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ON α-$\mathscr{I}_s$-OPEN SETS AND α-$\mathscr{I}_s$-CONTINUOUS FUNCTIONS

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Abstract. In this paper, we study the concepts of α-$\mathscr{I}_s$-open sets and α-$\mathscr{I}_s$-continuous functions introduced in [14] and some properties of the functions. Also we introduce notion of α-$\mathscr{I}_s$-open and α-$\mathscr{I}_s$-closed functions.

Keywords: α-$\mathscr{I}_s$-open set; α-$\mathscr{I}_s$-continuous function; Topological space.

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1. INTRODUCTION

In 1965, Njastad [13] introduced α-open sets and studied some of their properties. Later in 1983, Mashhour *et al.* [12] defined and studied α-continuity and α-openness in topological spaces. Where as in 2002, Hatir and Noiri [4] have introduced α-$\mathscr{I}$-open sets and α-$\mathscr{I}$-continuous and obtain decomposition of continuity in ideal topological spaces. Recently we introduce α-$\mathscr{I}_s$-open sets and α-$\mathscr{I}_s$-continuous and obtain its decomposition.

In this paper we obtain several characterization of α-$\mathscr{I}_s$-open sets and α-$\mathscr{I}_s$-continuous functions. Also we introduce α-$\mathscr{I}_s$-open and α-$\mathscr{I}_s$-closed functions.

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2. PRELIMINARIES

Let (X,τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X,τ), $cl(A)$ and $int(A)$ denote the closure and interior of A in (X,τ) respectively.

An ideal $\mathscr{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies: (1) $A\in\mathscr{I}$ and $B\subseteq A$ implies $B\in\mathscr{I}$ (2)$A\in\mathscr{I}$ and $B\in\mathscr{I}$ implies $A\cup B\in\mathscr{I}$.

If $(X,\tau)$is a topological space and $\mathscr{I}$ is an ideal on X, then $(X,\tau,\mathscr{I})$ is called an ideal topological space or an ideal space.

Let $P(X)$ be the power set of X. Then the operator $()^*:P(X)\to P(X)$, called a local function [9] of A with respect to τ and $\mathscr{I}$, is defined as follows: for $A\subseteq X$, $A^*(\mathscr{I},\tau)=\{x\in X: U\cap A\notin\mathscr{I}$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(\mathscr{I},\tau)$ in case there is no confusion. For every ideal topological space $(X,\tau,\mathscr{I})$ there exists topology τ^* finer than τ, generated by $\beta(\mathscr{I},\tau)=\{U\setminus J: U\in\tau$ and $J\in\mathscr{I}\}$ but in general $\beta(\mathscr{I},\tau)$ is not always a topology. Additionally $cl^*(A)=A\cup A^*$ defines Kuratowski closure operator for a topology τ^* finer than τ. Throughout this paper X denotes the ideal topological space $(X,\tau,\mathscr{I})$ and also $int^*(A)$ denotes the interior of A with respect to τ^*.

Definition 2.1. Let (X,τ) be a topological space. A subset A of X is said to be semi-open [10] if there exists an open set U in X such that $U\subseteq A\subseteq cl(U)$. The complement of a semi-open set is said to be semi-closed. The collection of all semi-open (resp. semi-closed) sets in X is denoted by SO(X) (resp. SC(X)). The semi-closure of A in (X,τ) is denoted by the intersection of all semi-closed sets containing A and is denoted by $scl(A)$.

Definition 2.2. For $A\subseteq X$,$A_*(\mathscr{I},\tau)$ = $\{ x\in X/U\cap A\notin\mathscr{I}$for every $U\in SO(X)\}$ is called the semi-local function [7] of A with respect to $\mathscr{I}$ and τ, where $SO(X,x)=\{U\in SO(X): x\in U\}$. We simply write A instead of $A_*(\mathscr{I},\tau)$ in case there is no ambiguity.

It is given in [2] that $\tau^{*s}(\mathscr{I})$ is a topology on X, generated by the sub basis $\{U-E: U\in SO(X)$ and $E\in I\}$ or equivalently $\tau^{*s}\mathscr{I}=\{U\subseteq X: cl^{*s}(X-U)=X-U\}$. The closure operator cl^{*s} for a topology $\tau^{*s}(\mathscr{I})$ is defined as follows: for A $\subseteq X, cl^{*s}(A)=A\cup A_*$ and int^{*s} denotes the interior of the set A in $(X,\tau^{*s},\mathscr{I})$. It is known that $\tau\subseteq\tau^*(\mathscr{I})\subseteq\tau^{*s}(\mathscr{I})$. A subset A of

$(X,\tau,\mathscr{I})$ is called semi-$*$-perfect [8] if $A=A_*$. $A\subseteq(X,\tau,\mathscr{I})$ is called $*$-semi dense in-itself [8](resp. semi-$*$-closed [8]) if $A\subset A_*$ (resp. $A_*\subseteq A$)

Lemma 2.3. [7] *Let $(X,\tau,\mathscr{I})$ be an ideal topological space and A, B subsets of X. Then for the semi-local function the following properties hold:*

(1) If $A\subseteq B$ then $A_*\subseteq B_*$.

(2) If $U\in\tau$ then $U\cap A_*\subseteq(U\cap A)_*$

(3) $A_*=scl(A_*)\subseteq scl(A)$ and A_* is semi-closed in X.

(4) $(A_*)_*\subseteq A_*$.

(5) $(A\cup B)_*=A_*\cup B_*$.

(6) If $\mathscr{I}=\{\phi\}$, then $A_*=scl(A)$.

Definition 2.4. A subset A of a topological space X is said to be

(1) α-open [13]if $A\subseteq int(cl(int(A)))$,

(2) pre-open [11] if $A\subseteq int(cl(A))$,

(3) β-open [1] if $A\subseteq cl(int(cl(A)))$.

Definition 2.5. A subset A of an ideal topological space $(X,\tau,\mathscr{I})$ is said to be

(1) α-$\mathscr{I}$-open [4]if $A\subseteq int(cl^*(int(A)))$,

(2) pre-$\mathscr{I}$-open [3] if $A\subseteq int(cl^*(A))$,

(3) semi-$\mathscr{I}$-open [4] if$A\subseteq cl^*(int(A))$.

Definition 2.6. A subset A of an ideal topological space $(X,\tau,\mathscr{I})$ is said to be

(1) α-$\mathscr{I}_s$-open [14]if $A\subseteq int(cl^{*s}(int(A)))$,

(2) pre-$\mathscr{I}_s$-open [14] if $A\subseteq int(cl^{*s}(A))$,

(3) semi-$\mathscr{I}_s$-open [14] if $A\subseteq cl^{*s}(int(A))$.

Lemma 2.7. [8] *Let $(X,\tau,\mathscr{I})$ be an ideal space and $A\subseteq Y\subseteq X$, where Y is an α-open in X. Then $A_*(\mathscr{I}_Y,\tau|_Y)=A_*(\mathscr{I},\tau)\cap Y$.*

The family of all α-$\mathscr{I}_s$-open(resp. semi -$\mathscr{I}_s$-open, pre -$\mathscr{I}_s$-open) sets in an ideal topological space $(X,\tau,\mathscr{I})$ is denoted by αISO(X)(resp. SISO(X), PISO(X)).

3. α-$\mathscr{I}_s$-OPEN SETS

Lemma 3.1. *Let $(X,\tau,\mathscr{I})$ be an ideal topological space. A subset A of X is α-$\mathscr{I}_s$-open if and only if it is semi-$\mathscr{I}_s$-open and pre-$\mathscr{I}_s$- open.*

Proof. Necessity: This is obvious.

Sufficiency: Let A be semi-$\mathscr{I}_s$-open and pre-$\mathscr{I}_s$- open. Then we have $A \subseteq int(cl^{*s}(A)) \subseteq int(cl^{*s}(cl^{*s}(int(A)))) = int(cl^{*s}(int(A)))$. This shows that A is α-$\mathscr{I}_s$-open.

Lemma 3.2. *Let $(X,\tau,\mathscr{I})$ be an ideal topological space and A be a subset of X. Then the following properties hold.*

(1) If U is open in (X, τ, $\mathscr{I}$), then $U \cap cl^{*s}(A) \subseteq cl^{*s}(U \cap A)$.

(2) If $A \subseteq X_\circ \subseteq X$, then $cl^{*s}_{X_\circ}(A) = cl^{*s}(A) \cap X_\circ$.

Proof. (1) By Lemma 2.3, if $U \in \tau$ then $U \cap A_* \subseteq (U \cap A)_*$ for any subset A of X. Thus we have $U \cap cl^{*s}(A) = U \cap (A_* \cup A) = (U \cap A_*) \cup (U \cap A) \subseteq (U \cap A)_* \cup (U \cap A) = cl^{*s}(U \cap A)$.

(2) By Lemma 2.7 that $A_*(\mathscr{I}_Y, \tau|_Y) = A_*(\mathscr{I},\tau) \cap Y$. Thus we have $cl^{*s}_{X_\circ}(A) = A_*(\mathscr{I}_{X_\circ}, \tau|_{X\circ}) \cup A = (A_*(\mathscr{I},\tau) \cap X_\circ) \cup A = (A_* \cup A) \cap (X_\circ \cup A) = cl^{*s}(A) \cap X_\circ$.

Proposition 3.3. *Let $(X,\tau,\mathscr{I})$ be an ideal topological space*

(1) If $V \in SISO(X)$ and $A \in \alpha ISO(X)$, then $V \cap A \in SISO(X)$.

(2) If $V \in PISO(X)$ and $A \in \alpha ISO(X)$, then $V \cap A \in PISO(X)$.

Proof. (1) Let $V \in SISO(X)$ and $A \in \alpha ISO(X)$. By using Lemma 3.2

$$V \cap A \subseteq cl^{*s}(int(V)) \cap int(cl^{*s}(int(A))) \subseteq cl^{*s}(int(V) \cap int(cl^{*s}(int(A))))$$

$$\subseteq cl^{*s}(int(V) \cap cl^{*s}(int(A))) \subseteq cl^{*s}(cl^{*s}(int(V) \cap int(A))) \subseteq cl^{*s}(int(V)).$$

This shows that $V \cap A \in SISO(X)$.

(2) Let $V \in PISO(X)$ and $A \in \alpha ISO(X)$. Then $V \cap A \subseteq int(cl^{*s}(V)) \cap int(cl^{*s}(int(A))) = int[cl^{*s}(V) \cap cl^{*s}(int(A))] \subseteq int[cl^{*s}((cl^{*s}(V)) \cap int(A))] \subseteq int[cl^{*s}(cl^{*s}(V \cap int(A)))] \subseteq int(cl^{*s}(V \cap A))$. This shows that $V \cap A \in PISO(X)$.

Proposition 3.4. *Let $(X,\tau,\mathscr{I})$ be an ideal topological space*

(1) If $A, B \in \alpha ISO(X)$, then $A \cap B \in \alpha ISO(X)$.

(2) If $A_\alpha \in \alpha ISO(X)$ for each $\alpha \in \Delta$, then $\bigcup_{\alpha\in\Delta} A_\alpha \in \alpha ISO(X)$.

Proof. (1) Let $A, B \in \alpha ISO(X)$. By Lemma 3.1 A abd B are semi-$\mathscr{I}_s$-open and pre-$\mathscr{I}_s$-open and by Proposition 3.3, $A \cap B$ is semi-$\mathscr{I}_s$-open and pre-$\mathscr{I}_s$-open. Therefore, by Lemma 3.1 $A \cap B \in \alpha ISO(X)$.

(2) Let $A_\alpha \in \alpha ISO(X)$ for each $\alpha \in \Delta$. Then we have

$$A_\alpha \subseteq int(cl^{*s}(int(A_\alpha))) \subseteq int\left(cl^{*s}\left(int\left(\bigcup_{\alpha\in\Delta} A_\alpha\right)\right)\right)$$

and hence

$$\bigcup_{\alpha\in\Delta} A_\alpha \subseteq int\left(cl^{*s}\left(int\left(\bigcup_{\alpha\in\Delta} A_\alpha\right)\right)\right).$$

This shows that $\bigcup\limits_{\alpha\in\Delta} A_\alpha \in \alpha ISO(X)$.

Corollary 3.5. *Let $(X, \tau, \mathscr{I})$ be an ideal topological space. Then the family $\alpha ISO(X)$ is a topology for X such that $\tau \subseteq \alpha ISO(X) \subseteq \tau_\alpha$, where τ_α denotes the family of α-open sets of X.*

Proof. Since $\phi, X \in \alpha ISO(X)$, this is an immediate consequence of Proposition 3.4 and [[14] Proposition 3.2].

Theorem 3.6. *Let $(X, \tau, \mathscr{I})$ be an ideal topological space. If $A \in \alpha ISO(X)$ and $A \subseteq X_\circ \in \alpha ISO(X)$, then $A \in \alpha ISO(X_\circ)$.*

Proof. By using Lemma 3.2 we obtain,

$$A \subseteq int(cl^{*s}(int(A \cap X_\circ))) \cap X_\circ = int_{X_\circ}[int(cl^{*s}(int(A \cap X_\circ))) \cap X_\circ]$$

$$\subseteq int_{X_\circ}[cl^{*s}(int(A \cap X_\circ)) \cap X_\circ] = int_{X_\circ}[cl^{*s}_{X_\circ}(int(A) \cap int(X_\circ))]$$

$$\subseteq int_{X_\circ}[cl^{*s}_{X_\circ}(int(A) \cap X_\circ)] = int_{X_\circ}[cl^{*s}_{X_\circ}(int_{X_\circ}(int(A) \cap X_\circ))]$$

$$\subseteq int_{X_\circ}(cl^{*s}_{X_\circ}(int_{X_\circ}(A))).$$

This shows that $A \in \alpha ISO(X_\circ)$.

Theorem 3.7. *Let $(X, \tau, \mathscr{I})$ be an ideal topological space. If $X_\circ \in \alpha ISO(X)$ and $A \in \alpha ISO(X_\circ)$, then $A \in \alpha ISO(X)$.*

Proof. Since $A \in \alpha ISO(X_\circ)$, $A \subseteq int_{X_\circ}(cl^{*s}_{X_\circ}(int_{X_\circ}(A))) = X_\circ \cap U$ for some $U \in \tau$. Since $X_\circ \in \alpha ISO(X)$, by Lemma 3.2,

$$A \subseteq X_\circ \cap U \subseteq U \cap int(cl^{*s}(int(X_\circ))) \subseteq int(cl^{*s}(int(U \cap X_\circ)))$$

$$= int(cl^{*s}(int(int_{X_\circ}(cl^{*s}_{X_\circ}(int_{X_\circ}(A)))))) \subseteq int(cl^{*s}(int((cl^{*s}(int_{X_\circ}(A)) \cap X_\circ)))$$

$$\subseteq int(cl^{*s}(int(cl^{*s}(int_{X_\circ}(A))))) \subseteq int(cl^{*s}((int_{X_\circ}(A)))).$$

Since $int_{X_\circ}(A) = V \cap X_\circ$, for some $V \in \tau$. Therefore we have

$$A \subseteq int(cl^{*s}(V \cap X_\circ)) \subseteq int(cl^{*s}(V \cap int(cl^{*s}(int(X_\circ)))))$$

$$\subseteq int(cl^{*s}(int(cl^{*s}(int(V \cap X_\circ))))) \subseteq int(cl^{*s}(int(A))).$$

This shows that $A \in \alpha ISO(X)$.

4. α-$\mathscr{I}_s$-CONTINUOUS FUNCTIONS

Definition 4.1. [14] A function $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ is called α-$\mathscr{I}_s$-continuous(resp. semi -$\mathscr{I}_s$-continuous, pre -$\mathscr{I}_s$-continuous) if the inverse image of each open set of Y is α-$\mathscr{I}_s$-open(resp. semi-$\mathscr{I}_s$-open, pre -$\mathscr{I}_s$-open).

Theorem 4.2. *Let $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ be a function.Then the following statements are equivalent.*

(1) f is α-$\mathscr{I}_s$-continuous,

(2) For each $x \in X$ and each open set $V \subseteq Y$ containing $f(x)$, there exists $W \in \alpha ISO(X)$ such that $x \in W$, $f(W) \subseteq V$,

(3) The inverse image of each closed set in Y is α-$\mathscr{I}_s$-closed,

(4) $cl(int^{*s}(cl(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$ for each $B \subseteq Y$,

(5) $f(cl(int^{*s}(cl(A)))) \subseteq cl(f(A))$ for each $A \subseteq X$.

Proof. (1) $\Rightarrow$ (2) Let $x \in X$ and V be any open set of Y containing $f(x)$. Set $W = f^{-1}(V)$, then by Definition 4.1, W is an α-$\mathscr{I}_s$-open set containing x and $f(W) \subseteq V$.

$(2) \Rightarrow (3)$ Let F be a closed set of Y. Set $V = Y - F$, then V is open in Y. Let $x \in f^{-1}(V)$, by (2)there exists an α-$\mathscr{I}_s$-open set W of X containing x such that $f(W) \subseteq V$. Thus we obtain $x \in W \subseteq int(cl^{*s}(int(W))) \subseteq int(cl^{*s}(int(f^{-1}(V))))$ and hence $f^{-1}(V) \subseteq int(cl^{*s}(int(f^{-1}(V))))$. This shows that $f^{-1}(V)$ is α-$\mathscr{I}_s$-open in X. Hence $f^{-1}(F) = X - f^{-1}(Y-F) = X - f^{-1}(V)$ is α-$\mathscr{I}_s$-closed in X.

$(3) \Rightarrow (4)$ Let B be any subset of Y. Since $cl(B)$ is closed in Y, by(3) $f^{-1}(cl(B))$ is α-$\mathscr{I}_s$-closed and $X - f^{-1}(cl(B))$ is α-$\mathscr{I}_s$-open. Thus $X - f^{-1}(cl(B)) \subseteq int(cl^{*s}(int(X - f^{-1}(cl(B)))))) = X - cl(int^{*s}(cl(f^{-1}(cl(B)))))$. Hence we obtain $cl(int^{*s}(cl(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$.

$(4) \Rightarrow (5)$ Let A be a subset of X. By(4) we have $cl(int^{*s}(cl(A))) \subseteq cl(int^{*s}(cl(f^{-1}(f(A)))))$ $\subseteq f^{-1}(cl(f(A)))$ and hence $f(cl(int^{*s}(cl(A)))) \subseteq cl(f(A))$.

$(5) \Rightarrow (1)$ Let V be any open set of Y . Then by(5), $f(cl(int^{*s}(cl(f^{-1}(Y-V))))) \subseteq cl(f(f^{-1}(Y-V))) \subseteq cl(Y-V) = Y-V$. Therefore we have $cl(int^{*s}(cl(f^{-1}(Y-V)))) \subseteq f^{-1}(Y-V) \subseteq X - f^{-1}(V)$. consequently we obtain that $f^{-1}(V) \subseteq int(cl^{*s}(int(f^{-1}(V))))$. This shows that $f^{-1}(V)$ is α-$\mathscr{I}_s$-open. Thus f is α-$\mathscr{I}_s$-continuous.

Corollary 4.3. *Let $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ be α-$\mathscr{I}_s$-continuous. Then*

(1) $f(cl^{*s}(U)) \subseteq cl(f(U))$ for each $U \in PISO(X)$,

(2) $cl^{*s}(f^{-1}(V)) \subseteq f^{-1}(cl(V))$ for each $V \in PISO(Y)$.

Proof. (1) Let $U \in PISO(X)$. Then $U \subseteq int(cl^{*s}(U))$. Therefore by Theorem 4.2, we have $f(cl^{*s}(U)) \subseteq f(cl(U)) \subseteq f(cl(int(cl^{*s}(U)))) \subseteq f(cl(int^{*s}(cl(U)))) \subseteq cl(f(U))$.

(2) Let $V \in PISO(Y)$. By Theorem 4.2, $cl^{*s}(f^{-1}(V)) \subseteq cl(f^{-1}(V)) \subseteq cl(f^{-1}(int(cl^{*s}(V))))$ $\subseteq cl(int(cl^{*s}(int[f^{-1}(int(cl^{*s}(V)))]))) \subseteq cl(int^{*s}(cl[f^{-1}(int(cl^{*s}(V)))])) \subseteq f^{-1}(cl(int(cl^{*s}(V)))) \subseteq f^{-1}(cl(V))$.

Theorem 4.4. *A function $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ is α-$\mathscr{I}_s$-continuous if and only if semi-$\mathscr{I}_s$-continuous and pre-$\mathscr{I}_s$-continuous.*

Proof. This is an immediate consequence of Lemma 3.1.

Theorem 4.5. *A function $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ is α-$\mathscr{I}_s$-continuous if and only if $f : (X, \alpha ISO(X)) \to (Y, \sigma)$ is continuous.*

Proof. This is an immediate consequence of Corollary 3.5.

Theorem 4.6. *A function* $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ *is* α-$\mathscr{I}_s$*-continuous if and only if the graph function* $g : X \to X \times Y$*, defined by* $g(x) = (x, f(x))$ *for each* $x \in X$*, is* α-$\mathscr{I}_s$*-continuous.*

Proof. Necessity. Suppose that f is α-$\mathscr{I}_s$-continuous. Let $x \in X$ and W be any open set of $X \times Y$ containing $g(x)$. Then there exists a basic open set $U \times V$ such that $g(x) = (x, f(x)) \in U \times V \subseteq W$. Since f is α-$\mathscr{I}_s$-continuous, there exists an α-$\mathscr{I}_s$-open set $U_\circ$ of X containing x such that $f(U_\circ) \subseteq V$. By Proposition 3.4 $U_\circ \cap U \in \alpha ISO(X, \tau)$ and $g(U_\circ \cap U) \subseteq U \times V \subseteq W$. This shows that g is α-$\mathscr{I}_s$-continuous.

Sufficiency Suppose that g is α-$\mathscr{I}_s$-continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then $X \times V$ is open in $X \times Y$ and by α-$\mathscr{I}_s$-continuity of g,there exists $U \in \alpha ISO(X, \tau)$ containing x such that $g(U) \subseteq X \times V$. Therefore, we obtain $f(U) \subseteq V$. This shows that f is α-$\mathscr{I}_s$-continuous.

Theorem 4.7. *If* $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ *is an* α-$\mathscr{I}_s$*-continuous function and* $X_\circ \in \alpha ISO(X)$*, then the restriction* $f|_{X_\circ} : (X_\circ, \tau|_{X_\circ}, \mathscr{I}_{X_\circ}) \to (Y, \sigma)$ *is* α-$\mathscr{I}_s$*-continuous.*

Proof. Let V be any open set of (Y, σ). Since f is α-$\mathscr{I}_s$-continuous, $f^{-1}(V)$ is α-$\mathscr{I}_s$-open in $(X, \tau, \mathscr{I})$ and by Proposition 3.4 $f^{-1}(V) \cap X_\circ = (f|_{X_\circ})^{-1}(V) \in \alpha ISO(X)$. Moreover by Theorem 3.6 we have $(f|_{X_\circ})^{-1}(V) \in \alpha ISO(X_\circ)$. This shows that $f|_{X_\circ}$ is α-$\mathscr{I}_s$-continuous.

Theorem 4.8. *Let* $(X, \tau, \mathscr{I})$ *be an ideal topological space and* $\{V_\alpha / \alpha \in \Delta\}$ *a cover of X by* α-$\mathscr{I}_s$*-open sets of* $(X, \tau, \mathscr{I})$*. A function* $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ *is* α-$\mathscr{I}_s$*-continuous if and only if the restriction* $f|_{V_\alpha} : (V_\alpha, \tau|_{V_\alpha}, \mathscr{I}_{V_\alpha}) \to (Y, \sigma)$ *is* α-$\mathscr{I}_s$*-continuous for each* $\alpha \in \Delta$.

Proof. Necessity. Let f be α-$\mathscr{I}_s$-continuous. Then by Theorem 4.7 $f|_{V_\alpha}$ is α-$\mathscr{I}_s$-continuous for each $\alpha \in \Delta$.

Sufficiency. Let $f|_{V_\alpha}$ be α-$\mathscr{I}_s$-continuous for each $\alpha \in \Delta$. For any open set V of (Y, σ), $(f|_{V_\alpha})^{-1}(V) \in \alpha ISO(V_\alpha)$ for each $\alpha \in \Delta$ and hence $f^{-1}(V) = \bigcup \left\{ (f|_{V_\alpha})^{-1}(V) \Big/ \alpha \in \Delta \right\} \in \alpha ISO(X)$. By Proposition 3.4 and Theorem 3.7. This shows that f is α-$\mathscr{I}_s$-continuous.

5. α-$\mathscr{I}_s$-OPEN FUNCTIONS

Definition 5.1. A function $f:(X,\tau)\to(Y,\sigma,\mathscr{J})$ is called α-$\mathscr{I}_s$-open(resp. semi-$\mathscr{I}_s$-open, pre-$\mathscr{I}_s$-open) if the image of each open set in X is an α-$\mathscr{I}_s$-open (resp. semi-$\mathscr{I}_s$-open, pre-$\mathscr{I}_s$-open) set of Y.

Definition 5.2. A function $f:(X,\tau)\to(Y,\sigma,\mathscr{J})$ is called α-$\mathscr{I}_s$-closed(resp. semi-$\mathscr{I}_s$-closed, pre-$\mathscr{I}_s$-closed) if the image of each closed set in X is an α-$\mathscr{I}_s$-closed (resp. semi-$\mathscr{I}_s$-closed, pre-$\mathscr{I}_s$-closed) set of Y.

Remark 5.3. *(1) Every α-$\mathscr{I}_s$-open (resp. α-$\mathscr{I}_s$-closed) function is semi-$\mathscr{I}_s$-open (resp. semi-$\mathscr{I}_s$-closed) and the converses are false in general.*

(2) Every α-$\mathscr{I}_s$-open (resp. α-$\mathscr{I}_s$-closed) function is pre-$\mathscr{I}_s$-open (resp. pre-$\mathscr{I}_s$-closed) and the converses are false in general.

(3) Every open function is α-$\mathscr{I}_s$-open but the converse is not true in general.

Example 5.4. *Let X = {a, b, c, d}, $\tau=\{\phi,X,\{a,b,c\}\},\sigma=\{\phi,X,\{c\},\{a,b\},\{a,b,c\}\}$ and $\mathscr{J}=\{\phi,\{a\}\}$. Define a function $f:(X,\tau)\to(X,\sigma,\mathscr{J})$ as follows $f(a)=a,f(b)=b,f(c)=d,f(d)=c$. Then f is semi-$\mathscr{I}_s$-open, but it is not α-$\mathscr{I}_s$-open.*

Example 5.5. *Let X = {a, b, c, d}, $\tau=\{\phi,X,\{a\},\{a,b\}\},\sigma=\{\phi,X,\{c\},\{a,b\},\{a,b,c\}\}$ and $\mathscr{J}=\{\phi,\{a\}\}$. Define a function $f:(X,\tau)\to(X,\sigma,\mathscr{J})$ as follows $f(a)=c,f(b)=b,f(c)=d,f(d)=a$. Then f is pre-$\mathscr{I}_s$-open, but it is not α-$\mathscr{I}_s$-open.*

Example 5.6. *Let X = {a, b, c}, $\tau=\{\phi,X,\{a\},\{a,c\}\},\sigma=\{\phi,X,\{a\},\{a,c\}\}$ and $\mathscr{J}=\{\phi,\{c\}\}$. The identity function $f:(X,\tau)\to(X,\sigma,\mathscr{J})$ is α-$\mathscr{I}_s$open, but it is not open.*

Theorem 5.7. *A function $f:(X,\tau)\to(Y,\sigma,\mathscr{J})$ is called α-$\mathscr{I}_s$-open if and only if it is semi-$\mathscr{I}_s$-open and pre-$\mathscr{I}_s$-open.*

Proof. This is an immediate consequence of Lemma 3.1.

Theorem 5.8. *A function $f:(X,\tau)\to(Y,\sigma,\mathscr{J})$ is called α-$\mathscr{I}_s$-open if and only if for each subset $W\subseteq Y$ and each closed set F of X containing $f^{-1}(W)$, there exists a α-$\mathscr{I}_s$-closed set $H\subseteq Y$ containing W such that $f^{-1}(H)\subseteq F$.*

Proof: Necessity. Let $H = Y - f(X-F)$. Since $f^{-1}(W) \subseteq F$, we have $f(X-F) \subseteq Y - W$. Since f is α-$\mathscr{I}_s$-open, then H is α-$\mathscr{I}_s$-closed and $f^{-1}(H) = X - f^{-1}(f(X-F)) \subseteq X - (X-F) = F$.

Sufficiency. Let U be any open set of X and $W = Y - f(U)$. Then $f^{-1}(W) = X - f^{-1}(f(U)) \subseteq X - U$ and $X - U$ is closed. By the hypothesis, there exists an α-$\mathscr{I}_s$-closed set H of Y containing W such that $f^{-1}(H) \subseteq X - U$. Then we have $f^{-1}(H) \cap U = \phi$ and $H \cap f(U) = \phi$. Therefore we obtain $Y - f(U) \supseteq H \supseteq W = Y - f(U)$ and $f(U)$ is α-$\mathscr{I}_s$-open in Y. This shows that f is α-$\mathscr{I}_s$-open.

Corollary 5.9. *If $f : (X, \tau) \to (Y, \sigma, \mathscr{J})$ is α-$\mathscr{I}_s$-open then the following properties hold.*

(1) $f^{-1}(cl(int^{*s}(cl(B)))) \subseteq cl(f^{-1}(B))$ for each set $B \subseteq Y$,

(2) $f^{-1}(cl^{*s}(V)) \subseteq cl(f^{-1}(V))$ for each pre-open set V of Y.

Proof. (1) Let B be any subset of Y. Then $cl(f^{-1}(B))$ is closed in X. By Theorem 5.8, there exists an α-$\mathscr{I}_s$-closed set $H \subseteq Y$ containing B such that $f^{-1}(H) \subseteq cl(f^{-1}(B))$. Since $Y - H$ is α-$\mathscr{I}_s$-open, $f^{-1}(Y-H) \subseteq f^{-1}(int(cl^{*s}(int(Y-H))))$ and $X - f^{-1}(H) \subseteq f^{-1}(Y - (cl(int^{*s}(cl(H))))) = X - f^{-1}(cl(int^{*s}(cl(H))))$. Thus we obtain that $f^{-1}(cl(int^{*s}(cl(B)))) \subseteq f^{-1}(cl(int^{*s}(cl(H)))) \subseteq f^{-1}(H) \subseteq cl(f^{-1}(B))$. Therefore we have $f^{-1}(cl(int^{*s}(cl(B)))) \subseteq cl(f^{-1}(B))$.

(2) Let V be any pre-open set of Y. By (1) we obtain $f^{-1}(cl^{*s}(V)) \subseteq f^{-1}(cl(V)) \subseteq f^{-1}(cl(int(cl(V)))) \subseteq f^{-1}(cl(int^{*s}(cl(V)))) \subseteq cl(f^{-1}(V))$.

Theorem 5.10. *A function $f : (X, \tau) \to (Y, \sigma, \mathscr{J})$ is called pre-$\mathscr{I}_s$-open(resp. semi-$\mathscr{I}_s$-open) if and only if for each subset $W \subseteq Y$ and each closed set F of X containing $f^{-1}(W)$, there exists a pre-$\mathscr{I}_s$-closed(resp. semi-$\mathscr{I}_s$-closed) set $H \subseteq Y$ containing W such that $f^{-1}(H) \subseteq F$.*

Proof. The Proof is similar to Theorem 5.8.

Corollary 5.11. *Let $f : (X, \tau) \to (Y, \sigma, \mathscr{J})$ be a function*

(1) If f is pre-$\mathscr{I}_s$-open, then $f^{-1}(cl(int^{*s}(B))) \subseteq cl(f^{-1}(B))$ for any subset B of Y.

(2) If f is semi-$\mathscr{I}_s$-open, then $f^{-1}(int^{*s}(cl(B))) \subseteq cl(f^{-1}(B))$ for any subset B of Y.

Proof. The Proof is similar to Corollary 5.9.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ. 12 (1983), 77-90.

[2] M.E. Abd El-Monsef, E.F. Lashien and A.A. Nasef, Some topological operators via ideals, Kyungpook Math. J. 32 (1992), 273-284.

[3] J. Dontchev, Idealization of Ganster-Reilly decomposition theorems, http://arxiv.org/abs/ Math. GN/9901017, 5 Jan. 1999(Internet).

[4] E. Hatir and T. Noiri, On decompositions of continuity via idealization, Acta. Math. Hungar. 96 (2002), 341-349.

[5] E. Hatir and T. Noiri, On α-I-continuous and α-I-open functions, Acta. Math. Hungar. 105 (2004), 27-37.

[6] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97 (1990), 295-310.

[7] M. Khan and T. Noiri, Semi-local functions in ideal topological spaces, J. Adv. Res. Pure Math. 2 (2010), 36-42.

[8] M. Khan and T. Noiri, On gI-closed sets in ideal topological spaces, J. Adv. Stud. Topology 1 (2010), 29-33.

[9] K. Kuratowski, Topology, Vol. I, Academic press, New York, 1966.

[10] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.

[11] A. S. Mashhour, M. E. Abd. El-Monsef and S. N. El-deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egypt, 53 (1982), 47-53.

[12] A. S. Mashhour, I. N. Hasanein and S. N. El-deeb, On α-continuous and α-open mappings, Acta. Math. Hungar. 41 (1983), 213-218.

[13] O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15 (1965), 961-970.

[14] R. Santhi and M. Rameshkumar, A decomposition of continuity in ideal by using semi-local functions, Asian J. Math. Appl. (2014), 1-11.

[15] R. Santhi and M. Rameshkumar, On semi-$\mathscr{I}_s$-open sets and semi-$\mathscr{I}_s$-continuous functions, IOSR J. Math. 7 (2013) 38-42.

[16] R. Vaidyanathaswamy, Set Topology, Chelsea Publishing Company, 1960.