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SUBDIVISION OF SUPER GEOMETRIC MEAN LABELING FOR SOME MORE GRAPHS

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Abstract. Let f: V(G) $\rightarrow$ {1,2,…,p+q} be an injective function. For a vertex labeling "f", the induced edge labeling f* (e=uv) is defined by, f*(e) = $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$. Then "f" is called a Super Geometric Mean Labeling if {f(V(G))}$\cup${f(e):e$\in$E(G)}={1,2,...,p+q}, A graph which admits Super Geometric mean labeling is called Super Geometric mean graph. In this paper we prove that the Subdivision of Super Geometric mean labeling for some standard graphs.

Key words: graph; subdivision graph; geometric mean graph; triangular snake and quadrilateral snake.

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1. Introduction

All graphs here will be finite undirected and simple. Let V(G) and E(G) will denote the vertex set and edge set of a graph G. The cardinality of the vertex set of a graph G is denoted by p and the cardinality of its edge set is denoted by q. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of "Geometric mean labeling" was introduced and the basic results proved in [6]. The concept of "Mean labeling" on subdivision was introduced in [4]. We investigate the Super Geometric mean labeling behaviour of S(G) for some standard graphs.

The definitions and other informations which are necessary for our present investigation.

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Definition 1.1 A graph G = (V,E) with p vertices and q edges is called a Geometric mean graph if it is possible to label to the vertices $x \in V$ with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labled with, f(e=uv) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$, then the edge labels are distinct. In this case "f" is called **Geometric mean labeling** of G.

Definition 1.2 Let f : V(G) → {1,2,…,p+q} be an injective function. For a vertex labeling "f" the induced edge labeling f*(e=uv) is defined by, f*(e) =$\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then "f" is called a Super Geometric mean labeling if {f(V(G))} ∪{f(e) : e∈E(G)} = {1,2,…,p+q}. A graph which admits Super Geometric mean labeling is called **Super Geometric mean graph.**

Definitions 1.3 If e=uv is an edge of G and w is not a vertex of G then e is said to be subdivided when it it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the **Subdivision** graph of G and is denoted by S(G).

Definition 1.4 A **Path** P_n is a walk in which all the vertices are distinct.

Definition 1.5 The graph obtained by attaching C_m to an end vertex of P_n is called a **Kite** graph.

Definition 1.6 A graph **$P_n A K_{1,2}$** is obtained by attaching $K_{1,2}$ to each vertex of P_n.

Definition 1.7 The graph **$P_n A K_{1,3}$** is obtained by attaching $K_{1,3}$ to each vertex of P_n.

Definition 1.8 A **Triangular Snake** T_n is obtained from a Path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n-1$. That is every edge of a Path is replaced by a triangle C_3.

Definition 1.9 A **Quadrilateral Snake** Q_n is obtained from a Path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} to new vertices v_i and w_i respectively and joining v_i and w_i. That is every edge of a Path is replaced by a cycle C_4.

Now we shall use frequent reference to the following theorems.

Theorem 1.10 [6]: Any Path is a Geometric mean graph.

Theorem 1.11 [6]: Kite graphs are Geometric mean graphs.

Theorem 1.12 [6]: $P_n A K_{1,2}$ is a Geometric mean graph.

Theorem 1.13 [6]: $P_n A K_{1,3}$ is a Geometric mean graph.

Theorem 1.14[6]: Triangular snakes are Geometric mean graphs.

Theorem 1.15[6]: Quadrilateral snakes are Geometric mean graphs.

2. Main Results

Theorem 2.1 Let $G = P_n \mathbb{A} C_3$ be a graph obtained by attaching C_3 to each vertex of a Path P_n. Let G_1 be the graph obtained by subdividing the edges of P_n of G. Then G_1 is a Super Geometric mean graph.

Proof: Let G be a graph obtained by attaching C_3 to each vertex of a Path P_n.

Let P_n be a Path $u_1u_2\ldots u_n$.

Let u_i, v_i, w_i, $1\leq i\leq n$ be the vertices of C_3.

Let G_1 be the graph obtained by subdividing all the edges of the Path G.

Let t_i, $1\leq i\leq n-1$ be vertices which subdivide u_i and u_{i+1}.

Define a function, $f: V(G_1) \rightarrow \{1,2,\ldots,p+q\}$by,

$f(v_1) = 1$

$f(v_i) = 9i-9,\ 2\leq i\leq n$

$f(w_i) = 9i-5,\ 1\leq i\leq n$

$f(u_i) = 9i-3,\ 1\leq i\leq n$

$f(t_i) = 9i-1,\ 1\leq i\leq n-1$

Edges are labeled with,

$f(v_1w_1) = 2$

$f(v_i w_i) = 9i-8,\ 2\leq i\leq n$

$f(v_iu_i) = 9i-6,\ 1\leq i\leq n,$

$f(u_i w_i) = 9i-4,\ 1\leq i\leq n$

$f(u_it_i) = 9i-2,\ 1\leq i\leq n-1$

$f(t_iu_{i+1}) = 9i+2,\ 1\leq i\leq n-1$

Thus both vertices and edges together get distinct labels from $\{1,2,3,\ldots,p+q\}$.

Hence G_1 is a Super Geometric mean graph.

Example 2.2 A Subdivision of each edge of a Path of $P_5 \mathbb{A} C_3$ is displayed below.

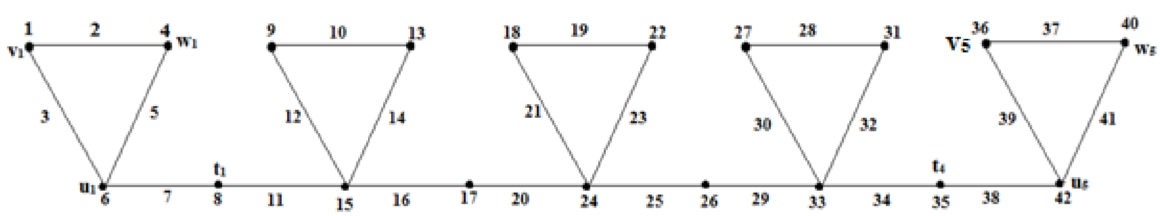

Figure: 1

Theorem 2.3 Let P_n be a Path and G be the graph obtained from P_n by attaching C_3 in both end edges of P_n. Let G_1 be the graph obtained by subdividing the edges of P_n of G. Then G_1 is a Super Geometric mean graph.

Proof: Let P_n be a Path $u_1u_2\ldots u_n$ and u_1xu_2, $u_{n-1}\, yu_n$ be the triangles at the end edges of P_n.

Let G be the graph obtained from P_n by attaching C_3 in both end edges of P_n.

Let G_1 be the graph obtained by subdividing the edges of P_n of G.

Let w_i, $1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Define a function, f: $V(G_1) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(x) = 1$

$f(u_i) = 4i$, $1 \le i \le n-1$

$f(u_n) = 4n+1$

$f(w_i) = 4i+2$, $1 \le i \le n-1$

$f(y) = 4n+3$

Edges are labeled with,

$f(xu_1) = 2$

$f(xu_2) = 3$

$f(u_iw_i) = 4i+1$, $1 \le i \le n-1$

$f(w_i\, u_{i+1}) = 4i+3$, $1 \le i \le n-2$

$f(w_{n-1}\, u_n) = 4n$

$f(yu_6) = 4n-1$

$f(yu_7) = 4n+2$

In view of the above labeling pattern, "f" provides a Super Geometric mean labeling of G_1. Hence G_1 is a Super Geometric mean graph.

Example 2.4 Let G be the graph obtained from P_7 by attaching C_3 in both end edges of P_7. The Subdivision of each edge of P_7 of G is given below.

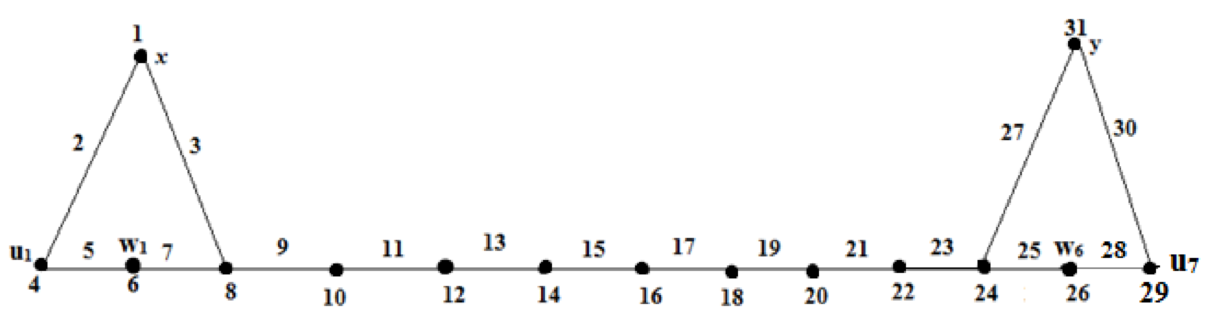

Figure: 2

Theorem 2.5 Let G be a graph obtained by attaching C_3 to an end edge of P_n. Let G_1 be the graph obtained by subdividing the edges of P_n of G. Then G_1 is a Super Geometric mean graph.

Proof: Let P_n be a Path $u_1u_2\ldots u_n$ and $u_{n-1}\,xu_n$ be the triangle at the end edge of P_n.

Let G be a graph obtained by attaching C_3 to an end edge of P_n.

Let G_1 be the graph obtained by subdividing the edges of P_n of G.

Let w_i, $1\leq i\leq n-1$ be the vertices which subdivide u_i and u_{i+1}

Define a function, $f: V(G_1) \rightarrow \{1,2,\ldots,p+q\}$by,

$f(u_i) = 4i-3,\ 1\leq i\leq n-1$

$f(u_n) = 4n-2$

$f(w_i) = 4i-1,\ 1\leq i\leq n-1$

$f(x) = 4n$

Edges are labeled with,

$f(u_i\, w_i) = 4i-2,\ 1\leq i\leq n-1$

$f(w_i\, u_{i+1}) = 4i,\ 1\leq i\leq n-2$

$f(w_{n-1}\, u_n) = 4n-3$

$f(u_4\, x) = 4n-4$

$f(u_5\, x) = 4n-1$

Thus both vertices and edges together get distinct labels from $\{1,2,3,\ldots,p+q\}$.

Hence G_1 is a Super Geometric mean graph.

Example 2.6 Let G be the graph obtained from P_5 by attaching C_3 to an end edge of P_5. The subdivision of each edge of P_5 of G is shown below.

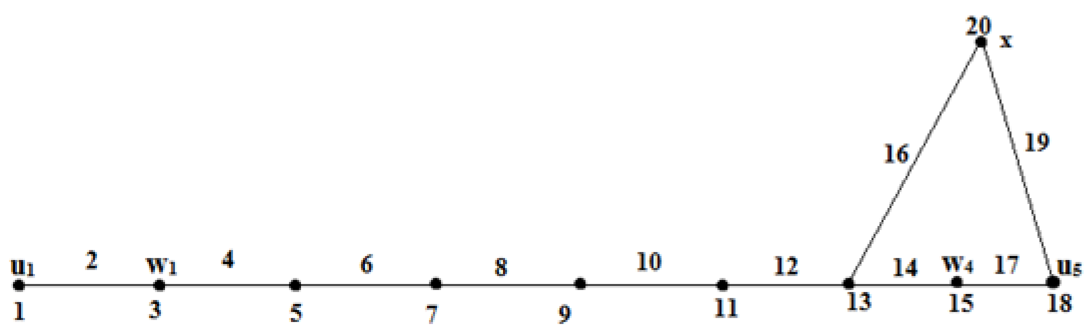

Figure: 3

Theorem 2.7 Let G be a graph obtained by attaching C_4 to an end edge of P_n. Let G_1 be the graph obtained by subdividing the edges of P_n of G. Then G_1 is a Super Geometric mean graph.

Proof: Let P_n be a Path $u_1u_2\dots u_n$ and $u_{n-1}\ u_n\ xy$ be the cycle C_4.

Let G be a graph obtained by attaching C_4 to an end edge of P_n.

Let G_1 be the graph obtained by subdividing the edges of P_n of G.

Let w_i, $1\le i\le n-1$ be the vertices which subdivide u_i and u_{i+1}

Define a function, f: $V(G_1) \rightarrow \{1,2,\dots,p+q\}$by,

$f(u_i) = 4i-3$, $1\le i\le n-1$

$f(u_n) = 4n$

$f(w_i) = 4i-1$, $1\le i\le n-2$

$f(w_{n-1}) = 4n-4$

$f(x) = 4n+2$

$f(y) = 4n-3$

Edges are labeled with,

$f(u_iw_i) = 4i-2$, $1\le i\le n-1$

$f(w_i\ u_{i+1}) = 4i$, $1\le i\le n-2$

$f(w_{n-1}\ u_n) = 4n-2$

$f(u_{n-1}\ y) = 4n-5$

$f(yx) = 4n-1$

$f(xu_n) = 4n+1$

Thus we get distinct edge labels.

Hence G_1 is a Super Geometric mean graph.

Example: 2.8 Let G be the graph obtained from P_5 by attaching C_4 to an end edge of P_5. The subdivision of each edge of P_5 of G is displayed below.

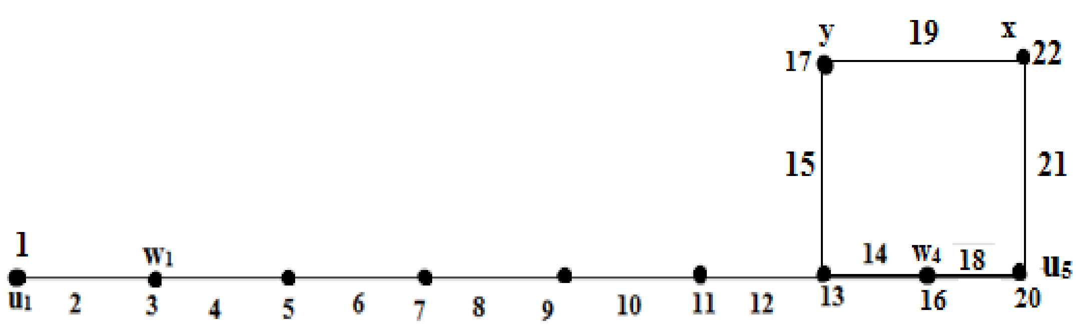

Figure: 4

Theorem: 2.9 Let P_n be the Path $u_1u_2\ldots u_n$. Let G be the graph obtained by attaching $K_{1,2}$ at each vertex of P_n. Let G_1 be the graph obtained by subdividing the Path P_n. Then G_1 is a Super Geometric mean graph.

Proof: Let P_n be the Path $u_1u_2\ldots u_n$.

Let v_i and w_i, $1\leq i\leq n$ be the vertices of $K_{1,2}$, which are attached to each u_i of P_n.

Let G_1 be the graph obtained by subdividing the edges of P_n.

Let t_i, $1\leq i\leq n-1$ be the vertices which subdivide u_i and u_{i+1}.

Then the graph G_1 contains 4n-1 vertices and 4n-2 edges and the graph G_1 is given below.

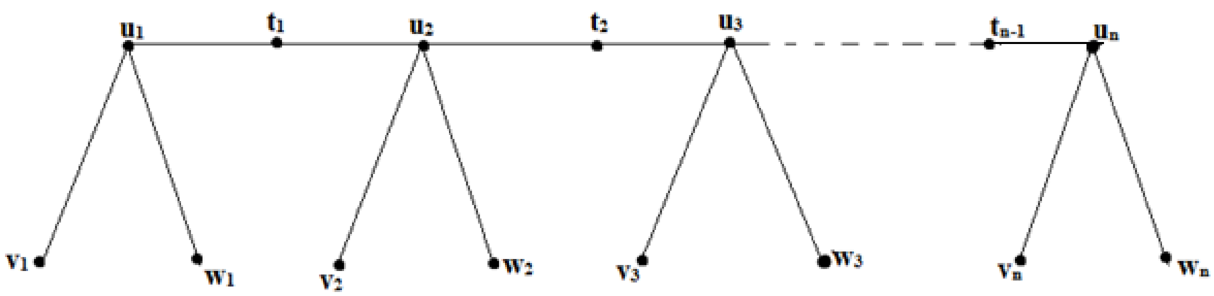

Figure: 5

Define a function, $f: V(G_1) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_i) = 8i-5, \quad 1\leq i\leq n$

$f(v_1) = 1$

$f(v_i) = 8i-9, \quad 2\leq i\leq n$

$f(w_i) = 8i-3, \quad 1\leq i\leq n$

$f(t_i) = 8i+1, \quad 1\leq i\leq n-1$

Edges are labled with,

$f(v_1u_1) = 2$

$f(v_iu_i) = 8i-8, \quad 2\leq i\leq n$

$f(w_iu_i) = 8i-4, \quad 1\leq i\leq n$

$f(u_i t_i) = 8i-2, \quad 1\leq i\leq n-1$

$f(t_iu_{i+1}) = 8i+2, \quad 1\leq i\leq n-1$

This gives a Super Geometric mean labeling of G_1.

Example 2.10 $S(P_5AK_{1,2})$ is shown below.

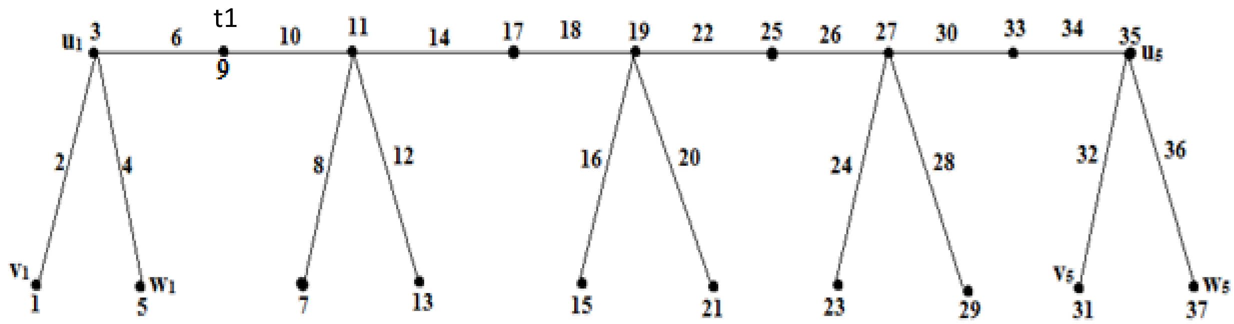

Figure: 6

Theorem 2.11 Let P_n be the Path $u_1u_2\ldots u_n$. Let G be the graph obtained by attaching $K_{1,3}$ at each vertex of P_n. Let G_1 be the graph obtained by subdividing the Path P_n. Then G_1 is a Super Geometric mean graph.

Proof: Let P_n be the Path $u_1u_2\ldots u_n$.

Let v_i, w_i, z_i, $1\le i\le n$ be the vertices of $K_{1,3}$, which are attached to each vertex u_i of P_n.

Let G_1 be the graph obtained by subdividing the edges of P_n.

Let t_i, $1\le i\le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Then the graph G_1 contains 5n-1 vertices and 5n-2 edges and the graph G_1 is shown below.

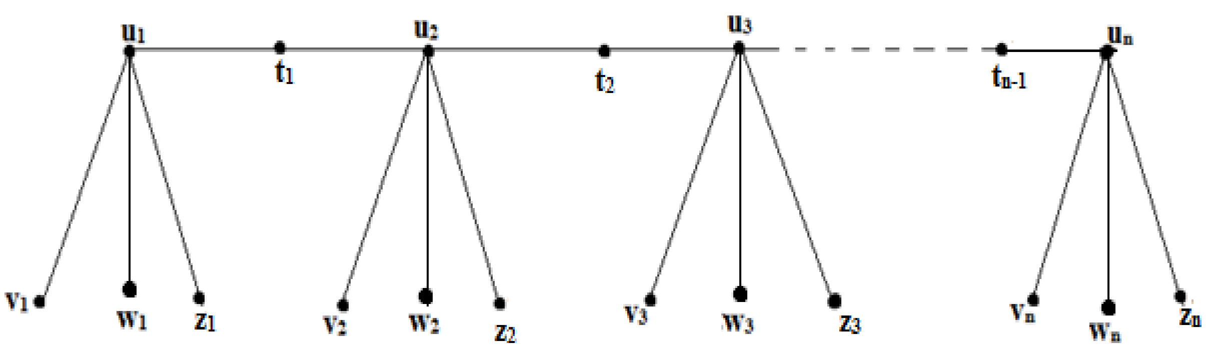

Figure: 7

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_i) = 10i-5, \quad 1\le i\le n$

$f(v_1) = 1$

$f(v_i) = 10i-11, \ 2\le i\le n$

$f(w_i) = 10i-7$, $1\leq i\leq n$

$f(z_i) = 10i-3$, $1\leq i\leq n$

$f(t_i) = 10i$, $1\leq i\leq n-1$

Edges are labeled with,

$f(v_1u_1) = 2$

$f(v_iu_i) = 10i-9$, $2\leq i\leq n$

$f(w_i\, u_i) = 10i-6$, $1\leq i\leq n$

$f(z_iu_i) = 10i-4$, $1\leq i\leq n$

$f(u_i\, t_i) = 10i-2$, $1\leq i\leq n-1$

$f(t_i\, u_{i+1}) = 10i+2$, $1\leq i\leq n-1$

$\therefore \{f(V(G_1))\} \cup \{f(e) : e\in E(G)\}=\{1,2,\ldots,p+q\}$

Thus G_1 is a Super Geometric mean graph.

Example 2.12 $S(P_5 A K_{1,3})$ is displayed below

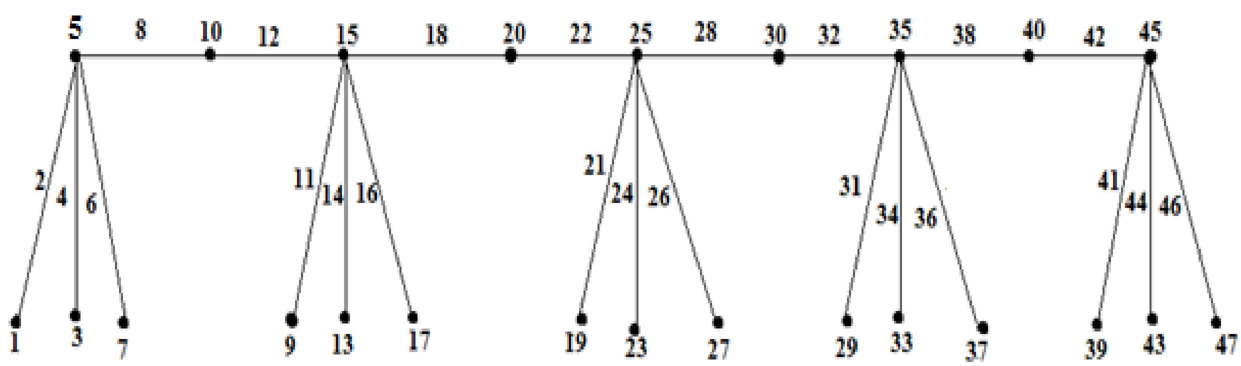

Figure: 8

Theorem 2.13

Subdivision of Triangular snake is a Super Geometric mean graph.

Proof:

Let T_n be a Triangular snake which is obtained from a Path $P_n = u_1u_2\ldots u_n$ by joining u_i and u_{i+1} to a new vertex v_i, $1\leq i\leq n-1$.

Let $S(T_n) = T_N$ be a graph obtained by subdividing all the edges of T_n.

Here we consider the following cases.

Case :1

Let T_N be a graph which is obtained by subdividing each edge of P_n.

Let w_i, $1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Define a function f: $V(T_N) \to \{1,2,\ldots,p+q\}$ by,

$f(v_1) = 1$

$f(v_i) = 7i-4,\ 2 \le i \le n-1$

$f(u_1) = 4$

$f(u_i) = 7i-6,\ 2 \le i \le n$

$f(w_i) = 7i-1,\ 1 \le i \le n-1$

Edges are labeled with,

$f(u_1w_1) = 5$

$f(u_iw_i) = 7i-3,\ 2 \le i \le n-1$

$f(w_i\, u_{i+1}) = 7i,\ 1 \le i \le n-1$

$f(u_i\, v_i) = 7i-5,\ 1 \le i \le n-1$

$f(u_2v_1) = 3,$

$f(u_{i+1}\, v_i) = 7i-2,\ 2 \le i \le n-1$

The labeling pattern is shown in the following figure.

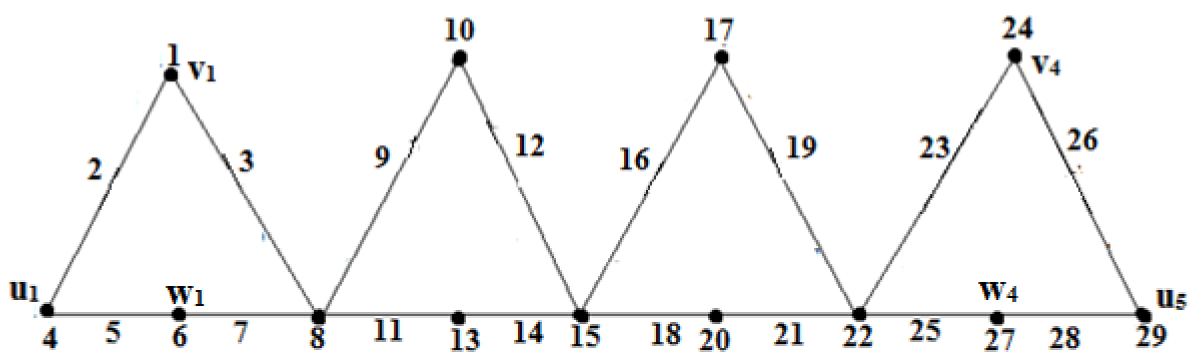

Figure: 9

From the above labeling pattern, we get distinct edge labels.

Hence T_N is a Super Geometric mean graph.

Case: 2

Let T_N be the graph obtained by subdividing the edges u_iv_i and $u_{i+1}\, v_i$.

Let x_i and y_i, $1 \le i \le n-1$ be the vertices which subdivide the edges u_iv_i and $u_{i+1}\, v_i$, respectively.

Define a function f: $V(T_N) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_i) = 9i-8, \quad 1 \leq i \leq n$

$f(v_i) = 9i-3, 1 \leq i \leq n-1$

$f(x_1) = 4$

$f(x_i) = 9i-6, 2 \leq i \leq n-1$

$f(y_i) = 9i-1, 1 \leq i \leq n-1$

Edges are labeled with,

$f(u_1u_2) = 3$

$f(u_i\, u_{i+1}) = 9i-4, 2 \leq i \leq n-1$

$f(u_ix_i) = 9i-7, 1 \leq i \leq n-1$

$f(y_i\, u_{i+1}) = 9i, 1 \leq i \leq n-1$

$f(x_1v_1) = 5$

$f(x_iv_i) = 9i-5, 2 \leq i \leq n-1$

$f(u_ix_i) = 9i-7, 1 \leq i \leq n-1$

$f(y_iv_i) = 9i-2, 1 \leq i \leq n-1$

The labeling pattern is shown in the following figure.

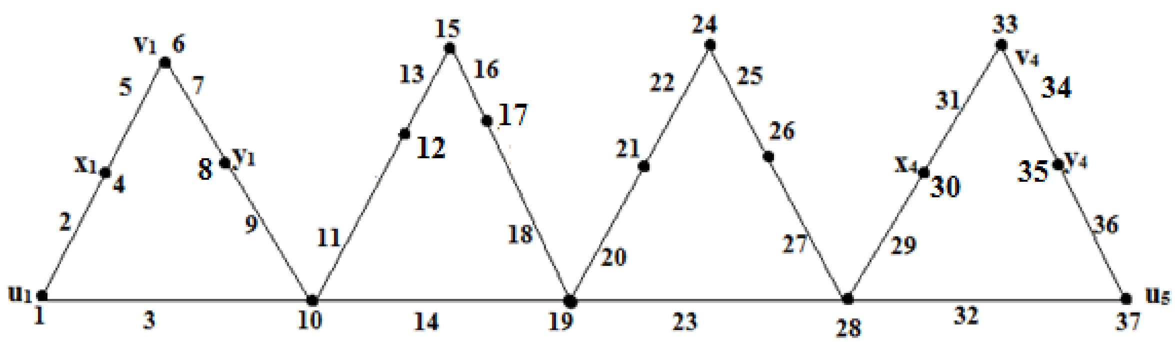

Figure: 10

From the above labeling pattern, we get distinct edge labels.

Hence T_N is a Super Geometric mean graph.

Case: 3

Let T_N be the graph obtained by subdividing all the edges of T_n.

Let w_i, $1 \leq i \leq n-1$ be the vertices which subdivide u_i and u_{i+1}.

Let x_i and y_i, $1\le i\le n-1$ be the vertices which subdivide the edges u_iv_i and $u_{i+1} v_i$ respectively.

Define a function, $f: V(T_N) \to \{1,2,\ldots,p+q\}$ by,

$f(v_1) = 1$

$f(v_i) = 10i-5,\ 2\le i\le n-1$

$f(u_1) = 6$

$f(u_i) = 11i-10,\ 2\le i\le n$

$f(w_1) = 8$

$f(x_i) = 11i-7,\ 1\le i\le n-1$

$f(y_i) = 11i-1,\ 1\le i\le n-1$

Edges are labeled with,

$f(u_1w_1) = 7$

$f(u_i w_i) = 11i-8,\ 2\le i\le n-1$

$f(w_i u_{i+1}) = 11i-2,\ 1\le i\le n-1$

$f(u_1x_1) = 5$

$f(u_i x_i) = 11i-9,\ 2\le i\le n-1$

$f(u_{i+1} y_i) = 11i,\ 1\le i\le n-1$

$f(x_1v_1) = 2$

$f(x_iv_i) = 11i-6,\ 2\le i\le n-1$

$f(y_1v_1) = 3$

$f(y_iv_i) = 11i-3,\ 2\le i\le n-1$

The labeling pattern is shown in the following figure.

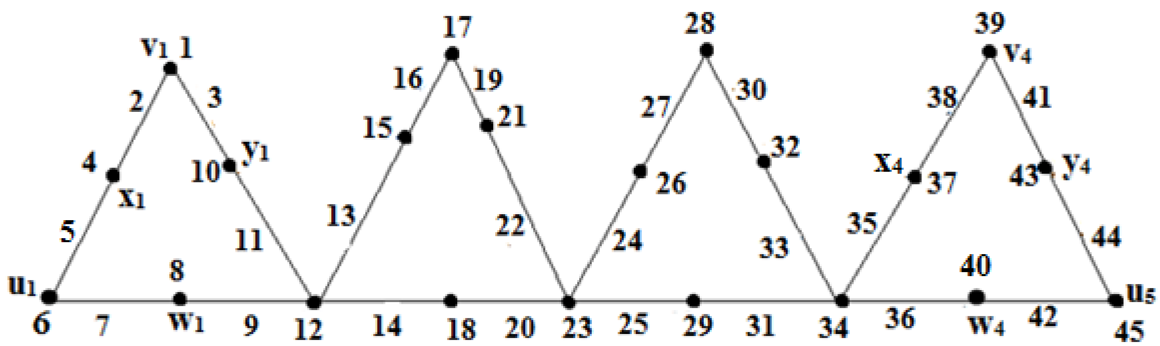

Figure: 11

From the above labeling pattern, we get distinct edge labels.

Hence T_N is a Super Geometric mean graph.

From the cases 1,2 and 3 it can be verified that $S(T_n) = T_N$ is a Super Geometric mean graph.

Theorem 2.14 Subdivision of any Quadrilateral snake is a Super Geometric mean graph.

Proof: Let Q_n be a Quadrilateral snake which is obtained from a Path $P_n = u_1u_2\ldots u_n$ by joining u_i and u_{i+1} to new vertices v_i and w_i respectively and joining v_i and w_i $1\leq i\leq n-1$.

Let $S(Q_n) = Q_N$ be a graph obtained by subdividing all the edges of Q_n.

Here we consider the following cases.

Case: 1 Let Q_N be the graph which is obtained by subdividing each edge of P_n.

Let t_i, $1\leq i\leq n-1$ be the vertices which subdivide u_i and u_{i+1}

Define a function $f: V(Q_N) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_i) = 9i-8, \quad 1\leq i\leq n$

$f(t_i) = 9i-1, \quad 1\leq i\leq n-1$

$f(v_1) = 4$

$f(v_i) = 9i-6, \quad 2\leq i\leq n-1$

$f(w_i) = 9i-3, \quad 1\leq i\leq n-1$

Edges are labeled with,

$f(u_1t_1) = 3$

$f(u_it_i) = 9i-5, \quad 2\leq i\leq n-1$

$f(t_iu_{i+1}) = 9i, \quad 1\leq i\leq n-1$

$f(u_{i+1}w_i) = 9i-2, 1\leq i\leq n-1$

$f(v_iw_i) = 9i-4, \quad 1\leq i\leq n-1$

The labeling pattern is displayed in the following figure.

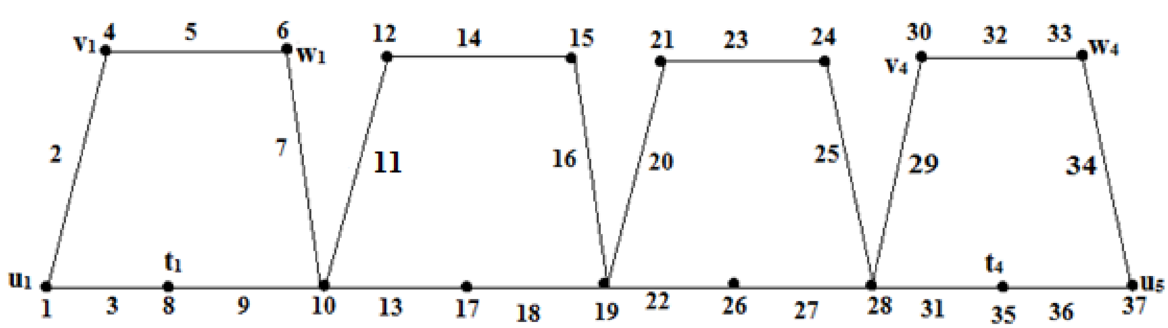

Figure: 12

From the above labeling pattern, we get distinct edge labels

Hence Q_N is a Super Geometric Mean Graph

Case: 2

Let Q_N be the graph which is obtained by subdividing all the edges of Q_n.

Let t_i, $1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Let x_i and y_i, $1 \le i \le n-1$ be the vertices which subdivide the edges $u_i v_i$ and $u_{i+1} w_i$ respectively.

Let z_i, $1 \le i \le n-1$ be the vertices which subdivide $v_i w_i$.

Define a function f: $V(Q_N) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_i) = 15i-14, \ 1 \le i \le n$

$f(t_i) = 15i-1, 1 \le i \le n-1$

$f(x_1) = 4$

$f(x_i) = 15i-12, 2 \le i \le n-1$

$f(y_i) = 15i-3, 1 \le i \le n-1$

$f(v_1) = 6$

$f(v_i) = 15i-10, 2 \le i \le n-1$

$f(w_i) = 15i-5, 2 \le i \le n-1$

$f(z_i) = 15i-7, 1 \le i \le n-1$

Edges are labeled with,

$f(u_1t_1) = 3$

$f(u_it_i) = 15i-8, 2 \le i \le n-1$

$f(t_iu_{i+1}) = 15i, 1 \le i \le n-1$

$f(v_1z_1) = 7$

$f(v_iz_i) = 15i-9, \quad 2 \le i \le n-1$

$f(z_iw_i) = 15i-6, \quad 1 \le i \le n-1$

$f(u_i x_i) = 15i-13, 1 \le i \le n-1$

$f(x_1v_1) = 5$

$f(x_iv_i) = 15i-11, \quad 2 \le i \le n-1$

$f(u_{i+1} y_i) = 15i-2, 1 \le i \le n-1$

$f(y_iw_i) = 15i-4, \quad 1 \le i \le n-1$

The labeling pattern is shown in the following figure.

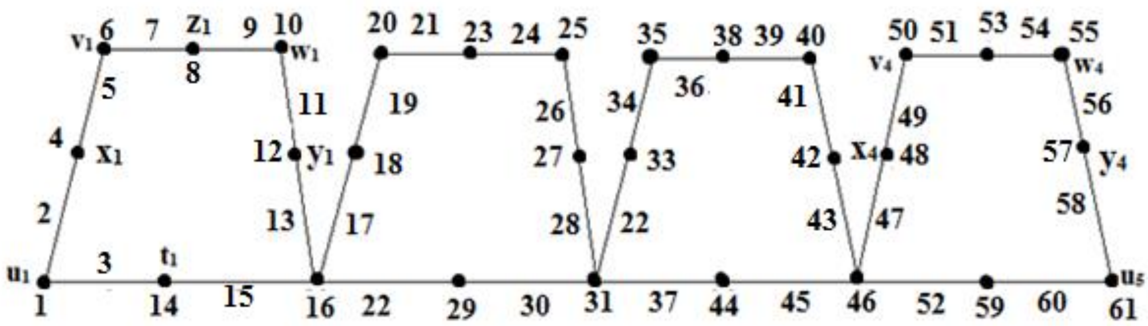

Figure: 13

From the above labeling pattern, we get distinct edge labels.

Hence Q_N is a Super Geometric mean graph.

From case 1 and case 2, it can be verified that, $S(Q_n) = Q_N$ is a Super Geometric mean graph.

Conflict of Interests

The authors declare that there is no conflict of interests.

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