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DARBOUX ROTATION AXIS OF A NULL CURVE IN MINKOWSKI 3-SPACE

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Abstract. The Darboux rotation for a null curve in Minkowski 3- space is decomposed into two simultaneous rotations. The axes of these simultaneous rotations are joined by a simple mechanism. One of these axes is a parallel of the principal normal vector of the curve, the direction of the other axis is the direction of the Darboux vector of the curve.

Keywords: Darboux vector, Darboux rotation axis,Minkowski Space.

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1. Introduction

In differential geometry, especially the theory of space curves, the Darboux vector is the areal velocity vector of the Frenet frame of a space curve. It is named after Gaston Darboux who discovered it. It is also called angular momentum vector, because it is directly proportional to angular momentum.

Now, we will define Darboux vector in Minkowski 3-space. While point P draws the curve, the vectors T,N,B change. Hence these vectors generate spherical images of the curve. Assume that Frenet trihedron $\{T,N,B\}$ of the curve makes an instantaneous helix motion around an axis

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at each s time. This axis is called Darboux axis with the parameter s at point $\alpha(s)$. The vector giving orientation and direction of this axis is called Darboux vector at point $\alpha(s)$ of the curve [6].

In this paper, the Darboux rotation for a null curve in Minkowski 3- space is decomposed into two simultaneous rotations. The axes of these simultaneous rotations are joined by a simple mechanism. One of these axes is a parallel of the principal normal vector of the curve, the direction of the other axis is the direction of the Darboux vector of the curve.

2. Preliminaries

Let consider a 3-dimensional Minkowski space E_1^3 which is defined as a usual 3-dimensional vector space consisting of $\{(x_1,x_2,x_3)\,|x_1,x_2,x_3 \in \mathbb{R}\}$vectors, but with a linear connection D corresponding to its Minkowski metric $\langle,\rangle$ given by

$$\langle x,y\rangle = -x_1y_1 + x_2y_2 + x_3y_3.$$

where $x = (x_1,x_2,x_3)$ and $y = (y_1,y_2,y_3)$.

The set of all null vectors of E_1^3 forms the light cone in coordinates;

$$\left\{(x_1,x_2,x_3)\,|x_1^2 = x_2^2 + x_3^2, x_1 \neq 0\right\}.$$

In E_1^3, the rules of calculus are the same as in Euclidean space.Thus, we can speak of immersions and regular or smooth curves just as in the Euclidean case.However, since any E_1^3 can have a null curve (for example, any null vector can generate a null curve), the calculus of null curves is quite different than the non-null curves.

To derive the Frenet type equations for a null curve α, defined by $\alpha : [a,b] \longrightarrow E_1^3$, Cartan [3] has shown that with respect to an affine parameter,say s,and a positively oriented set

$\{\alpha'(s), \alpha''(s), \alpha'''(s)\}$, $\forall s \in [a,b]$, there exists a local frame $F = \{T = \alpha', N, B\}$, called Cartan frame satisfying

$$\langle T,T\rangle = \langle N,N\rangle = 0, \qquad \langle T,N\rangle = 1$$
$$\langle B,T\rangle = \langle B,N\rangle = 0, \qquad \langle B,B\rangle = 1.$$

with the vector product $\wedge$ given by

$$T \wedge B = -T$$
$$T \wedge N = -B$$
$$B \wedge N = -N.$$

The Cartan equations are given by

$$T' = D_T T = \kappa B$$
$$N' = D_T N = -\tau B$$
$$B' = D_T B = -\tau T + \kappa N,$$

where κ and τ are the curvatures and torsion functions of α with respect to F [4].

3. Main results

Let $\{T,N,B\}$ be the Frenet trihedron of the differentiable null curve in E_1^3. The equations (2.4) form a rotation motion with Darboux vector,

$$w = \tau T + \kappa N.$$

Also momentum rotation vector is expressed as follows:

$$T' = w \wedge T, \ N' = w \wedge N, \ B' = w \wedge B$$

and

$$T \wedge B = -T, \ T \wedge N = -B, \ B \wedge N = -N.$$

The Darboux rotation of Frenet frame can be separated into two rotation motions T tangent vector rotates with κ angular speed around the normal vector N, that is

$$T' = (\kappa N) \wedge T$$

and N' rotates with a τ angular speed round the tangent vector T , that is

$$N' = (\tau T) \wedge N.$$

The separation of the rotation motion of the momentum Darboux axis into two rotation motions can be indicated like: the vector $\frac{w}{\|w\|}$ rotates with a

$$W = \frac{\tau'\kappa - \kappa'\tau}{2\kappa\tau} = 2\frac{\left(\frac{\tau}{\kappa}\right)'}{\left(\frac{\tau}{\kappa}\right)}.$$

angular speed round binormal B, also;

$$\left(\frac{w}{\|w\|}\right)' = (WB) \wedge \frac{w}{\|w\|}.$$

and binormal vector B rotates with a angular speed $\|w\|$ round Darboux axis $\frac{w}{\|w\|}$, also

$$N' = w \wedge N..$$

From now on we shall do a further study of momentum Darboux axis. We obtain the unit vector E from Darboux vector,

$$E = \frac{w}{\|w\|} = \frac{\tau T + \kappa N}{\sqrt{2\kappa\tau}}..$$

From $w' = \tau' T + \kappa' N$, differentiation of E

$$E' = \left(\frac{w}{\|w\|}\right)' = \left(\frac{\kappa\tau' - \kappa'\tau}{(2\kappa\tau)^{\frac{3}{2}}}\right)(\tau T - \kappa N)$$

is found. From this,

$$E' = -W(E \wedge B)$$

is written. According to the second Frenet formula,

$$B' = \|w\| (E \wedge B)$$

and

$$(E \wedge B)' = -WE - \|w\| B$$

are obtained.These three equations are in the form of the Frenet formulas that is

$$\begin{aligned} B' &= \quad \|w\| (E \wedge B) \\ (E \wedge B)' &= -WE - \|w\| B \\ E' &= \quad -W (E \wedge B) \end{aligned}$$

where the first coefficient is $\|w\| = \sqrt{2\kappa\tau} > 0$ and second coefficient

$$W = \frac{\tau'\kappa - \kappa'\tau}{2\kappa\tau} = 2\frac{\left(\frac{\tau}{\kappa}\right)'}{\left(\frac{\tau}{\kappa}\right)}$$

is related only to harmonic curvature $\left(\frac{\tau}{\kappa}\right)$.Thus, the vectors $\{B, (E \wedge N), E\}$define a rotation motion together with the rotation vector,

$$w_1 = WB + \|w\| E = WB + w.$$

Corollary 3.1. This rotation motion of Darboux axis can be separated into two rotation motions again. Here rotation vector w_1 is the addition of the rotation vectors of the rotation motions. If we continue in the similar way, the rotation motion of Darboux axis is will be in a consecutive manner. In this way, the series of Darboux vectors are obtained. That is

$$w_0 = w, w_1, ...$$

Conflict of Interests

The authors declare that there is no conflict of interests.

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