)$.

Similarly, form (IFM3), we have $\lambda_A(x)\ge\min\{\lambda_A(0*(y*x)),\lambda_A(y*0)\}$, hence,

$\lambda_A(x)\ge\min\{\lambda_A(x*y),\lambda_A(y)\}=\min\{\lambda_A(0),\lambda_A(y)\}=\lambda_A(y)$.

Lemma 4.9. Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy medial ideal of *X*, if the inequality $x*y\le z$ hold in *X*, then

$\mu_A(x)\le\max\{\mu_A(y),\mu_A(z)\}$, $\lambda_A(x)\ge\min\{\lambda_A(y),\lambda_A(z)\}$, for all $x,y,z\in X$.

Proof. Let $x,y,z\in X$ be such that $x*y\le z$. Thus, put $z=0$ in (IFM2), (using lemma2.5 and lemma 4.8), we get, $\mu_A(x)\le\max\{\mu_A(0*(y*x),\mu_A(y*0)\}=\max\{\mu_A(x*y),\mu_A(y)\}\le$ $\overbrace{\max\{\mu_A(z),\mu_A(y)\}}^{\text{since } \mu_A(x*y)\ge\mu_A(z)}$. Similarly we can prove that, $\lambda_A(x)\ge\min\{\lambda_A(z)),\lambda_A(y)\}$.

Theorem 4.10. Every doubt intuitionistic fuzzy medial ideal of X is doubt intuitionistic fuzzy subalgebra of X.

Proof. Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy medial ideal of *X*. Since $x*y\le x$, for all $x,y\in X$, then $\mu_A(x*y)\le\mu_A(x)$, $\lambda_A(x*y)\ge\lambda_A(x)$.Put z = 0 in (IFM2), (IFM3), we have $\mu_A(x*y)\le\mu_A(x)\le\max\{\mu_A(0*(y*x)),\mu_A(y*0)\}=\max\{\mu_A(x*y),\mu_A(y)\}$

$\le\max\{\mu_A(x),\mu_A(y)\}$ .Now

$\lambda_A(x*y)\ge\lambda_A(x)\ge\min\{\lambda_A(0*(y*x)),\lambda_A(y*0)\}=\min\{\lambda_A(x*y),\lambda_A(y)\}$

$\ge\min\{\lambda_A(x),\lambda_A(y\}$.

Then $A=(\mu_A,\lambda_A)$ is doubt intuitionistic fuzzy subalgebra of *X*.

The converse of theorem 4.10 may not be true. For example, the doubt intuitionistic fuzzy subalgebra $A=(\mu_A,\lambda_A)$ in example 4.3 is not doubt intuitionistic fuzzy medial ideal of X since $\mu_A(1)=0.5>0.2=\max\{\mu_A(4*(4*1)),\mu_A(4*4)\}$.

Theorem 4.11. Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy medial ideal (subalgebra) of X ,such that $\mu_A(x)\le\max\{\mu_A(y),\mu_A(z)\}$, $\lambda_A(x)\ge\min\{\lambda_A(y),\lambda_A(z)\}$, and the inequality $x*y\le z$ are satisfied for all $x,y,z\in X$. Then $A=(\mu_A,\lambda_A)$ is doubt intuitionistic fuzzy medial ideal(subalgebra) of X.

Proof. Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy ideal (subalgebra) of X. Recall that $\mu_A(0)\le\mu_A(x)$ and $\lambda_A(0)\ge\lambda_A(x)$, for all $x\in X$. Since, $x*(z*(y*x))=(y*x)*(z*x)\le y*z$, it follows from the hypothesis that $\mu_A(x)\le\max\{\mu_A(z*(y*x)),\mu_A(y*z)\}$, $\lambda_A(x)\ge\min\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}$.Hence $A=(\mu_A,\lambda_A)$ is doubt intuitionistic fuzzy medial ideal of X.

Definition 4.12. Let $A=(\mu_A,\lambda_A)$ be an intuitionistic fuzzy set of X, we define the following: For any $t\in[0,1]$ and nonempty fuzzy sets μ , λ in X,

the set $L(\mu,t):=\{x\in X \mid \mu(x)\le t\}$ is called t-level cut of μ , and

the set $U(\lambda,s):=\{x\in X \mid \lambda(x)\ge s\}$ is called s-level cut of λ .

Theorem 4.13. An IFS $A=(\mu_A,\lambda_A)$ is doubt intuitionistic fuzzy medial ideal of X if and only if for all $s,t\in[0,1]$, the set $L(\mu_A,t)$ and $U(\lambda_A,s)$ are either empty or medial ideals of X.

Proof. Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy medial ideal of X and $L(\mu_A,t)\ne\phi\ne U(\lambda_A,s)$. Since $\mu_A(0)\le t$ and $\lambda_A(0)\ge s$, let $x,y,z\in X$ be such that $z*(y*x)\in L(\mu_A,t)$ and $y*z\in L(\mu_A,t)$, then $\mu_A(z*(y*x))\le t$ and $\mu_A(y*z)\le t$, it follows that $\mu_A(x)\le\max\{\mu_A(x*(y*z)),\mu_A(y*z)\}\le t$, we get $x\in L(\mu_A,t)$. Hence $L(\mu_A,t)$ is a medial ideal of X. Now let $x,y,z\in X$ be such that $z*(y*x)\in U(\lambda_A,s)$ and $y*z\in U(\lambda_A,s)$, then $\lambda_A(z*(y*x))\ge s$ and $\lambda_A(y*z)\ge s$ which imply that $\lambda_A(x)\ge\min\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}\ge s$. Thus $x\in U(\lambda_A,s)$ and therefore $U(\lambda_A,s)$ is a medial ideal of X.

Conversely, assume that for each $s,t \in [0,1]$, the sets $L(\mu_A,t)$ and $U(\lambda_A,s)$ are either empty or medial ideal of *X*. For any $x \in X$, let $\mu_A(x)=t$ and $\lambda_A(x)=s$. Then $x \in L(\mu_A,t) \cap U(\lambda_A,s)$ and so $L(\mu_A,t) \neq \phi \neq U(\lambda_A,s)$. Since $L(\mu_A,t)$ and $U(\lambda_A,s)$ are medial ideals of *X*, therefore $0 \in L(\mu_A,t) \cap U(\lambda_A,s)$. Hence $\mu_A(0) \leq t = \mu_A(x)$ and $\lambda_A(0) \geq s = \lambda_A(x)$ for all $x \in X$. If there exist $x', y', z' \in X$ be such that $\mu_A(x') > \max\{\mu_A(z' * (y' * x')), \mu_A(y' * z')\}$. Then by taking $t_0 := \frac{1}{2}\{\mu_A(x') + \max\{\mu_A(z' * (y' * x'), \mu_A(y' * z')\}\}$, we get

$$\mu_A(x') > t_0 > \max\{\mu_A(z' * (y' * x')), \mu_A(y' * z')\}$$

and hence $x' \notin L(\mu_A,t_0)$, $z' * (y' * x') \in L(\mu_A,t_0)$ and $y' * z' \in L(\mu_A,t_0)$, i.e. $L(\mu_A,t_0)$ is not a medial ideal of *X*, which make a contradiction. Finally assume that there exist $a,b,c \in X$ such that $\lambda_A(a) < \min\{\lambda_A(c * (b * a)), \lambda_A(b * c)\}$.

Then by taking $s_0 := \frac{1}{2}\{\lambda_A(a) + \min\{\lambda_A(c * (b * a), \lambda_A(b * c)\}\}$, we get

$$\min\{\lambda_A(c * (b * a)), \lambda_A(b * c)\} > s_0 > \lambda_A(a)$$

Therefore, $(c * (b * a)) \in U(\lambda_A,s_0)$ and $b * c \in U(\lambda_A,s_0)$, but $a \notin U(\lambda_A,s_0)$, which make a contradiction. This completes the proof.

5. The image and the pre- image of doubt intuitionistic fuzzy medial ideal under Homomorphism of BCI-algebras

Definition 5.1. Let $(X,*,0)$ and $(Y,*',0')$ be BCI-algebras. A mapping $f : X \to Y$ is said to be a homomorphism if $f(x * y) = f(x) *' f(y)$ for all $x, y \in X$.

Theorem. Let f be a homomorphism of BCI- algebra X into BCI -algebra Y, then

(i) If $\mathbf{0}$ is the identity in X, then $f(0) = 0'$ is the identity in Y.

(ii) If S is subalgebra of X, then $f(S)$ is sub-algrbra of Y.

(iii) If I is an medial- ideal of X, then $f(I)$ is an medial- ideal in Y.

(iv) If B is a sub-algebra of Y, then $f^{-1}(B)$ is a subalgebra - algebra of X .

Proof. Clear.

Let $f: X \to Y$ be a homomorphism of BCI-algebras for any I F S $A = (\mu_A, \lambda_A)$ in *Y*, we define new I F S $A^f = (\mu_A^f, \lambda_A^f)$ in *X* by $\mu_A^f(x) := \mu_A(f(x))$, and $\lambda_A^f(x) := \lambda_A(f(x))$ for all $x \in X$.

Theorem 5.2. Let $f: X \to Y$ be a homomorphism of BCI-algebras. If $A = (\mu_A, \lambda_A)$, is doubt intuitionistic fuzzy medial ideal of *Y*, then $A^f = (\mu_A^f, \lambda_A^f)$ is doubt intuitionistic fuzzy medial ideal of *X*.

Proof. $\mu_A^f(x) := \mu_A(f(x)) \geq \mu_A(0) = \mu_A(f(0)) = \mu_A^f(0)$, and

$\lambda_A^f(x) := \lambda_A(f(x)) \leq \lambda_A(0) = \lambda_A(f(0)) = \lambda_A^f(0)$, for all $x, y \in X$. Now

$\mu_A^f(x) := \mu_A(f(x)) \leq \max\{\mu_A(f(z) * (f(y) * f(x))), \mu_A(f(y) * f(z))\}$

$= \max\{\mu_A(f(z) * f(y * x)), \mu_A(f(y * z))\} = \max\{\mu_A(f(z * (y * x)), \mu_A(f(y * z))\}$

$= \max\{\mu_A^f(z * (y * x)), \mu_A^f(y * z)\}$, and

$\lambda_A^f(x) := \lambda_A(f(x)) \geq \min\{\lambda_A(f(z) * (f(y) * f(x)), \lambda_A(f(y * z))\}$

$= \min\{\lambda_A(f(z) * f(y * x)), \lambda_A(f(y * z))\} = \min\{\lambda_A(f(z * (y * x)), \lambda_A(f(y * z))\}$

$= \min\{\lambda_A^f(z * (y * x)), \lambda_A^f(y * z)\}$. Hence $A^f = (\mu_A^f, \lambda_A^f)$ is doubt intuitionistic fuzzy medial ideal in *X*.

Theorem 5.3. Let $f: X \to Y$ be an epimorphism of BCI-algebras . If $A^f = (\mu_A^f, \lambda_A^f)$ is doubt intuitionistic fuzzy medial ideal of *X*, then $A = (\mu_A, \lambda_A)$ is doubt intuitionistic fuzzy medial ideal in *Y*.

Proof . For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$ .Then

$$\mu_A(a) = \mu_A(f(x)) = \mu_A^f(x) \geq \mu_A^f(0) = \mu_A(f(0)) = \mu_A(0),$$

$$\lambda_A(a) = \lambda_A(f(x)) = \lambda_A^f(x) \leq \lambda_A^f(0) = \lambda_A(f(0)) = \lambda_A(0).$$

Let $a, b, c \in Y$ be such that $f(x) = a, f(y) = b, f(z) = c$, for some $x, y, z \in X$. It follows that $\mu_A(a) = \mu_A(f(x)) = \mu_A^f(x) \leq \max\{\mu_A^f(z * (y * x)), \mu_A^f(y * z)\}$

$= \max\{\mu_A(f(z * (y * x)), \mu_A(f(y * z))\} = \max\{\mu_A(f(z) * f(y * x)), \mu_A(f(y) * f(z))\}$

$= \max\{\mu_A(f(z) * (f(y) * f(x))), \mu_A(f(y) * f(z))\} = \max\{\mu_A(c * (b * a)), \mu_A(b * c)\}$,and

$\lambda_A(a) = \lambda_A(f(x)) = \lambda_A^f(x) \geq \min\{\lambda_A^f(z*(y*x)), \lambda_A^f(y*z)\}$

$= \min\{\lambda_A(f(z*(y*x)), \lambda_A(f(y*z))\} = \min\{\lambda_A(f(z)*f(y*x)), \lambda_A(f(y)*f(z))\}$

$= \min\{\lambda_A(f(z)*(f(y)*f(x))), \lambda_A(f(y)*f(z))\} = \min\{\lambda_A(c*(b*a)), \lambda_A(b*c)\}$.

This completes the proof.

6. Product of doubt intuitionistic fuzzy medial ideals

Definition 6.1. Let μ and λ be two fuzzy sets in the set *X*. the product $\lambda \times \mu : X \times X \to [0,1]$ is defined by $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$, for all $x, y \in X$.

Definition 6.2. Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ be two I F S of *X*, the doubt product $A \times B = (X \times X, \mu_A \times \mu_B, \lambda_A \times \lambda_B)$ is defined by $\mu_A \times \mu_B(x, y) = \max\{\mu_A(x), \mu_B(y)\}$ and $\lambda_A \times \lambda_B(x, y) = \min\{\lambda_A(x), \lambda_B(y)\}$, where $\mu_A \times \mu_B : X \times X \to [0,1]$, for all $x, y \in X$.

Remark 6.3.Let *X* and *Y* be BCI-algebras, we define* on $X \times Y$ by:

For every $(x, y), (u, v) \in X \times Y$, $(x, y)*(u, v) = (x*u, y*v)$. Clearly $(X \times Y; *, (0,0))$ is BCI-algebra.

Proposition 6.4.Let $A = (X, \lambda_A, \mu_A)$, $B = (X, \lambda_B, \mu_B)$ be doubt intuitionistic fuzzy medial ideals of *X*, then $A \times B$ is doubt intuitionistic fuzzy medial ideal of $X \times X$.

Proof.

$\mu_A \times \mu_B(0,0) = \max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(y)\} = \mu_A \times \mu_B(x, y)$, and

$\lambda_A \times \lambda_B(0,0) = \min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(y)\} = \lambda_A \times \lambda_B(x, y)$, for all $x, y \in X$

let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then

$\max\{(\mu_A \times \mu_B)((z_1, z_2)*((y_1, y_2)*(x_1, x_2))), (\mu_A \times \mu_B)((y_1, y_2)*(z_1, z_2))\}$

$= \max\{(\mu_A \times \mu_B)((z_1, z_2)*(y_1*x_1, y_2*x_2)), (\mu_A \times \mu_B)(y_1*z_1, y_2*z_2)\}$

$= \max\{(\mu_A \times \mu_B)(z_1*(y_1*x_1), z_2*(y_2*x_2)), (\mu_A \times \mu_B)(y_1*z_1, y_2*z_2)\}$

$= \max\{\max\{\mu_A(z_1*(y_1*x_1)), \mu_B(z_2*(y_2*x_2))\}, \max\{\mu_A(y_1*z_1), \mu_B(y_2*z_2)\}\}$

$= \max\{\max\{\mu_A(z_1*(y_1*x_1)), \mu_A(y_1*z_1)\}, \max\{\mu_B(z_2*(y_2*x_2)), \mu_B(y_2*z_2)\}$

$= \max\{\max\{ \mu_A(z_1 * (y_1 * x_1)), \mu_A(y_1 * z_1)\}, \max\{\mu_B(z_2 * (y_2 * x_2)), \mu_B(y_2 * z_2)\}$

$\geq \max\{\mu_A(x_1), \mu_B(x_2) = (\mu_A \times \mu_B)(x_1, x_2)$.

and

$\min\{(\lambda_A \times \lambda_B)((z_1, z_2) * ((y_1, y_2) * (x_1, x_2))), (\lambda_A \times \lambda_B)((y_1, y_2) * (z_1, z_2))\}$

$= \min\{(\lambda_A \times \lambda_B)((z_1, z_2) * (y_1 * x_1, y_2 * x_2)), (\lambda_A \times \lambda_B)(y_1 * z_1, y_2 * z_2)\}$

$= \min\{(\lambda_A \times \lambda_B)(z_1 * (y_1 * x_1), z_2 * (y_2 * x_2)), (\lambda_A \times \lambda_B)(y_1 * z_1, y_2 * z_2)\}$

$= \min\{\min\{ \lambda_A(z_1 * (y_1 * x_1)), \lambda_B(z_2 * (y_2 * x_2))\}, \min\{\lambda_A(y_1 * z_1), \lambda_B(y_2 * z_2)\}\}$

$= \min\{\min\{ \lambda_A(z_1 * (y_1 * x_1)), \lambda_A(y_1 * z_1)\}, \min\{\lambda_B(z_2 * (y_2 * x_2)), \lambda_B(y_2 * z_2)\}$

$= \min\{\min\{ \lambda_A(z_1 * (y_1 * x_1)), \lambda_A(y_1 * z_1)\}, \min\{\lambda_B(z_2 * (y_2 * x_2)), \lambda_B(y_2 * z_2)\}$

$\leq \min\{\lambda_A(x_1), \lambda_B(x_2)\} = (\lambda_A \times \lambda_B)(x_1, x_2)$.

This completes the proof.

Definition 6.5. Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ be doubt intuitionistic fuzzy sub-sets of a BCI-algebra *X*. for $s, t \in [0,1]$ the set $L(\mu_A \times \mu_B, t) := \{(x, y) \in X \times X \mid (\mu_A \times \mu_B)(x, y) \leq t\}$ is called t-level of $(\mu_A \times \mu_B)(x, y)$ and the set $U(\lambda_A \times \lambda_B, s) := \{(x, y) \in X \times X \mid (\lambda_A \times \lambda_B)(x, y) \geq s\}$ is called s-level of $(\lambda_A \times \lambda_B)(x, y)$.

Theorem 6.6. A doubt intuitionistic fuzzy set $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are doubt intuitionistic fuzzy medial ideal of *X* if and only if the non-empty set *t*-level cut $L(\mu_A \times \mu_B, t)$ and the non-empty *s*-level cut $U(\lambda_A \times \lambda_B, s)$ are medial ideals of $X \times X$ for any $s, t \in [0,1]$.

Proof. Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ be doubt intuitionistic fuzzy medial ideals of *X*, therefore for any $(x, y) \in X \times X$, we have

$\mu_A \times \mu_B(0,0) = \max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(y)\} = \mu_A \times \mu_B(x, y)$ and for $t \in [0,1]$, if $(\mu_A \times \mu_B)(x_1, x_2) \leq t$, therefore $(x_1, x_2) \in L(\mu_A \times \mu_B, t)$.

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ be such that $((z_1, z_2) * ((y_1, y_2) * (x_1, x_2))) \in L(\mu_A \times \mu_B, t)$, and $(y_1, y_2) * (z_1, z_2) \in L(\mu_A \times \mu_B, t)$.

Now

$(\mu_A \times \mu_B)(x_1, x_2) \leq$

$$\max\{(\mu_A\times\mu_B)((z_1,z_2)*((y_1,y_2)*(x_1,x_2))),(\mu_A\times\mu_B)((y_1,y_2)*(z_1,z_2))\}$$
$$=\max\{(\mu_A\times\mu_B)((z_1,z_2)*(y_1*x_1,y_2*x_2)),(\mu_A\times\mu_B)(y_1*z_1,y_2*z_2)\}$$
$$=\max\{(\mu_A\times\mu_B)(z_1*(y_1*x_1),z_2*(y_2*x_2)),(\mu_A\times\mu_B)(y_1*z_1,y_2*z_2)\}$$
$$\le\min\{t,t\}=t,$$

Therefore $(x_1,x_2)\in L((\mu_A\times\mu_B)(x,y),t)$, hence is $L((\mu_A\times\mu_B)(x,y),t)$ a medial ideal of $X\times X$.

Similarly ,we can prove that $U((\lambda_A\times\lambda_B)(x,y),s)$ is a medial ideal of $X\times X$.

This completes the proof.

7. Doubt intuitionistic fuzzy magnified translation medial ideals in BCI-algebras

Let $A=(\mu_A,\lambda_A)$ be doubt intuitionistic fuzzy subset of a set X, $\alpha\in[0,1-\sup\{\mu(x),x\in X\}]$, $\beta\in(0,1]$. An object having the form $A_\beta^\alpha=((\mu_A)_\beta^\alpha,(\lambda_A)_\beta^\alpha)$ is called doubt intuitionistic fuzzy magnified translation of A if $(\mu_A)_\beta^\alpha(x)=\beta\mu_A(x)+\alpha$, $(\lambda_A)_\beta^\alpha(x)=\beta\lambda_A(x)+\alpha$, be such that, $\alpha\in[0,1-\sup\{\mu(x),\forall x\in X\}$, $\beta\in(0,1-2\alpha]$.In particular if $\beta=1$,then $A_1^\alpha=((\mu_A)_1^\alpha,(\lambda_A)_1^\alpha)$ is called doubt intuitionistic fuzzy translation of A.If $\alpha=0$, then $A_\beta^0=((\mu_A)_\beta^0,(\lambda_A)_\beta^0)$ is called doubt intuitionistic fuzzy multiplication of A.

Example7.1. Consider the BCI-algebra $X=\{0,1,2,3\}$ in example 4.7. Define a fuzzy Subsets μ_A,λ_A of X by

X	0	1	2	3
μA	0.1	0.4	0.5	0.8
λB	0.9	0.6	0.2	0.2

Since α ϵ [0, 1 - sup{μ(x), xϵ X}], β ϵ (0, 1-2 α] , then α ϵ [0, 1 - 0.8]= [0, 0.2], If we take α =0.1 , therefore $\beta \in (0,1-2\alpha] = (0,0.2]$. Hence we can take $\alpha = 0.1$, $\beta = 0.2$ and therefore we get the following table :

X	0	1	2	3
μ_A	0.1	0.4	0.5	0.8
λ_B	0.9	0.6	0.2	0.2
$(\mu_A)_{0.2}^{0.1}(x)$	0.12	0.16	0.20	0.26
$(\lambda_A)_{0.2}^{0.1}(x)$	0.28	0.22	0.14	0.14

it is easy to show that $A_{0.2}^{0.1} = ((\mu_A)_{0.2}^{0.1}, (\lambda_A)_{0.2}^{0.1})$, is a doubt intuitionistic fuzzy magnified translation of medial ideal on X.

Theorem 7.2.The doubt intuitionistic fuzzy magnified translation $A_\beta^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ of $A = (\mu_A, \lambda_A)$ is doubt intuitionistic fuzzy medial ideal of X if and only if $A = (\mu_A, \lambda_A)$ is doubt intuitionistic fuzzy medial ideal of X.

Proof. Let $A = (\mu_A, \lambda_A)$ be doubt intuitionistic fuzzy medial ideal of X. Then A is a non-empty intuitionistic fuzzy subset of X, and hence A_β^α is also non-empty. Now for $x \in X$ we have

$$(\mu_A)_\beta^\alpha(0) = \beta\mu_A(0) + \alpha \le \beta\mu_A(x) + \alpha = (\mu_A)_\beta^\alpha(x) \ ,$$

$$(\lambda_A)_\beta^\alpha(0) = \beta\lambda_A(0) + \alpha \ge \beta\lambda_A(x) + \alpha = (\lambda_A)_\beta^\alpha(x) .$$

And

$$(\mu_A)_\beta^\alpha(x) = \beta\mu_A(x) + \alpha \le \ \beta(\max\{\mu_A(z*(y*x), \mu_A(y*z)\}) + \alpha$$
$$= \ \max\{\beta\mu_A(z*(y*x) + \alpha, \beta\mu_A(y*z) + \alpha\}$$
$$= \max\{(\mu_A)_\beta^\alpha(z*(y*x)), (\mu_A)_\beta^\alpha(y*z)\}$$

and

$$(\lambda_A)_\beta^\alpha(x) = \beta\lambda_A(x) + \alpha \ge \ \beta(\min\{\lambda_A(z*(y*x), \lambda_A(y*z)\}) + \alpha$$
$$= \ \min\{\beta\lambda_A(z*(y*x) + \alpha, \beta\lambda_A(y*z) + \alpha\}$$
$$= \min\{(\lambda_A)_\beta^\alpha(z*(y*x)), (\lambda_A)_\beta^\alpha(y*z)\} .$$

Hence $A_{\beta}^{\alpha}=((\mu_A)_{\beta}^{\alpha},(\lambda_A)_{\beta}^{\alpha})$ is doubt intuitionistic fuzzy magnified translation medial ideal of X.

Conversely, let $A_{\beta,}^{\alpha}=((\mu_A)_{\beta}^{\alpha},(\lambda_A)_{\beta}^{\alpha})$ be doubt intuitionistic fuzzy magnified translation medial ideal of X. Then

$$(\mu_A)_{\beta}^{\alpha}(0)\le(\mu_A)_{\beta}^{\alpha}(x)\text{. i.e } \beta\mu_A(0)+\alpha\le\beta\mu_A(x)+\alpha\text{, therefore } \mu_A(0)\le\mu_A(x)$$

Now

$$(\lambda_A)_{\beta}^{\alpha}(0)=\beta\lambda_A(0)+\alpha\ge\beta\lambda_A(x)+\alpha=(\lambda_A)_{\beta}^{\alpha}(x)\text{, i.e } \beta\lambda_A(0)+\alpha\ge\beta\lambda_A(x)+\alpha,$$

therefore $\lambda_A(0)\ge\lambda_A(x)$ .Now for all $x,y,z\in X$, we have

$$\begin{aligned}\beta\mu_A(x)+\alpha=(\mu_A)_{\beta}^{\alpha}(x)\le&\ \max\{(\mu_A)_{\beta}^{\alpha}(z*(y*x)),(\mu_A)_{\beta}^{\alpha}(y*z)\}\\ &=\max\{\beta\mu_A(z*(y*x)+\alpha,\beta\mu_A(y*z)+\alpha\}\\ &=\beta(\max\{\mu_A(z*(y*x),\mu_A(y*z)\})+\alpha\end{aligned}$$

therefore , $\mu_A(x)\le\max\{\mu_A(z*(y*x),\mu_A(y*z)\}$ and

$$\begin{aligned}\beta\lambda_A(x)+\alpha=(\lambda_A)_{\beta}^{\alpha}(x)\ge&\ \min\{(\lambda_A)_{\beta}^{\alpha}(z*(y*x)),(\lambda_A)_{\beta}^{\alpha}(y*z)\}\\ &=\min\{\beta\lambda_A(z*(y*x)+\alpha,\beta\lambda_A(y*z)+\alpha\}\\ &=\beta(\min\{\lambda_A(z*(y*x),\lambda_A(y*z)\})+\alpha\end{aligned}$$

i.e. $\lambda_A(x)\ge\min\{\lambda_A(z*(y*x),\lambda_A(y*z)\}$. Hence $A=(\mu_A,\lambda_A)$ is doubt intuitionistic fuzzy medial ideal of X.

Lemma 7.3. If $A_{\beta}^{\alpha}=((\mu_A)_{\beta}^{\alpha},(\lambda_A)_{\beta}^{\alpha})$ is doubt intuitionistic fuzzy magnified translation medial ideal and $x\le y$ in *X*, then $(\mu_A)_{\beta}^{\alpha}(x)\ge(\mu_A)_{\beta}^{\alpha}(y)$, $(\lambda_A)_{\beta}^{\alpha}(x)\le(\lambda_A)_{\beta}^{\alpha}(y)$. That is $(\mu_A)_{\beta}^{\alpha}$ is order reserving and $(\lambda_A)_{\beta}^{\alpha}$ is order preserving.

Proof. Let $x,y\in X$ be such that $x\le y$, by lemma 4.7, we have $\mu_A(x)\le\mu_A(y)$, and

$$(\mu_A)_{\beta}^{\alpha}(x)=\beta\mu_A(x)+\alpha\le\beta\mu_A(y)+\alpha\ =(\mu_A)_{\beta}^{\alpha}(y)$$

Similarly,

$$(\lambda_A)_{\beta}^{\alpha}(x)=\beta\lambda_A(x)+\alpha\ge\beta\lambda_A(y)+\alpha=(\lambda_A)_{\beta}^{\alpha}(x)\,.$$

Lemma 7.4.If $A_\beta^\alpha=((\mu_A)_\beta^\alpha,(\lambda_A)_\beta^\alpha)$ is doubt intuitionistic fuzzy magnified translation medial ideal and the inequality $x*y\le z$ hold in X, then

$(\mu_A)_\beta^\alpha(x)\le\max\{(\mu_A)_\beta^\alpha(y),(\mu_A)_\beta^\alpha(z)\}$, $(\lambda_A)_\beta^\alpha(x)\ge\min\{(\lambda_A)_\beta^\alpha(y),(\lambda_A)_\beta^\alpha(z)\}$.

Proof. Let $x,y,z\in X$ be such that $x*y\le z$. Thus, by lemma 4.9, we have

$$(\mu_A)_\beta^\alpha(x)=\beta\mu_A(x)+\alpha\le\beta(\max\{\mu_A(y),\mu_A(z)\})+\alpha$$
$$=\max\{\beta\mu_A(y)+\alpha,\beta\mu_A(z)+\alpha\}$$
$$=\max\{(\mu_A)_\beta^\alpha(y),(\mu_A)_\beta^\alpha(z)\}.$$

Similarly, we can prove that, $(\lambda_A)_\beta^\alpha(x)\ge\min\{(\lambda_A)_\beta^\alpha(y),(\lambda_A)_\beta^\alpha(z)\}$.

Definition 7.5. Let $f:X\to Y$ be a homomorphism of BCI-algebras , for any doubt intuitionistic fuzzy magnified translation $A_\beta^\alpha=((\mu_A)_\beta^\alpha,(\lambda_A)_\beta^\alpha)$ of A in Y. We define doubt intuitionistic fuzzy magnified translation $(A_\beta^\alpha)^f=((\mu_A^f)_\beta^\alpha,(\lambda_A^f)_\beta^\alpha)$ in X by $(\mu_A^f)_\beta^\alpha(x)=(\mu_A)_\beta^\alpha(f(x))$ and $(\lambda_A^f)_\beta^\alpha(x)=(\lambda_A)_\beta^\alpha(f(x))$, for all x∈X.

Theorem 7.6. Let $f:X\to Y$ be a homomorphism of BCI-algebras. If $A_\beta^\alpha=((\mu_A)_\beta^\alpha,(\lambda_A)_\beta^\alpha)$ is doubt intuitionistic fuzzy magnified translation medial ideal, then $(A_\beta^\alpha)^f=((\mu_A^f)_\beta^\alpha,(\lambda_A^f)_\beta^\alpha)$ is doubt intuitionistic fuzzy magnified translation medial ideal of X.

Proof. For all $x, y, z\in X$, we have

$$(\mu_A^f)_\beta^\alpha(x):=(\mu_A)_\beta^\alpha(f(x))\ge(\mu_A)_\beta^\alpha(0)=(\mu_A)_\beta^\alpha(f(0))=(\mu_A^f)_\beta^\alpha(0),$$

and

$$(\lambda_A^f)_\beta^\alpha(x):=(\lambda_A)_\beta^\alpha(f(x))\le(\lambda_A)_\beta^\alpha(0)=(\lambda_A)_\beta^\alpha(f(0))=(\lambda_A^f)_\beta^\alpha(0),$$

Now

$$(\mu_A^f)_{\beta,\alpha}^\alpha(x)=(\mu_A)_\beta^\alpha(f(x))$$
$$\le\max\{(\mu_A)_\beta^\alpha(f(z)*(f(y)*f(x))),(\mu_A)_\beta^\alpha(f(y)*f(z))\}$$
$$=\max\{(\mu_A)_\beta^\alpha(f(z)*(f(y*x))),(\mu_A)_\beta^\alpha(f(y*z))\}$$
$$=\max\{(\mu_A)_\beta^\alpha(f(z*(y*x))),(\mu_A)_\beta^\alpha(f(y*z))\}$$

$$= \max\{(\mu_A^f)_\beta^\alpha(z*(y*x)), (\mu_A^f)_\beta^\alpha(y*z)\},$$

and

$$(\lambda_A^f)_\beta^\alpha(x) = (\lambda_A)_\beta^\alpha(f(x)) \geq \min\{(\lambda_A)_\beta^\alpha(f(z)*(f(y)*f(x))), (\lambda_A)_\beta^\alpha(f(y)*f(z))\}$$

$$= \min\{(\lambda_A)_\beta^\alpha(f(z)*(f(y*x))), (\lambda_A)_\beta^\alpha(f(y*z))\}$$

$$= \min\{(\lambda_A)_\beta^\alpha(f(z*(y*x))), (\lambda_A)_\beta^\alpha(f(y*z))\}$$

$$= \min\{(\lambda_A^f)_\beta^\alpha(z*(y*x)), (\lambda_A^f)_\beta^\alpha(y*z)\}.$$

Then $(A_\beta^\alpha)^f = ((\mu_A^f)_\beta^\alpha, (\lambda_A^f)_\beta^\alpha)$ is doubt intuitionistic fuzzy magnified translation medial ideal of *X*.

Theorem 7.7. Let $f : X \to Y$ be an epimorphism of BCI-algebras and $A_\beta^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ a doubt intuitionistic fuzzy magnified translation in *Y*. If $(A_\beta^\alpha)^f = ((\mu_A^f)_\beta^\alpha, (\lambda_A^f)_\beta^\alpha)$ is doubt intuitionistic fuzzy medial ideal of *X*, then $A_\beta^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ is doubt intuitionistic fuzzy medial ideal in *Y*.

Proof. For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$ .Then

$(\mu_A)_\beta^\alpha(a) = (\mu_A)_\beta^\alpha(f(x)) = (\mu_A^f)_\beta^\alpha(x) \geq (\mu_A^f)_\beta^\alpha(0) = (\mu_A)_\beta^\alpha(f(0)) = (\mu_A)_\beta^\alpha(0)$, and

$(\lambda_A)_\beta^\alpha(a) = (\lambda_A)_\beta^\alpha(f(x)) = (\lambda_A^f)_\beta^\alpha(x) \leq (\lambda_A^f)_\beta^\alpha(0) = (\lambda_A)_\beta^\alpha(f(0)) = (\lambda_A)_\beta^\alpha(0)$. Now,

let $a, b, c \in Y$, and $f(x) = a, f(y) = b, f(z) = c$, for some $x, y, z \in X$. It follows that

$$(\mu_A)_\beta^\alpha(a) = (\mu_A)_\beta^\alpha(f(x)) = (\mu_A^f)_\beta^\alpha(x)$$

$$\leq \max\{(\mu_A^f)_\beta^\alpha(z*(y*x)), (\mu_A^f)_\beta^\alpha(y*z)\}$$

$$= \max\{(\mu_A)_\beta^\alpha(f(z*(y*x))), (\mu_A)_\beta^\alpha(f(y*z))\}$$

$$= \max\{(\mu_A)_\beta^\alpha(f(z)*(f(y*x))), (\mu_A)_\beta^\alpha(f(y*z))\}$$

$$= \max\{(\mu_A)_\beta^\alpha(f(z)*(f(y)*f(x))), (\mu_A)_\beta^\alpha(f(y)*f(z))\}$$

$$= \max\{(\mu_A)_\beta^\alpha(c*(b*a)), (\mu_A)_\beta^\alpha(b*c)\}, \text{and}$$

$$(\lambda_A)_\beta^\alpha(a) = (\lambda_A)_\beta^\alpha(f(x)) = (\lambda_A^f)_\beta^\alpha(x)$$

$$\geq \min\{(\lambda_A^f)_\beta^\alpha(z*(y*x)), (\lambda_A^f)_\beta^\alpha(y*z)\}$$

$$= \min\{(\lambda_A)_\beta^\alpha(f(z*(y*x))), (\lambda_A)_\beta^\alpha(f(y*z))\}$$

$$= \min\{(\lambda_A)_\beta^\alpha(f(z)*(f(y*x))), (\lambda_A)_\beta^\alpha(f(y*z))\}$$

$$= \min\{(\lambda_A)_\beta^\alpha(f(z)*(f(y)*f(x))),(\lambda_A)_\beta^\alpha(f(y)*f(z))\}$$

$$= \min\{(\lambda_A)_\beta^\alpha(c*(b*a)),(\lambda_A)_\beta^\alpha(b*c)\}$$

This completes the proof.

Conclusion

we have studied doubt intuitionistic fuzzy magnified translation medial ideal in BCI-algebras *X*. Also we discussed few results of doubt intuitionistic fuzzy magnified translation medial ideal under homomorphism of BCI-algebras , the image and the pre- image of doubt intuitionistic fuzzy magnified translation medial ideal in BCI-algebras are defined. How the image and the pre-image of doubt intuitionistic fuzzy magnified translation medial ideal in BCI-algebras become doubt intuitionistic fuzzy magnified translation medial ideal in BCI-algebras are studied. Moreover, the product of doubt intuitionistic fuzzy magnified translation medial ideal in BCI-algebras is established. Furthermore, we construct some algorithms applied to medial -ideals in BCI-algebras.

The main purpose of our future work is to investigate the foldedness of doubt intuitionistic fuzzy magnified translation medial ideal in BCI-algebras and bipolar intuitionistic fuzzy magnified translation medial ideal in BCI-algebras.

Appendix B. Algorithms

Algorithm for BC I-algebras

Input (X : set, $*$: binary operation)

Output (" X is a BCI -algebra or not")

Begin

If $X = \phi$ then go to (1.);

EndIf

If $0 \notin X$ then go to (1.);

EndIf

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If $x_i * x_i \neq 0$ then

Stop: = true;

EndIf

$j := 1$

While $j \leq |X|$ and not (Stop) do

If $(x_i * (x_i * y_j)) * y_j \neq 0$, then

Stop: = true;

EndIf

EndIf

$k := 1$

While $k \leq |X|$ and not (Stop) do

If $((x_i * y_j) * (x_i * z_k)) * (z_k * y_i) \neq 0$, then

Stop: = true;

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

(1.) Output (" X is not a BCI-algebra")

Else

Output (" X is a BCI -algebra")

EndIf

Algorithm for fuzzy subsets

Input (X :BCI-algebra, $\mu : X \rightarrow [0,1]$);

Output (“ A is a fuzzy subset of X or not”)

Begin

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If ($\mu(x_i) < 0$) or ($\mu(x_i) > 1$) then

Stop: = true;

EndIf

EndIf While

If Stop then

Output (“ μ is a fuzzy subset of X ”)

Else

Output (“ μ is not a fuzzy subset of X ”)

EndIf

End

Algorithm for medial -ideals

Input (X :BCI-algebra, I :subset of X);

Output (“ I is an medial -ideals of X or not”);

Begin

If $I = \phi$ then go to (1.);

EndIf

If $0 \notin I$ then go to (1.);

EndIf

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

$j := 1$

While $j \leq |X|$ and not (Stop) do

$k := 1$

While $k \leq |X|$ and not (Stop) do

If $z_k * (y_j * x_i) \in I$ and $y_j * z_k \in I$ then

If $x_i \notin I$ then

Stop: = true;

EndIf

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

Output ("I is is an medial -ideals of X ")

Else

(1.) Output ("I is not is an medial -ideals of X ")

EndIf

End .

Algorithm for doubt intuitionistic fuzzy medial ideal of X

Input (X :BCI-algebra, $*$:binary operation, μ, λ fuzzy subsets of X);

Output ("$A = (\mu, \lambda)$ is a doubt intuitionistic fuzzy medial ideal of X or not")

Begin

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If $\mu(0) > \mu(x_i), \lambda(0) < \lambda(x_i)$ then

Stop: = true;

EndIf

$j := 1$

While $j \leq |X|$ and not (Stop) do

$k := 1$

While $k \leq |X|$ and not (Stop) do

If $\mu_A(x_i) > \max\{\mu_A(z_k * (y_j * x_i)), \mu_A(y_j * z_k)\}$,

$\lambda_A(x_i) < \min\{\lambda_A(z_k * (y_j * x_i)), \lambda_A(y_j * z_k)\}$, then

Stop: = true;

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

Output (“ $A=(\mu,\lambda)$ is not a doubt intuitionistic fuzzy medial ideal of X ”)

Else

Output (“ $A=(\mu,\lambda)$ is a doubt intuitionistic fuzzy medial ideal of X ”)

EndIf

End.

Conflict of Interests

The author declares that there is no conflict of interests.

Acknowledgment

The authors are greatly appreciate the referees for their valuable comments and suggestions for improving the paper.

REFERENCES

[1] K. Atanassov, intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1) (1986), 87-96

[2] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, fuzzy sets and systems, 61 (2) 1994, 137-142.

[3] T. Bej and M. Pal, Doubt intuitionistic fuzzy ideals in BCK=BCI-algebras, Annals of fuzzy mathematics and informatics, Vol.8, no.4, 2014, pp.593-605.

[4] R.Biswas, Fuzzy sub- group and anti fuzzy sub- group, Fuzzy sets and systems, 35(1990), 121 – 124.

[5] S. K. Datta, R. Rammohunpur, D. Darjeeling, fuzzy magnified translation rings, International Journal of Pure and Applied Mathematics Volume 50 No. 4 2009, 523-528

[6] K. Iseki, An algebra related with a propositional calculus XIV[J]. Proc.Japan Academy, 42(1966), 26-29.

[7] K. Iseki and S. Tanaka, An introduction to the theory of BCK-algebras, Math. Japon. 23(1978), 1-26.

[8] K. Iseki, On BCI-algebras, Math. Sem. Notes 8 (1980) 125-130.

[9] K. Iseki and S. Tanaka, An introduction to the theory of BCK-algebras, Math. Japonica 23 (1978) 1-26.

[10]Kyoung Ja Lee, Young Bae Jun, and Myung Im Doh, fuzzy translations and fuzzy multiplications of BCK/BCI-algebras, Commun. Korean Math. Soc. 24(2009), no. 3, pp.353-360.

[11] Y. B. Jun, Traslations of fuzzy ideals in BCK=BCI-algebras, Hacettepe J. Math. and Stat.38(3) (2011) 349-358.

[12] Y. B. Jun, Doubt fuzzy BCK=BCI-algebras, Soochow J. Math. 20(3) (1994) 351-358.

[13]Y.B.Jun and S.Z.Song, Fuzzy set theory applied to implicative ideal in BCK-algebras, Bull. Korean Math. Soc 43(2006), No. 3, 461 – 470.

[14] B.Jun, Fuzzy sub-implicative ideals of BCI-algebras, Bull. Korean Math, 39(2002). No.2, 185 – 198.

[15] J. Meng, On ideals in BCK-algebras, Math. Japon., 40 (1994), 143–154.

[16] J.Meng , An ideal characterization of commutative BCI-algebras , Pusam Kyongnam Math.J. 9(1) (1993) 1 – 6.

[17] J.Meng, X.L.Xin, Implicative BCI-algebras, Pure and Appl. Math.8:2(1992), 99 – 103.

[18] J. Meng, Y. B. Jun, and H. S. Kim, Fuzzy implicative ideals of BCK-algebras, Fuzzy

Sets and Systems 89 (1997), 243{248.

[19] J.Meng, X.L.Xin, Characterization of atoms in BCI-algebras, Math. Japon. 37(1992), 359 – 362.

[20] J. Meng and Y.B. Jun, Notes on medial BCI-algebras, Comm. Korean Math. Soc. 8(1) (1993), 33-37.

[21] J. Meng and Y.B. Jun, BCK-algebras, Kyung Moon Sa Co. (1994).

[22] Amany M. Menshawy, fuzzy medial ideals characterized by its intuitionistic, International Journal of algebra, 7 (2013), no.10, 479-486.

[23] Samy M. Mostafa, Young Bae Jun and Amany El-menshawy, Fuzzy medial ideals in BCI-algebras, fuzzy math., 7(1999), no.2 , 445-457.

[24] Samy M.Mostafa, Fuzzy implicative ideals in BCK-algebras, Fuzzy sets and systems, 87(1997), 361 – 368.

[25] P. K. Sharma, On Intuitionistic Fuzzy Magnified translation in Rings, International Journal of Algebra, 5(2011), no. 30, 1451 – 1458.

[26] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl., 35 (1971), 512–517.

[27] O. G. Xi, Fuzzy BCK-algebras, Math. Japon. 36(5) (1991), 935-942.

[28] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.