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ANALYSIS OF MULTIPHASE FLOW IN POROUS MEDIA FOR SLIGHTLY COMPRESSIBLE FLOW AND ROCK

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Abstract. Multiphase flow in porous media is a matter of different complexities with a long, rich history in the field of fluid mechanics. This is a subject with important technical applications, most notably in oil recovery from petroleum reservoirs and so on. The single-phase fluid flow through a porous medium is well characterized by Darcy's law. In the petroleum industry and in other technical applications, transport is modeled by postulating a multiphase generalization of the Darcy's law. In this; distinct pressures are defined for each constituent phase with the difference known as capillary pressure, determined by the interfacial tension, micro pore geometry and surface chemistry of the solid medium. For flow rates, relative permeability is defined that relates the volume flow rate of each fluid to its pressure gradient. In the present paper, there is an analysis about the mathematical laws and equations for the slightly compressible flow and rock and some important results have been founded. The results show that the velocity of the fluid at any phase varies inversely with the viscosity of the fluid. The capillary pressure of the capillary tube varies inversely with the radius of tube, and increases with increase in the surface tension of the fluid. It also varies inversely with the radii of curvature of the interface of the fluid. The pressure exerted by the fluid varies directly with its velocity and varies inversely with the absolute permeability of the porous medium.

Keywords: Multiphase flow, Porous media, Darcy's law, Compressible flow.
2010 AMS Subject Classification: 76S05

1. Introduction

The Fluid mechanics studies about the moving matters that include liquids, gases, plasmas and other solids. From a 'fluid-mechanical' point of view, the matter can be considered of consisting of fluid and solid, in a one-fluid system the difference between these two states being that a solid

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can resist shear stress by a static deformation, which is impossible in fluid. But we can see that a distinction between the gas and liquid states of matter is impossible thermodynamically if temperature is above the critical point. The essential differences between these two phases are only due to the distinction between their equilibrium density and compressibility, below the critical point. For the gaseous phase molecules move almost freely relative to each other but in the controlled volume, the number of molecules changes continuously. Due to this microscopic uncertainty, fluid density has no precise meaning unless the control volume is comparatively large for the intermolecular spacing. But, if the chosen is sufficiently large, then there could be variation in the bulk density of the molecules due to other effects. A reasonable value for such a volume is around 10^{-9} mm^3 for liquids and gases at the atmospheric pressure. The practical problems are due to the physical dimensions, which are much larger than this size so that they display fluid properties with practically continuous spatial variation. These fluids can be called as continuum, and well-defined derivatives of the relevant variables can be used to explain its dynamical properties. The local equilibrium of these fluids are produced and maintained by collisions between the molecules that occur in a characteristic time scale τ. In classical fluids, τ is of the order of 10^{-10} to 10^{-14} seconds in general. As discussed by *U. Aaltosalmi et. al*. [1], [8], the mean free path λ between collisions of the molecules is the related length scale. For the systems of macroscopic motion, the states of equilibrium are not homogeneous. The states of equilibrium due to the disturbances vary in time and length scales given by t and λ, and the variables of the system have slow temporal variations at the long wavelengths. For these variables, there are large amount of equilibrating collisions, and dynamical disturbances and these are small in space at all times for these variables. To determine whether the above conditions for the fluid dynamics are satisfied, the Knudsen number will be evaluated for the problem. The Knudsen number K_n is the ratio of the molecular mean free path λ to a physical length scale λ_0 of the obstacles and flow channels and is given by *U. Aaltosalmi et.al.* [1], [8] i.e.,

$$K_n = \lambda / \lambda_0 \tag{1}$$

The multiphase flow term is generally used for any fluid flow consisting of more than one phase or component. The flows considered in the multiphase have some level of phase or component separation at a scale well above the molecular level.

That drops an enormous spectrum of different multiphase flows. In other words, it can be classified according to the state of the different phases or components which refers to gas, solids, particle or bubbly flows etc. many texts exist that limit their attention in this way. Some theories deal for a specific type of fluid flow with low Reynolds number suspension flows, dusty gas dynamics etc. But others are related with a specific application such as slurry flows, cavitations flows, aerosols, debris flows and fluidized beds etc. In the multiphase flow there are a broad range of flows with their applications. Every processing technology deals with multiphase flow, from cavitations pumps and turbines to electro photographic processes to paper making the pellet form of almost raw plastics. The amount of granular material, coal, grain, ore etc., is required to flow. Clearly the ability to predict the fluid flow behavior of these processes is central to the efficiency and effectiveness of those processes. For example, the effective flow of toner is a major factor in the quality and speed of electro photographic printers. Multiphase flows founds everywhere of our environment whether one considers rain, snow, fog, avalanches, mud slides, sediment transport, debris flows, and countless other natural phenomenon. In medical sciences some biological and medical flows are also multiphase, from blood flow to semen, laser surgery cavitations etc. In general, two types of flows are identified as disperse flows and separated flows. The disperse flows consisting of finite particles, drops or bubbles distributed in a connected volume of continuous phase. Whereas the separated flows consisting of two or more continuous streams of different fluids separated by interfaces.

On the study of the multiphase flows, there is the need of a model and predict the detailed behavior of those flows and the phenomenon that they show. *C. E. Brenen et. al.* [6] had given the three methods in which these models are explained as given:

(a) Experimentally by using laboratory-sized models equipped with appropriate instrumentation.
(b) Theoretically by using mathematical equations and models for the flows.
(c) Computationally by using the power and size of computers to illustrate the complexity of the flows.

So it is clear that in some applications in which full-scale laboratory models are needed. But, in many cases, the laboratory model must have a very different scale than the prototype and then a theoretical or computational model is better for extrapolation to the scale of the prototype. There are also some cases, in which a laboratory model is not suitable due to different reasons. It may be possible in many cases at some distant time to code the Navier-Stokes equations for all the phases and to compute the details of the multiphase flow. If any one or both of them phases become turbulent then the magnitude of the challenge becomes astronomical. So that the simplifications are necessary in the realistic models of most of the multiphase flows.

In the disperse flows two types of models are generally used, trajectory models and two-fluid models. In trajectory models, the motion of the disperse phase is analyzed by either the motion of the actual particles or the motion of larger particles. The details of the flow around each of the particles are included into assumed drag, lift and moment forces acting on and altering the trajectory of those particles. The thermal history of the particles can also be taken if it is suitable to do so. The trajectory models have been much useful for studying the granular flows due to the effects of the interstitial fluid. In the alternative approach, two-fluid models, the disperse phase is taken as a second continuous phase mixed and interacting with the continuous phase. So the equations for the conservation of mass, momentum and energy are derived for the two fluid flows which included interaction terms modeling the exchange of mass, momentum and energy between the two flows. *C. E. Brenen et. al.* [6] had solved these equations either by the theoretical method or by the computational method. So the two-fluid models neglect the discrete nature of the disperse phase and approximate its effects upon the continuous phase. The theory of separated flows can be solved as the single phase fluid flow equations in the two streams, coupling them through appropriate kinematic and dynamic conditions at the interface. Free streamline theory is an example of a successful implementation of the strategy, so the interface conditions used in this context are simple.

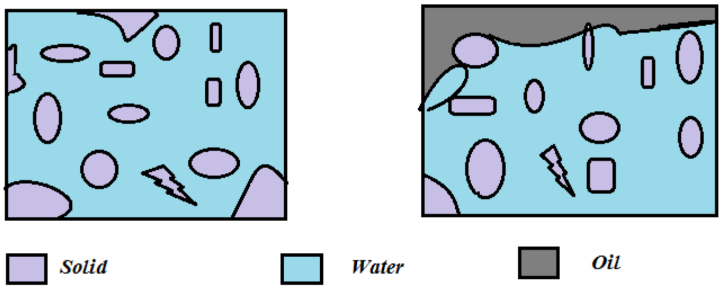


Figure-1 (Porous medium filled with water and oil)

Multiphase flow in porous media is a matter of different complexities with a long, rich history in the field of fluid mechanics. This is a subject with important technical applications, most notably in oil recovery from petroleum reservoirs and so on. The single-phase fluid flow through a porous medium is well characterized by Darcy’s law, and the main features of the subject have been well understood for 150 years. On the other hand, the modeling of multiphase flow remains an enormous technical difficulties and challenges. There is a big gap between the model equations used in industrial applications and the fundamental understanding of the detailed micro scale physics. In the petroleum industry and in other technical applications, transport is modeled by postulating a multiphase generalization of the *Darcy's law* by *P. M. Adler et.al.* [2]. In this, distinct pressures are defined for each constituent phase with the difference known as capillary pressure, determined by the interfacial tension, micro pore geometry and surface chemistry of the solid medium. For flow rates, relative permeability is defined that relates the volume flow rate of each fluid to its pressure gradient. The prediction of the relative permeability as a function of material properties, morphology and saturation are the primary challenges for scholars in the petroleum industry.

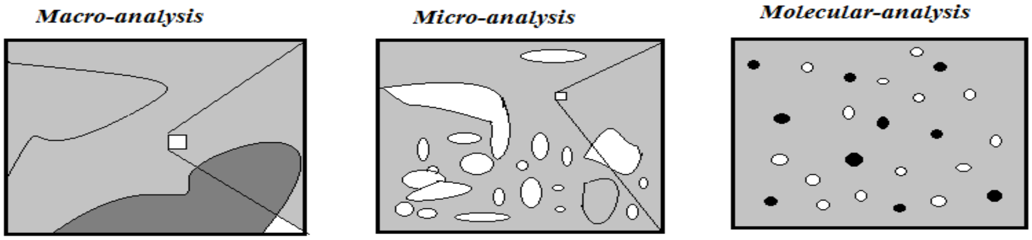


Figure-2 (Analytical scales in porous medium)

The classic reviews on multiphase flow in porous media have been given by *Wooding & Morel-Seytoux* [12] in 1976 and *Adler & Brenner* in 1988 [2], [6]. It has often been said that in general, all models are wrong but some are useful and this certainly holds true for the applications of Darcy's law with relative permeability to the multiphase flow. Firstly, the Darcy's law is linear, while the true governing equation is nonlinear as derived by experiment and micro-scale analysis. In addition to the surface tension; the fluid flow is often shear thinning even if the constituent fluids are Newtonian. The flow will lead to local jamming if the pressure gradient is not sufficient to overcome surface tension and remove individual droplets from a pore region. This jamming phenomenon gives rise to a distinct difference between flows at constant pressure gradient and flows at constant flow rate. Some other difficulties arise when compact droplets undergo break-up or combined and leading to changing droplet size distributions. The big difficulty of the problem is the issue of wet ability and dynamic contact lines moving over the highly irregular solid surface. Now we are not still in a position of real predictive capability for the dynamic contact lines moving through even the most ideal systems. The porous medium filled with water and oil is shown in figure-1 and was given by *P. Bastian &A. Corey* [5], [7] and the analysis at different scales is shown in figure-2 and was given by *P. Bastian & A. Corey* [5], [7].

2. Governing Equations

The motion of a fluid is given by the basic hydrodynamic equations; the equation of continuity is given by

$$\frac{\partial \rho}{\partial t} + \nabla .(\rho\, v) = 0 \tag{2}$$

That gives the conservation of mass and momentum given by the equations:

$$\frac{\partial (\rho v)}{\partial t} + \nabla .(\rho v v) = -\nabla p + \nabla .\tau + \rho g \tag{3}$$

Where ρ is the density, v is the velocity, p is the hydrostatic pressure, τ is the fluid shear tensor and g is acceleration due to external forces including gravity on the fluid. The equation of conservation of energy can be written as:

$$\rho \frac{d\hat{u}}{dt} + p(\nabla . v) = \nabla .(k \nabla T) + \phi \tag{4}$$

Where k the coefficient of thermal conductivity of the fluid, T is temperature, ϕ is the viscous dissipation function and $\hat{u}$ is the density of thermal energy. Here $\hat{u}$ is given by

$$\hat{u}=\hat{u}\ (p, T),$$

This is approximated as:

$$d\hat{u} \approx c_v\ dT,$$

Where c_v is the specific heat.

In the Newtonian fluids the viscous stresses varies with velocity derivatives *i.e.*,

$$\tau_{\alpha\beta} = \partial_\alpha u_\beta \tag{5}$$

Now the equation of momentum will be reduced to Navier-Stokes equation given by

$$\frac{\partial (\rho v)}{\partial t} + \nabla .(\rho v v) = -\nabla p + \mu \nabla^2 v + \rho g \tag{6}$$

Here μ denotes the viscosity of the fluid. For the incompressible fluids, ρ is constant, and then the equation takes the form

$$\frac{\partial v}{\partial t} + (v.\nabla) v = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 v + g \tag{7}$$

Where $^{\mu}/_{\rho} = \nu,$ denotes the kinematics viscosity of the fluid.

The equations for continuity and momentum are generally independent of T and can be solved separately from the equation of energy. The Navier-Stokes equation gives three nonlinear partial differential equations of second order for four unknown variables, *i.e.* pressure and three components of velocity that can be solved in space and time. The system is solved by using with the equation of continuity and using suitable boundary conditions, such as known velocity or pressure at the inlet and outlet or at free surfaces. At solid-fluid interfaces the so-called no-slip boundary condition is the characteristic of all viscous fluid flows. That means that at a solid wall the fluid velocity is zero with respect to the wall. Now we can solve the fluid-flow equations. If we assume the flow as frictionless then the Navier-Stokes equation reduces to the Euler equation given as:

$$g - \frac{1}{\rho}\nabla p = \frac{\partial v}{\partial t} + (v.\nabla)v \tag{8}$$

On integrating along the streamlines in the gravitational field given by

$$g = \hat{g}\hat{e}_{\alpha} \tag{9}$$

We get a relationship between pressure, velocity and the elevation of flow of the fluid; the Bernoulli equation for the incompressible, steady flow is given as:

$$p + \frac{1}{2}\rho v^2 + \rho g \alpha = C_1 \tag{10}$$

Where C_1 is the constant and α is the angle of elevation.

The equation of Bernoulli is related to the steady-flow energy equation and is used with some restrictions. For example; no shaft work or heat transfer is accepted on the streamline. For the case of stationary flow with very low inertial forces, the term on the left-hand side of eq.(6) can be neglected, and it is reduced to the Stokes equation *i.e.*,

$$\nabla p - \rho g = \mu \nabla^2 v \tag{11}$$

The equation (11) is most important in theoretical and experimental work related to fluid flows in porous media, for which the fluid velocities are low. In the flow systems given by the Stokes equation, the pressure drop varies with the fluid velocity, and only one experiment or analysis is used to cover this linear range. If the flow is symmetric and the force is reversed; the streamlines remain unchanged. For the Stokes flow the motion of fluid is smooth and regular *i.e.,* the laminar flow. If the inertial forces become comparable to viscous forces *i.e.,* on increasing the velocity of flow, the flow begins to have instabilities and enters the state of transition. On still increasing the velocity of flow then it will be turned in to the turbulent flow [1], [8].

The ratio of inertial and viscous forces gives us a primary parameter for the behavior of all the Newtonian fluids, and this ratio is the Reynolds number *i.e.*, dimensionless and is given as:

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \tag{12}$$

Where V is the characteristic velocity and L is the length scales of the flow.

The Reynolds number Re gives a criterion for dynamic similarity. For the two similar flow systems of different sizes; the fluids or flow rates have the same Reynolds number then they will have a similar pattern of flow. There exist also many other dimensionless numbers that are very important when results or experiments are applied to real systems. In many cases, we have advantage to express the relevant quantities and equations in the dimensionless form. For example, the dimensionless form of the continuity equation will be

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^*.(\rho^* v^*) = 0 \tag{13}$$

3. Mathematical Analysis and Discussions

The governing equations for the single phase flow of the fluid in porous medium are given by the conservation of mass, Darcy's law and the equation of state as taken by *C. Zhangxin et.al.* [10], [13]. The Darcy's law is given by

$$v = -\frac{K}{\mu}(\nabla p - \rho g) \tag{14}$$

Where *p (x, t)* is the fluid pressure and the unknown function to be determined by the flow model, *g* is the gravity vector pointing in the direction of gravity and *K(x)* is the symmetric tensor of absolute permeability and a parameter of the solid matrix only and may depend on position in the case of a heterogeneous porous medium. The *μ(x, t)* is dynamic viscosity of the given fluid and is taken either constant or a function of pressure. The Darcy's Law is valid for slow flow of a Newtonian fluid through a porous medium with rigid solid matrix given by *C. Zhangxin et.al.* [13]. No slip boundary conditions are taken at the fluid-solid boundary on the microscopic level. *C. Zhangxin et.al.* [13] had taken the assumptions that the mass fluxes due to dispersion and diffusion are very small and can be neglected and that of fluid-solid interface is a material surface with respect to fluid mass such that no mass of fluid can cross it. The spatial and temporal variables are taking as $x=(x, y, z)$ and t respectively. Let ϕ be the porosity of the porous medium, ρ be the density of fluid $v= (v_1, v_2, v_3)$ the superficial Darcy velocity and q be the external sources and sinks.

Let us suppose a cube in such a way that its faces are parallel to the coordinate axes. Its centroid is given by (x, y, z) and its length along coordinate axes are $\Delta x,\ \Delta y$ and $\Delta z.$ the components of mass flux of the fluid are ρv_1, ρv_2 and ρv_3. Now the mass inflow across the surface at $x-\frac{\Delta x}{2}$ per unit time is$(\rho v_1)_{(x-\frac{\Delta x}{2})}\Delta y\Delta z$. And the mass outflow at $x+\frac{\Delta x}{2}$ is$(\rho v_1)_{(x+\frac{\Delta x}{2})}\Delta y\Delta z$. In the similar way, the inflow and outflow masses across the y and z- directions will be $(\rho v_2)_{(y-\frac{\Delta y}{2})}\Delta x\Delta z,\ (\rho v_2)_{(y+\frac{\Delta y}{2})}\Delta x\Delta z$ and $(\rho v_3)_{(z-\frac{\Delta z}{2})}\Delta x\Delta y,\ (\rho v_3)_{(z+\frac{\Delta z}{2})}\Delta x\Delta y$ respectively. By taking the time derivative $\partial/\partial t$, the mass accumulation due to compressibility is $\frac{\partial(\rho\phi)}{\partial t}\Delta x\Delta y\Delta z$ and the mass outflow *i.e.*, mass decrement due to sink of strength q is (-$q\Delta x\Delta y\Delta z$). Now the sum of mass accumulation is equal to the differences of the mass inflows and outflows, so that

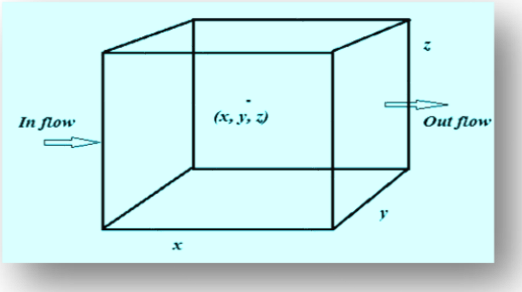


Figure-3 (Fluid flow in a volume element)

$$(15)\ \left[\frac{\partial(\rho\phi)}{\partial t}-q\right]\Delta x\Delta y\Delta z = \left[(\rho v_1)_{\left(x-\frac{\Delta x}{2}\right)}-(\rho v_1)_{\left(x+\frac{\Delta x}{2}\right)}\right]\Delta y\Delta z+\left[(\rho v_2)_{\left(y-\frac{\Delta y}{2}\right)}-(\rho v_2)_{\left(y+\frac{\Delta y}{2}\right)}\right]\Delta x\Delta z + \left[(\rho v)_{\left(z-\frac{\Delta z}{2}\right)}-(\rho v_3)_{\left(z+\frac{\Delta z}{2}\right)}\right]\Delta x\Delta y$$

$$(16)\ \frac{\partial(\rho\phi)}{\partial t}-q = \frac{[(\rho v_1)_{\left(x-\frac{\Delta x}{2}\right)}-(\rho v_1)_{\left(x+\frac{\Delta x}{2}\right)}]}{\Delta x}+\frac{[(\rho v_2)_{\left(y-\frac{\Delta y}{2}\right)}-(\rho v_2)_{\left(y+\frac{\Delta y}{2}\right)}]}{\Delta y} + \frac{[(\rho v_3)_{\left(z-\frac{\Delta z}{2}\right)}-(\rho v_3)_{\left(z+\frac{\Delta z}{2}\right)}]}{\Delta z}$$

Now taking the limit as $\Delta x \to 0, \Delta y \to 0, \Delta z \to 0$, we get

$$(17)\qquad \frac{\partial(\rho\phi)}{\partial t} = -\nabla.(\rho v)+\ q$$

This is the mass conservation equation for three dimensions space.

The equation (17) can be applied to one and two dimensional flows by introducing a factor Â defined as:

Â=Δy Δz, for one dimensional flow;
= Δz, for two dimensional flow;
=1, for three dimensional flows.

So by using this factor α̂, the equation (11) will be

$$(18)\qquad \hat{A}\frac{\partial(\rho\phi)}{\partial t} = -\nabla.\left(\hat{A}\rho u\right)+\ q\hat{A}$$

The ratio of fluid volume at reservoir conditions to the volume ay standard conditions is taken as the factor *B* i.e.,

V(p,T) / V$_s$ *=B(p,T),* where *p, T* denotes the pressure and temperature of the fluid respectively. The suffix‘s’ refers always to standard conditions. Now the equation (12) will be

$$(19)\qquad \hat{A}\frac{\partial\left(\frac{\phi}{B}\right)}{\partial t} = -\nabla.\left(\frac{\hat{A}v}{B}\right)+\frac{q\hat{A}}{\rho_s}$$

The mass conservation equation and Darcy’s law gives the relation

$$(20)\qquad v = -\frac{1}{\mu}K(\nabla p-\rho\hat{g}\nabla z)$$

Where *K* is the absolute permeability factor of the porous medium, μ is the fluid viscosity, $\hat{g}$ is the magnitude due to gravitational acceleration and *z* is the depth.

Here,

$$\nabla p = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right) \tag{21}$$

The z-coordinate is in the vertical downward direction. If we take $K=k_{ij}$ defined as:

$K=k_{ij}$,

$=k_{ii,}$ when $i=j$;

$=0$, when $i\neq j$.

When $k_{11}=k_{22}=k_{33}$, then the porous medium is known as isotropic otherwise anisotropic. From equations (17) and (20), we have

$$\frac{\partial(\rho\phi)}{\partial t} = \nabla.\left(\rho\frac{1}{\mu}K(\nabla p - \rho\hat{g}\nabla z)\right) + q \tag{22}$$

The equation of state in the terms of fluid compressibility c_f is

$$c_f = -\frac{1}{R}\left[\frac{\partial V}{\partial p}\right]_T = \frac{1}{\rho}\left[\frac{\partial \rho}{\partial p}\right]_T \tag{23}$$

Now we define a phase indicator function [5] as given

(24) $\gamma_\alpha=\gamma_\alpha (x, t) =1$, where x belongs to phase α at time t

$=0$, otherwise.

The saturation function $S_\alpha (x, t)$ at the phase α is defined as

$$S_\alpha(X,T) = \frac{Volume\ of\ phase\ \alpha\ in\ reservoir}{Volume\ of\ void\ space\ in\ reservoir} = \frac{\int \gamma_\alpha\,(x,t)dx}{\int \gamma\,(x,t)dx} \tag{25}$$

As defined the saturation, we have

$$\sum_\alpha S_\alpha(x,t) = 1;\ S_\alpha(x,t) \in [0,1] \tag{26}$$

If the porous medium fills the domain $D \subseteq R^3$, then the conservation of mass at α is given by

$$\frac{\partial(\phi\rho_\alpha S_\alpha)}{\partial t} + \nabla.(\rho_\alpha v_\alpha) = \rho_\alpha q_\alpha \tag{27}$$

Here $\rho_{\alpha,}$ $S_{\alpha,}$ v_α , q_α are the density, saturation, velocity and the source term at α respectively.

By the Darcy's law, in the single phase, the macroscopic phase velocity can be written in the term of macroscopic phase pressure by considering the concept that momentum transfer between the phases is negligible, is as given

$$v_\alpha = -\frac{1}{\mu_\alpha}K_\alpha(\nabla p_\alpha - \rho_\alpha\hat{g}) \tag{28}$$

The phase permeability K_α depending on the phase α can be written as:

$$K_\alpha = K_{r\alpha}(S_\alpha)\, K \tag{29}$$

Here $K_{r\alpha}$ is a scalar and is known as relative permeability. The absolute permeability K is independent of the fluid. Here, $0 \leq K_{r\alpha}(S_\alpha) \geq 1$.

From (28) and (29), we get the final expression of Darcy's law by *C. Zhangxin et.al.* [13] for the multiphase flow as given

$$v_\alpha = -\frac{1}{\mu_\alpha} K_{r\alpha}(S_\alpha)\, K(\nabla p_\alpha - \rho_\alpha \hat{g}) \tag{30}$$

$$v_\alpha = -\lambda_\alpha\, K(\nabla p_\alpha - \rho_\alpha \hat{g}) \tag{31}$$

Here $\frac{K_{r\alpha}}{\mu_\alpha} = \lambda_\alpha$, is known as the mobility [10].

P. Bastian & L. Richards [5], [9] analyzed that the pressure on the microscopic level has a jump discontinuity as the fluid passes from one phase to other phase. The height of the jump is the capillary pressure. This is given by a macroscopic capillary pressure on the macroscopic level and is given as:

$$p_{\gamma\beta\alpha} = p_\beta(x,t) - p_\alpha(x,t); \beta \neq \alpha \tag{32}$$

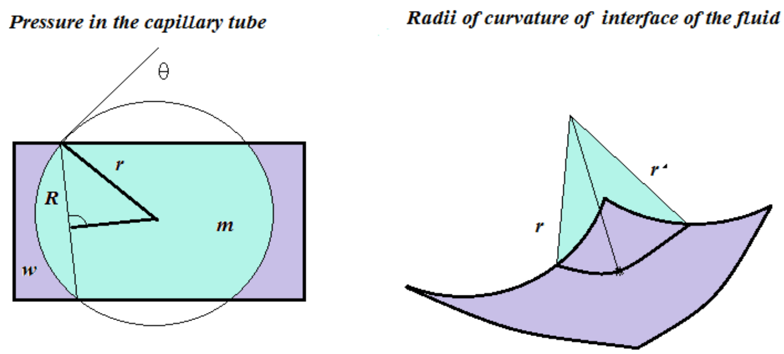
Where the macroscopic capillary pressure $p_{\gamma\alpha\beta}$ is the function of phase distribution at x and the time t, is given as:

$$p_{\gamma\beta\alpha}(x,t) = f(S_1, S_2, \dots\dots, S_n\,) \tag{33}$$

From the equations (32) and (33), it can be seen that only one phase pressure variable can be chosen independently and only *(n-1)* capillary pressure saturation relationships will be required for defining the other remaining phase pressures. Now we are taking a two phase flow system with a wetting phase w and a non–wetting phase m. For this case we want a single capillary pressure function p_c [5], [11], defined as:

$$p_c = \; p_m - p_w \tag{34}$$

The pressure p_m in the non-wetting phase is more than the pressure p_w in the wetting phase. So to derive a relation for the capillary pressure, *P. Bastian* [5] had taken a tube with a small diameter $2R$, which is filled with a wetting phase and a non-wetting phase as given in the figure-4.


Pressure in the capillary tube

Radii of curvature of interface of the fluid

Figure-4 (Capillary pressure in the tube and radii of curvature)

The curved interface has spherical shape with radius r as taken by *Bear and Bachmat* in 1991 [5]. The radii r and R are given by the relation

$$\cos\theta = \frac{R}{r}$$

Let us consider the infinitesimal increase in the radius r is dr. The work required to increase the area of the interface is given by

$$(35) \qquad \Delta W = \sigma\Delta A = \sigma\{A(r+dr) - A(r)\} = r\left(\frac{1}{2} - \frac{\theta}{\pi}\right) 8\pi r\, dr + O(dr^2)$$

Where σ is the surface tension.

Now the work done by capillary pressure of uniform capillary over the interface is

$$(36) \qquad \Delta W = P\, dr = p_c\, A(r) dr = p_c \left(\frac{1}{2} - \frac{\theta}{\pi}\right) 4\pi r^2\, dr$$

From (35) and (36), we get

$$(37) \qquad p_c = \frac{2\sigma\, cos\theta}{R}$$

Where σ and θ depend on the fluid and R depends on the porous medium.

Similarly the capillary pressure at any point of the interface to the surface tension and the principal radii of curvature at this point is given by the expression

$$(38) \qquad p_c = \sigma\left(\frac{1}{r} + \frac{1}{r'}\right)$$

Where r and r' are the principal radii of curvature at this point. The equation (38) is said to be Laplace's equation given by *P. Bastian* [5].

3.1 Equations for the slightly compressible flow and rock

C. Zhangxin et.al. [13] had taken the fluid compressibility c_f as a constant over a certain range of pressures [13] and integrating the equation (23), we have

$$\rho = \rho_0\, e^{c_f(p-p_0)} \tag{39}$$

Here ρ_0 is the density of the fluid at the pressure p_0. By the Taylor series expansion, we have

$$\rho = \rho_0 \left\{1 + c_f(p-p_0) + \frac{{c_f}^2(p-p_0)^2}{2!} + \cdots \right\} \tag{40}$$

Neglecting the terms of higher powers and taking approximations, we get

$$\rho \approx \{1 + c_f(p-p_0)\}$$

The compressibility is given as

$$c_R = \frac{1}{\phi}\frac{d\phi}{dp} \tag{41}$$

Again integrating both the side, we get

$$\phi = \phi_0\, e^{c_R(p-p_0)} \tag{42}$$

Here ϕ_0 is the porosity at the pressure p_0. Also by the Taylor series expansion, we have

$$\phi = \phi_0 \left\{1 + c_R(p-p_0) + \frac{{c_R}^2(p-p_0)^2}{2!} + \cdots \right\} \tag{43}$$

This can also be approximated as:

$$\phi \approx \phi_0 \{1 + c_R(p-p_0)\} \tag{44}$$

So we have

$$\frac{d\phi}{dp} = c_R\phi_0 \tag{45}$$

After taking the differentiation on the left-hand side of (22), we have

$$\left\{\phi\frac{\partial\rho}{\partial p} + \rho\frac{d\phi}{dp}\right\}\frac{\partial p}{\partial t} = \nabla.\left(\rho\frac{1}{\mu}K(\nabla p - \rho\hat{g}\nabla z)\right) + q \tag{46}$$

Put the approximated calculated values [13] in (32), we have

$$\rho\{\phi c_f + c_R\phi_0\}\frac{\partial p}{\partial t} = \nabla.\left(\rho\frac{1}{\mu}K(\nabla p - \rho\hat{g}\nabla z)\right) + q \tag{47}$$

Let us taking, the total compressibility c_t as given

$$c_t = c_f + \frac{\phi_0}{\phi} c_R \tag{48}$$

We have

$$\rho_0\, e^{c_f(p-p_0)} \phi c_t \frac{\partial p}{\partial t} = \nabla . \left(\rho_0\, e^{c_f(p-p_0)} \frac{1}{\mu} K \left(\nabla p - \rho_0\, e^{c_f(p-p_0)} \hat{g} \nabla z \right) \right) + \; q \tag{49}$$

4. Numerical Simulation

The table-1 gives the values of calculated pressure of the three fluids, kerosene oil, water and turpentine oil.

Table-1 (Numerical values of the pressure exerted by the kerosene oil, water and turpentine oil in a porous media having the values of K between $1 \text{x} 10^{-11}$ to $25 \text{x} 10^{-11}$ m^2)

S.No.	Fluid	Viscosity μ(Pa s)	Velocity v m/sec.	p(Pa) x10^7 for K=1 x$10^{-11} m^2$	p(Pa) x10^7 for K=2 x$10^{-11} m^2$	p(Pa) x10^7 for K=5 x$10^{-11} m^2$	p(Pa) x10^7 for K=10 x$10^{-11} m^2$	p(Pa)x10^7 for K=25x10^{-11} m^2
1.	Kerosene oil	640x10^{-6}	1	6.4	3.2	1.28	0.64	0.26
			2	12.8	6.4	2.56	1.28	0.52
			5	32	16	6.4	3.2	1.30
			10	64	32	12.8	6.4	2.60
			25	160	80	32	16	6.4
2.	Water	890x10^{-6}	1	8.9	4.45	2.97	0.89	0.36
			2	17.8	8.9	3.35	1.78	0.72
			5	44.5	22.25	8.9	4.45	1.80
			10	89.0	44.5	17.8	8.9	3.60
			25	222.5	111.25	44.5	22.25	9.00
3.	Turpantine oil	1375x10^{-6}	1	13.75	6.875	2.75	1.38	0.55
			2	27.5	13.75	5.5	2.75	1.1
			5	68.75	34.375	13.75	6.88	2.75
			10	137.5	68.75	27.5	13.75	5.5
			25	343.75	171.875	68.75	34.50	13.75

The graphical representations of variation of pressure with respect to permeability factor *K* and velocity *v* of the three fluids kerosene oil, water and turpentine oil are shown in figure-5 and figure-6 respectively.

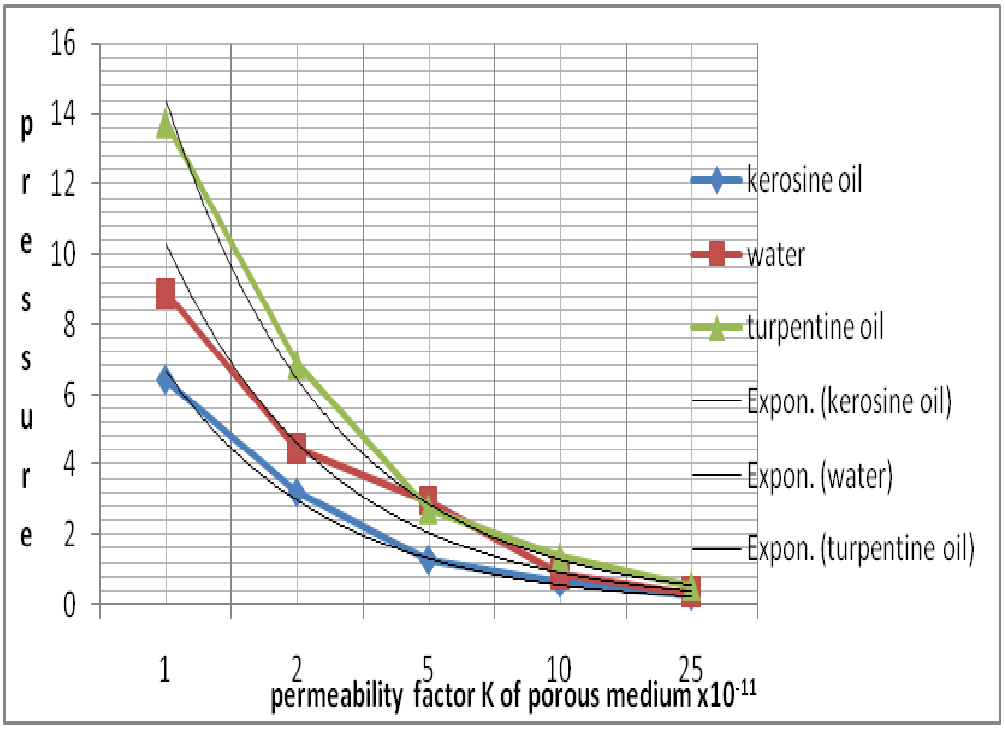


Figure-5 (Variation of pressure with respect to *K* for *v*=1m/sec.)

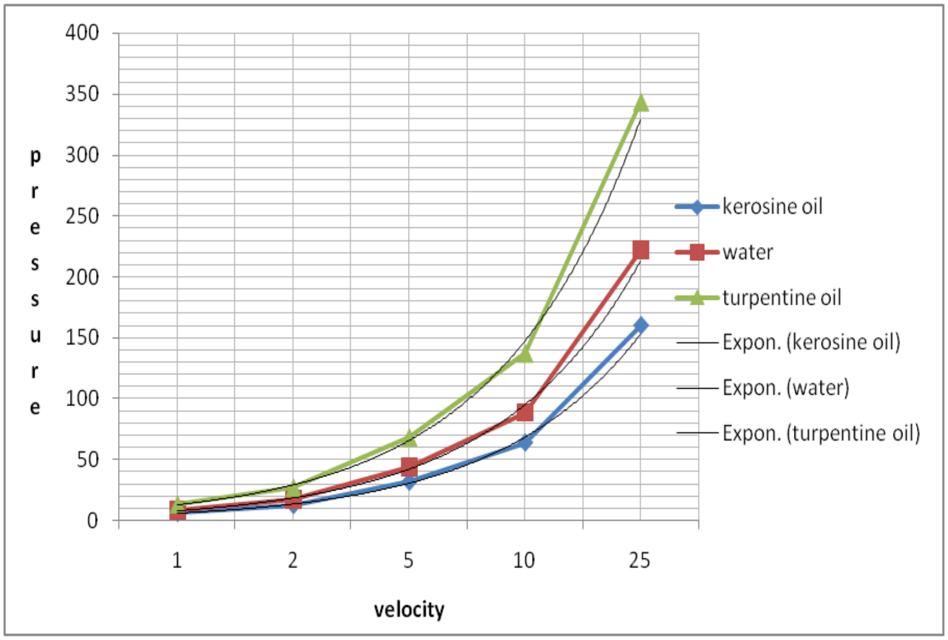


Figure-6 (Variation of pressure with respect to velocity for K=1m^2)

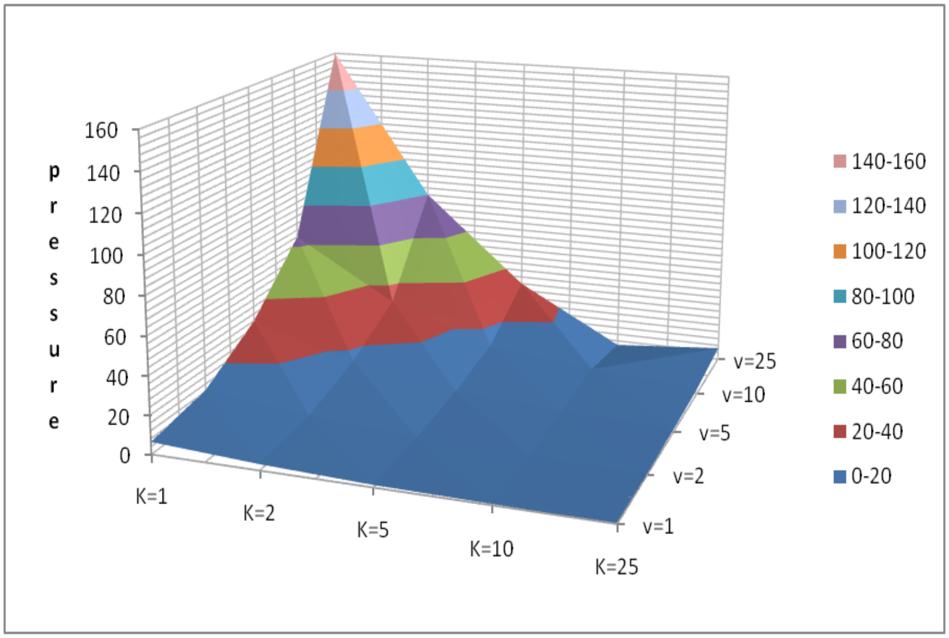


Figure-7 (Surface representation of pressure, velocity of fluid and permeability factor *K* of the porous medium for kerosene oil)

The surface represented by the figure-7 shows that the pressure on the fluid increases with velocity and decreases with the increasing values of permeability factor *K*.

5. Results and Conclusions

For the slightly compressible flow and rock, the motion always governed by the law of conservation of mass and Darcy's law given by equation (30), that shows the velocity of the fluid at phase α varies inversely with the viscosity of the fluid. The capillary pressure p_c varies inversely with the radius of tube, and increases with increase in the surface tension σ of the fluid. It also varies inversely with the radii of curvature of the interface of the fluid. The compressibilityc_R is not independent of the porosity ϕ of the fluid and varies directly with ϕ. The final partial differential equation for the slightly compressible flow and rock is given by equation (49). The variations of pressure exerted by fluid with respect to velocity of the fluid and permeability factor of the porous medium are shown by the figures-5, 6 and 7. These show that the pressure varies directly with velocity of fluid and varies inversely with permeability factor *K* of the medium.

Conflict of Interests

The authors declare that there is no conflict of interests.

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