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ON INTUITIONISTIC FUZZY R_1-SPACES

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Abstract: In this paper, we define some notions of the intuitionistic fuzzy R_1-spaces. We investigate some relations among them and we also investigate the relationship between intuitionistic fuzzy topological space and intuitionistic topological space. We show that R_1-spaces satisfy "good extensions" property. It is also shown that these notions are hereditary and projective.

Keywords: intuitionistic set; intuitionistic fuzzy set; intuitionistic topological space; intuitionistic fuzzy topological space; intuitionistic fuzzy R_1-spaces.

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I. INTRODUCTION

Atanassov [3] introduced the concepts of intuitionistic fuzzy sets which take into account both the degrees of membership and non-membership subject to the condition that their sum does not exceed 1. D. Coker subsequently initiated a study of intuitionistic fuzzy topological spaces. After then many topologists work in intuitionistic fuzzy topological spaces.

In this paper, we define some new notions of R_1-spaces using intuitionistic fuzzy sets and we investigate the properties of R_1-spaces.

Definition 1.1.[10] An intuitionistic set A is an object having the form $A = (x, A_1, A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A.

Throughout this paper, we use the simpler notation $A = (A_1, A_2)$ for an intuitionistic set.

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Remark 1.2.[10] Every subset A on a nonempty set X may obviously be regarded as an intuitionistic set having the form $A' = (A,\ A^C)$, where $A^C = X \setminus A$ is the complement of A in X.

Definition 1.3.[10] Let the intuitionistic sets A and B on X be of the forms $A = (A_1,\ A_2)$ and $B = (B_1,\ B_2)$ respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = (A_j^{(1)},\ A_j^{(2)})$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(c) $\overline{A} = (A_2,\ A_1)$, denotes the complement of A.

(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$.

(e) $\cup A_j = (\cup A_j^{(1)},\ \cap A_j^{(2)})$.

(f) $\phi_\sim = (\phi,\ X)$ and $X_\sim = (X,\ \phi)$.

Definition: 1.4.[8] An intuitionistic topology on a set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(1) $\phi_\sim, X_\sim \in \tau$.

(2) $G_1 \cap G_2 \in \tau$ for any $G_1,\ G_2 \in \tau$.

(3) $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$.

In this case, the pair $(X,\ \tau)$ is called an intuitionistic topological space (ITS, in short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS, in short) in X.

Definition 1.5.[3] Let X be a non empty set and I be the unit interval $[0, 1]$. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A : X \rightarrow I$ and $\nu_A : X \rightarrow I$ denote the degree of membership and the degree of non- membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$.

Let $I(X)$ denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, 1 - \mu_A)$.

Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$.

Definition 1.6.[3] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\nu_A, \mu_A)$.

(4) $A \cap B = (\mu_A \cap \mu_B; \nu_A \cup \nu_B)$.

(5) $A \cup B = (\mu_A \cup \mu_B; \nu_A \cap \nu_B)$.

(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

Definition 1.7.[8] Let $\{ A_i : i \in J \}$ be an arbitrary family of IFSs in X. Then

(a) $\cap A_i = (\cap \mu_{A_i}, \cup \upsilon_{A_i})$.

(b) $\cup A_i = (\cup \mu_{A_i}, \cap \upsilon_{A_i})$.

Definition 1.8.[9] An intuitionistic fuzzy topology (IFT, in short) on X is a family t of IFS's in X which satisfies the following axioms:

(1) $0_\sim, 1_\sim \in$ t.

(2) if $A_1, A_2 \in$ t, then $A_1 \cap A_2 \in$ t.

(3) if $A_i \in$ t for each i, then $\cup A_i \in$ t.

The pair (X, t) is called an intuitionistic fuzzy topological space (IFTS, in short). Let (X, t) be an IFTS. Then any member of t is called an intuitionistic fuzzy open set (IFOS, in short) in X. The complement of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS, in short) in X.

Definition 1.9.[3] Let X and Y be two nonempty sets and $f : X \rightarrow Y$ be a function. If $B = \{(y, \mu_B(y), \nu_B(y)) / y \in Y\}$ is an IFS in Y, then the pre image of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)), x \in X\}$ and the image of A under f, denoted by $f(A) = \{(y, f(\mu_A), f(\nu_A)), y \in Y\}$is an IFS of Y, where for each $y \in Y$

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise.} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ 1 & \text{otherwise.} \end{cases}$$

Definition 1.10.[6] Let $A = (X, \mu_A, \upsilon_A)$ and $B = (Y, \mu_B, \upsilon_B)$ be IFSs of X and Y respectively. Then the product of intuitionistic fuzzy sets A and B denoted by $A \times B$ is defined by $A \times B = \{X \times Y, \mu_A \times \mu_B, \upsilon_A \times \upsilon_B\}$ where $(\mu_A \times \mu_B)(x, y) = \min(\mu_A(x), \mu_B(y))$ and $(\upsilon_A \times \upsilon_B)(x,$

y) = max $(\upsilon_A(x), \upsilon_B(y))$ for all $(x, y) \in X \times Y$. Obviously, $0 \le \mu_A \times \mu_B + \upsilon_A \times \upsilon_B \le 1$. This definition can be extended to an arbitrary family of IFSs as follows:

If $A_i = ((\mu_{A_i}, \upsilon_{A_i}), i \in J)$ is a family of IFSs in X_i, then their product is defined as the IFSs in $\prod X_i$ given by $\prod A_i = (\prod \mu_{A_i}, \prod \upsilon_{A_i})$ where $\prod \mu_{A_i}(x) = \inf \mu_{A_i}(x_i)$, for all $x = \prod x_i \in X$ and $\prod \upsilon_{A_i}(x) = \sup \upsilon_{A_i}(x_i)$, for all $x = \prod x_i \in X$.

Definition 1.11.[6] Let (X_i, t_i), $i = 1, 2$ be two IFTSs, and then the product $t_1 \times t_2$ on $X_1 \times X_2$ is defined as the IFT generated by $\{\rho_i^{-1}(U_i) : U_i \in t_i, i = 1, 2\}$, where $\rho_i : X_1 \times X_2 \to X_i$, $i = 1, 2$ are the projection maps and the IFTS $(X_1 \times X_2, t_1 \times t_2)$ is called product IFTS.

Theorem 1.12.[1] Let (X, τ) be an intuitionistic topological space and let $t = \{1_A : A \in \tau\}$, $1_{(A_1, A_2)} = (1_{A_1}, 1_{A_2})$, then (X, t) is the corresponding intuitionistic fuzzy topological space of (X, τ).

2. INTUITIONISTIC FUZZY R_1−SPACES

Definition 2.1 An intuitionistic fuzzy topological space (X, t) is called

(1) IF−R_1(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1, \upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$.

(2) IF−R_1(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1, \upsilon_B(x) = 0$; $\mu_C(y) > 0$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

(3) IF−R_1(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) > 0, \upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

(4) IF−R_1(iv) if for all $x, y \in X, x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) > 0, \upsilon_B(x) = 0$; $\mu_C(y) > 0$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

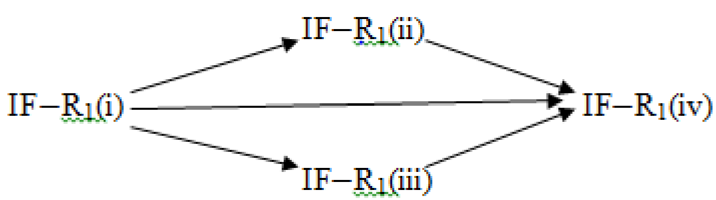
Definition 2.2. Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α −IF−R_1(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$.

(b) α −IF−R_1(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) \geq \alpha$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

(c) α −IF−R_1(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) > 0$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

Theorem2.3. Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implication:



Proof: Suppose (X, t) is IF−R_1(i) space. We shall prove that (X, t) is IF−R_1(ii). Let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is IF−R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Longrightarrow$ $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) > 0$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is IF−R_1(ii). Hence IF−R_1(i) $\Longrightarrow$ IF−R_1(ii).

Again, suppose (X, t) is IF−R_1(i)**.** We shall prove that (X, t) is IF−R_1(iii)., Let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is IF−R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Longrightarrow$ $\mu_B(x) > 0$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is IF−R_1(iii). Hence IF−R_1(i) $\Longrightarrow$ IF−R_1(iii)**.**

Furthermore, it can verify that IF−R_1(i) $\Longrightarrow$ IF−R_1(iv), IF−R_1(ii) $\Longrightarrow$ IF−R_1(iv**)** and IF−R_1(iii) $\Longrightarrow$ IF−R_1(iv)**.**

None of the reverse implications is true in general which can be seen from the following examples.

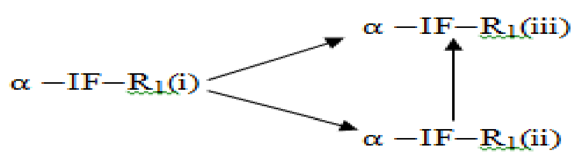
Example 2.3.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.4, 0), (y, 0, 0.2)\}$ and $B = \{(x, 1, 0), (y, 0, 0.5)\}$, $C = \{(x, 0, 0.6), (y, 0.7, 0)\}$. We see that the IFTS (X, t) is IF$-R_1$(ii) but not IF$-R_1$(i).

Example 2.3.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.3, 0), (y, 0, 0.2)\}$ and $B = \{(x, 0.5, 0), (y, 0, 0.7)\}$, $C = \{(x, 0, 0.6), (y, 1, 0)\}$. We see that the IFTS (X, t) is IF$-R_1$(iii) but not IF$-R_1$(i).

Example 2.3.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.1, 0), (y, 0, 0.7)\}$ and $B = \{(x, 1, 0), (y, 0, 0.6)\}$, $C = \{(x, 0, 0.3), (y, 0.9, 0)\}$. We see that the IFTS (X, t) is IF$-R_1$(ii) but not IF$-R_1$(iii).

Example 2.3.4. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.4)\}$ and $B = \{(x, 0.6, 0), (y, 0, 0.3)\}$, $C = \{(x, 0, 0.5), (y, 0.8, 0)\}$. We see that the IFTS (X, t) is IF$-R_1$(iii) but not IF$-R_1$(ii).

Theorem 2.4. Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is $\alpha -$IF$-R_1$(i) space. We shall prove that (X, t) is $\alpha -$IF$-R_1$(ii). Let $\alpha \in (0, 1)$. Again, let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is IF$-R_1$(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1, \upsilon_B(x) = 0; \mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Rightarrow$ $\mu_B(x) \geq \alpha$, $\upsilon_B(x) = 0; \mu_C(y) \geq \alpha, \upsilon_C(y) = 0$ for any $\alpha \in (0, 1)$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is $\alpha -$IF$-R_1$(ii). Hence $\alpha -$IF$-R_1$(i) $\Rightarrow$ $\alpha -$IF$-R_1$(ii).

Again, suppose (X, t) is $\alpha -$IF$-R_1$(ii). We shall prove that (X, t) is $\alpha -$IF$-R_1$(iii). Let $\alpha \in (0, 1)$. Again, let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is $\alpha -$IF$-R_1$(ii), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) \geq \alpha, \upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$ $\Rightarrow$ $\mu_B(x) > 0, \upsilon_B(x) = 0$;

$\mu_C(y) \geq \alpha, \upsilon_C(y) = 0$ and $B \cap C = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$. Which is α −IF−R_1(iii). Hence α −IF−R_1(ii) $\Longrightarrow$ α −IF−R_1(iii).

Furthermore, it can verify that α −IF−R_1(i) $\Rightarrow$ α −IF−R_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example 2.4.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.2, 0), (y, 0, 0.3)} and B = {(x, 0.6, 0), (y, 0, 0.4)}, C = {(x, 0, 0.5), (y, 0.7, 0)}. For α =0.4, we see that the IFTS (X, t) is α −IF−R_1(ii) but not α −IF−R_1(i).

.Example 2.4.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.1, 0), (y, 0, 0.4)} and B = {(x, 0.3, 0), (y, 0, 0.7)}, C = {(x, 0, 0.6), (y, 0.5, 0)}. For α =0.5, we see that the IFTS (X, t) is α −IF−R_1(iii) but not α −IF−R_1(i).

Theorem 2.5. Let (X, t) be an intuitionistic fuzzy topological space and $0 < \alpha \leq \beta < 1$, then

(a) β −IF−R_1(i) $\Longrightarrow$ α −IF−R_1(i).

(b) β −IF−R_1(ii) $\Longrightarrow$ α −IF−R_1(ii).

(c) β −IF−R_1(iii) $\Longrightarrow$ α − IF−R_1(iii).

Proof(a): Let $\beta \in (0, 1)$. Suppose the intuitionistic fuzzy topological space (X, t) is β −IF−R_1(i). We shall prove that (X, t) is α −IF−R_1(i). Let $x, y \in X, x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is β −IF−R_1(i), then $\exists\ B = (\mu_B, \upsilon_B),\ C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1,\ \nu_B(x) = 0;\ \mu_C(y) \geq \beta,\ \nu_C(y) = 0$ and $B \cap C = 0_{\sim} \Longrightarrow \mu_B(x) = 1,\ \nu_B(x) = 0;\ \mu_C(y) \geq \alpha,\ \nu_C(y) = 0$ and $B \cap C = 0_{\sim}$ as $0 < \alpha \leq \beta < 1$. Which is α −IF−R_1(i). Hence β −IF−R_1(i) $\Longrightarrow$ α −IF−R_1(i).

Furthermore, it can verify that β − IF − R_1(ii) $\Longrightarrow$ α − IF − R_1(ii) and β − IF − R_1(iii) $\Longrightarrow$ α −IF−R_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example 2.5.1. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A, B, C} where A = {(x, 0.3, 0), (y, 0, 0.7)} and B = {(x, 1, 0), (y, 0, 0.4)}, C = {(x, 0, 0.5), (y, 0.6, 0)}. For α = 0.5 and β = 0.7, it is clear that the IFTS (X, t) is α −IF−R_1(i) but not β −IF−R_1(i).

Example 2.5.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.4, 0), (y, 0, 0.2)\}$ and $B = \{(x, 0.7, 0), (y, 0, 0.5)\}$, $C = \{(x, 0, 0.3), (y, 0.6, 0)\}$. For $\alpha = 0.6$ and $\beta = 0.8$, it is clear that the IFTS (X, t) is $\alpha-IF-R_1(ii)$ but not $\beta-IF-R_1(ii)$.

Example 2.5.3. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.5, 0), (y, 0, 0.1)\}$ and $B = \{(x, 0.4, 0), (y, 0, 0.6)\}$, $C = \{(x, 0, 0.3), (y, 0.5, 0)\}$. For $\alpha = 0.4$ and $\beta = 0.6$, it is clear that the IFTS (X, t) is $\alpha-IF-R_1(iii)$ but not $\beta-IF-R_1(iii)$.

Theorem 2.6. Let $\alpha \in (0, 1)$ and let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$ then

(a) (X, t) is $IF-R_1(i) \Rightarrow (U, t_U)$ is $IF-R_1(i)$.

(b) (X, t) is $IF-R_1(ii) \Rightarrow (U, t_U)$ is $IF-R_1(ii)$.

(c) (X, t) is $IF-R_1(iii) \Rightarrow (U, t_U)$ is $IF-R_1(iii)$.

(d) (X, t) is $IF-R_1(iv) \Rightarrow (U, t_U)$ is $IF-R_1(iv)$.

(e) (X, t) is $\alpha-IF-R_1(i) \Rightarrow (U, t_U)$ is $\alpha-IF-R_1(i)$.

(f) (X, t) is $\alpha-IF-R_1(ii) \Rightarrow (U, t_U)$ is $\alpha-IF-R_1(ii)$.

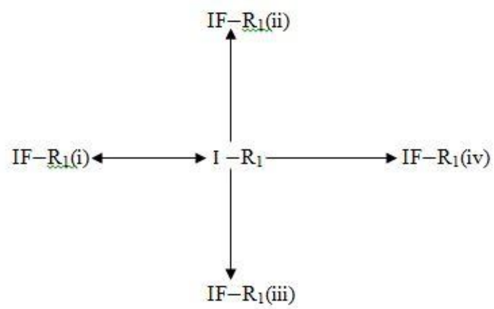
(g) (X, t) is $\alpha-IF-R_1(iii) \Rightarrow (U, t_U)$ is $\alpha-IF-R_1(iii)$.

The proofs (a), (b), (c), (d), (e), (f), (g) are similar. As an example we proved (a).

Proof (a): Suppose (X, t) is the intuitionistic fuzzy topological space and is also $IF-R_1(i)$. We shall prove that (U, t_U) is $IF-R_1(i)$. Let $x, y \in U$, $x \neq y$ with $A_U = (\mu_{A_U}, \nu_{A_U}) \in t_U$ such that $A_U(x) \neq A_U(y)$. Since $x, y \in U \subseteq X$, then $x, y \in X$, $x \neq y$ as $U \subseteq X$. Suppose $A = (\mu_A, \nu_A) \in t$ is the extension IFS of A_U on X, then $A(x) \neq A(y)$. Since (X, t) is $IF-R_1(i)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Rightarrow$ $(\mu_B|U)(x) = 1$, $(\nu_B|U)(x) = 0$; $(\mu_C|U)(y) = 1$, $(\nu_C|U)(y) = 0$ and $(\mu_B|U, \nu_B|U) \cap (\mu_C|U, \nu_C|U) = 0_\sim$. Hence $\{(\mu_B|U, \nu_B|U), (\mu_C|U, \nu_C|U)\} \in t_U$ $\Rightarrow$ (U, t_U) is $IF-R_1(i)$. Therefore (U, t_U) is $IF-R_1(i)$.

Definition 2.7 An intuitionistic topological space (ITS, in short) (X, τ) is called intuitionistic R_1–space ($I-R_1$ space) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $P = (P_1, P_2) \in \tau$ with ($x \in P_1$, $y \in P_2$) or ($y \in P_1$, $x \in P_2$) then $\exists$ $L = (L_1, L_2)$, $M = (M_1, M_2) \in \tau$ such that $x \in L_1$, $x \notin L_2$; $y \in M_1$, $y \notin M_2$ and $L \cap M = \phi_\sim$.

Theorem 2.8. Let (X, τ) be an intuitionistic topological space and let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let (X, τ) be $I-R_1$. We shall prove that (X, t) is IF−R_1(i). Suppose (X, τ) is $I-R_1$. Let $x, y \in X$, $x \neq y$ with $A = (\mu_A, \upsilon_A) \in t$ such that $A(x) \neq A(y)$. Since $A(x) \neq A(y)$, then let $(1_{C_1}(x) = 1, 1_{C_2}(y) = 1)$ or $(1_{C_1}(y) = 1, 1_{C_2}(x) = 1)$ $\Longrightarrow$ $(x \in C_1, y \in C_2)$ or $(y \in C_1, x \in C_2)$. Hence $(C_1, C_2) \in \tau$. Since (X, τ) is $I-R_1$, then $\exists$ $L = (L_1, L_2), M = (M_1, M_2) \in \tau$ such that $x \in L_1$, $x \notin L_2$; $y \in M_1$, $y \notin M_2$ and $L \cap M = \phi_\sim$ $\Longrightarrow$ $1_{L_1}(x) = 1$, $1_{L_2}(x) = 0$; $1_{M_1}(y) = 1$, $1_{M_2}(y) = 0$ and $L \cap M = \phi_\sim$. Let $\mu_B = 1_{L_1}$, $\upsilon_B = 1_{L_2}$ $\mu_C = 1_{M_1}$, $\upsilon_C = 1_{M_2}$ where $B = (\mu_B, \upsilon_B)$ and $C = (\mu_C, \upsilon_C)$. Which implies for all $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B), C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1, \upsilon_C(y) = 0$ and $B \cap C = 0_\sim$. Which is IF−R_1(i).

Conversely, let (X, t) be IF−R_1(i). We shall prove that (X, τ) is $I-R_1$. Suppose (X, t) is IF−R_1(i). Let $x, y \in X$, $x \neq y$ with $P = (P_1, P_2) \in \tau$ such that $(x \in P_1, y \in P_2)$ or $(y \in P_1, x \in P_2)$. Since $(x \in P_1, y \in P_2)$ or $(y \in P_1, x \in P_2)$ $\Longrightarrow$ $(1_{P_1}(x) = 1, 1_{P_2}(y) = 1)$ or $(1_{P_1}(y) = 1, 1_{P_2}(x) = 1)$. Hence $(1_{P_1}, 1_{P_2}) \in t$ and $(1_{P_1}, 1_{P_2})(x) \neq (1_{P_1}, 1_{P_2})(y)$. Since (X, t) is IF−R_1(i), then $\exists$ $(1_{C_1}, 1_{C_2}), (1_{D_1}, 1_{D_2}) \in t$ such that $1_{C_1}(x) = 1, 1_{C_2}(x) = 0$; $1_{D_1}(y) = 1, 1_{D_2}(y) = 0$ and $\{(1_{C_1}, 1_{C_2}) \cap (1_{D_1}, 1_{D_2})\} = 0_\sim$ $\Longrightarrow$ $x \in C_1$, $x \notin C_2$; $y \in D_1$ $y \notin D_2$ and $(C_1, C_2) \cap (D_1, D_2) = \phi_\sim$. Hence (X, τ) is $I-R_1$. Therefore $I-R_1$ $\Leftrightarrow$ IF−R_1(i).

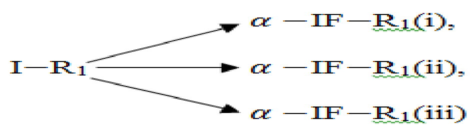
None of the reverse implications is true in general which can be seen from the following examples.

Example 2.8.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.1, 0), (y, 0, 0.5)}, B = {(x, 1, 0), (y, 0, 0.7)} and C = {(x, 0, 0.4), (y, 0.8, 0)}. We see that the IFTS (X, t) is IF$-R_1$(ii) but not corresponding I$-R_1$.

Example 2.8.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.2, 0), (y, 0, 0.3)}, B = {(x, 0.3, 0), (y, 0, 0.6)} and C = {(x, 0, 0.8), (y, 1, 0)}. We see that the IFTS (X, t) is IF$-R_1$(iii) but not corresponding I$-R_1$.

Example 2.8.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A= {(x, 0.4, 0), (y, 0, 0.3)} and B = {(x, 0.5, 0), (y, 0, 0.7)}, C = {(x, 0, 0.5), (y, 0.9, 0)}. We see that the IFTS (X, t) is IF$-R_1$(iv) but not corresponding I$-R_1$.

Theorem 2.9. Let (X, τ) be an intuitionistic topological space and let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let (X, τ) be I$-R_1$. We shall prove that (X, t) is IF$-R_1$(i). Let $\alpha \in (0, 1)$. Suppose (X, τ) is I$-R_1$. Let $x, y \in X$, $x \neq y$ with $A = (\mu_A, \upsilon_A) \in t$ such that $A(x) \neq A(y)$. Since $A(x) \neq A(y)$, then let $(1_{C_1}(x) = 1, 1_{C_2}(y) = 1)$ or $(1_{C_1}(y) = 1, 1_{C_2}(x) = 1)$ $\Longrightarrow$ $(x \in C_1, y \in C_2)$ or $(y \in C_1, x \in C_2)$. Hence $(C_1, C_2) \in \tau$. Since (X, τ) is I$-R_1$, then $\exists$ $L = (L_1, L_2)$, $M = (M_1, M_2) \in \tau$ such that $x \in L_1$, $x \notin L_2$; $y \in M_1$, $y \notin M_2$ and $L \cap M = \phi_\sim \Longrightarrow 1_{L_1}(x) = 1$, $1_{L_2}(x) = 0$; $1_{M_1}(y) = 1$, $1_{M_2}(y) = 0$ and $L \cap M = \phi_\sim$. Let $\mu_B = 1_{L_1}$, $\upsilon_B = 1_{L_2}$ $\mu_C = 1_{M_1}$, $\upsilon_C = 1_{M_2}$ where $B = (\mu_B, \upsilon_B)$ and $C = (\mu_C, \upsilon_C)$. Which implies for all $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Longrightarrow$ $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ for any $\alpha \in (0, 1)$. Which is $\alpha-$IF$-R_1$(i).

None of the reverse implications is true in general which can be seen by the following examples.

Example 2.9.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.3, 0), (y, 0, 0.4)}, B = {(x, 1, 0), (y, 0, 0.4)} and C = {(x, 0, 0.2), (y, 0.5, 0)}. For $\alpha = 0.5$, we see that the IFTS (X, t) is α −IF−R_1(i) but not corresponding I−R_1 .

Example 2.9.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.7, 0), (y, 0, 0.9)}, B ={(x, 0.6, 0), (y, 0, 0.4)} and C = {{(x, 0, 0.5), (y, 0.4, 0}. For $\alpha = 0.3$, we see that the IFTS (X, t) is α −IF−R_1(ii) but not corresponding I−R_1 .

Example 2.9.3 Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A={ (x, 0.2, 0), (y, 0, 0.6)} and B ={(x, 0.5, 0), (y, 0, 0.4)}, }, C ={(x, 0, 0.3), (y, 0.8, 0}. For $\alpha = 0.7$, we see that the IFTS (X, t) is α −IF−R_1(iii) but not corresponding I−R_1 .

Theorem 2.10. Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and $f: X \to Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF−R_1(i) $\Leftrightarrow$ (Y, s) is IF−R_1(i).

(2) (X, t) is IF−R_1(ii) $\Leftrightarrow$ (Y, s) is IF−R_1(ii).

(3) (X, t) is IF−R_1(iii) $\Leftrightarrow$ (Y, s) is IF−R_1(iii).

(4) (X, t) is IF−R_1(iv) $\Leftrightarrow$ (Y, s) is IF−R_1(iv).

(5) (X, t) is α −IF−R_1(i) $\Leftrightarrow$ (Y, s) is α −IF−R_1(i).

(6) (X, t) is α −IF−R_1(ii) $\Leftrightarrow$ (Y, s) is α −IF−R_1(ii).

(7) (X, t) is α −IF−R_1(iii) $\Leftrightarrow$ (Y, s) is α −IF−R_1(iii).

Proof: Suppose the intuitionistic fuzzy topological space (X, t) is IF−R_1(i). We shall prove that the intuitionistic fuzzy topological space (Y, s) is IF−R_1(i). Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ and $W = (\mu_W, \nu_W) \in s$ such that $W(y_1) \neq W(y_2)$. Since f is onto, then $\exists$ $x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. We have $A = (\mu_A, \nu_A) \in t$ such that $A = f^{-1}(W)$, that is $(\mu_A, \nu_A) = (f^{-1}(\mu_W), f^{-1}(\nu_W))$ as f is IF-continuous. Now, $A(x_1) = \{\mu_A(x_1) = (f^{-1}(\mu_W))(x_1) = \mu_W(f(x_1)) = \mu_W(y_1), \nu_A(x_1) = (f^{-1}(\nu_W))(x_1) = \nu_W(f(x_1)) = \nu_W(y_1)\}$ and $A(x_2) = \{\mu_A(x_2) = (f^{-1}(\mu_W))(x_2) = \mu_W(f(x_2)) = \mu_W(y_2), \nu_A(x_2) = (f^{-1}(\nu_W))(x_2) = \nu_W(f(x_2)) = \nu_W(y_2)\}$. Hence $A(x_1) \neq A(x_2)$ as $W(y_1) \neq W(y_2)$. Therefore, since (X, t) is IF−R_1(i), then $\exists$ $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ such that $\mu_B(x_1) = 1, \nu_B(x_1) = 0$; $\mu_C(x_2) = 1$, $\nu_C(x_2) = 0$ and $B \cap C = 0_\sim$. Put $U = f(B)$ and $V = f(C)$ where $U = (\mu_U, \nu_U)$, $V = (\mu_V, \nu_V) \in s$ as f is IF- continuous. Now, $\{\mu_U(y_1) = (f(\mu_B))(y_1) = \mu_B\left(f^{-1}(y_1)\right) = \mu_B(x_1) = 1, \nu_U(y_1) = (f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) =$

$0\}$; $\{\mu_V(y_2) = (f(\mu_C))(y_2) = \mu_C(f^{-1}(y_2)) = \mu_C(x_2) = 1, \nu_V(y_2) = (f(\nu_C))(y_2) = \nu_C(f^{-1}(y_2)) = \nu_C(x_2) = 0\}$ and $U \cap V = 0_\sim$. Hence $(U, V) \in s$. Therefore (Y, s) is IF$-$R$_1$(i).

Conversely, Suppose the intuitionistic fuzzy topological space (Y, s) is IF$-$R$_1$(i). We shall prove that the intuitionistic fuzzy topological space (X, t) is IF$-$R$_1$(i). Let $x_1, x_2 \in X$, $x_1 \neq x_2$ and $A = (\mu_A, \nu_A) \in t$ such that $A(x_1) \neq A(x_2)$. Since f is one-one, then $\exists\, y_i \in s$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. We have $W = (\mu_W, \nu_W) \in s$ such that $W = f(A)$, that is $(\mu_W, \nu_W) = (f(\mu_A), f(\nu_A))$ as f is IF-continuous. Now, $W(y_1) = \{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1), (f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1)\}$ and $W(y_2) = \{(f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2), (f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2)\}$. Hence $W(y_1) \neq W(y_2)$ as $A(x_1) \neq A(x_2)$. Since (Y, s) is IF$-$R$_1$(i), then $\exists\, U = (\mu_U, \nu_U)$, $V = (\mu_V, \nu_V) \in s$ such that $\mu_U(y_1) = 1, \nu_U(y_1) = 0$; $\mu_V(y_2) = 1$, $\nu_V(y_2) = 0$ and $U \cap V = 0_\sim$. Put $B = f^{-1}(U)$ and $C = f^{-1}(V)$ where $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ as f is IF- continuous. Now, $\{(f^{-1}(\mu_U))(x_1) = \mu_U(f(x_1)) = \mu_U(y_1) = 1, (f^{-1}(\nu_U))(x_1) = \nu_U(f(x_1)) = \nu_U(y_1) = 0\}$; $\{(f^{-1}(\mu_V))(x_2) = \mu_V(f(x_2)) = \mu_V(y_2) = 1, (f^{-1}(\nu_V))(x_2) = \nu_V(f(x_2)) = \nu_V(y_2) = 0\}$ and $B \cap C = 0_\sim$. Hence $(B, C) \in t$. Therefore (X, t) is IF$-$R$_1$(i).

Theorem: 2.11. Let $\{(X_m, t_m) : m \in J\}$ be a finite family of intuitionistic fuzzy topological spaces and let (X, t) be their product IFTS. Then each (X_m, t_m) is IF$-$R$_1$(i) if the product IFTS $(\prod X_m, \prod t_m)$ is IF$-$R$_1$(i).

Proof: Suppose (X, t) is IF$-$R$_1$(i). We shall prove that the intuitionistic fuzzy topological spaces (X_m, t_m) is IF$-$R$_1$(i), for all $m \in J$. Let for $j \in J$, choose $x_j, y_j \in X_j$, such that $x_j \neq y_j$. Now consider $x = \prod x_m$, $y = \prod y_m$ where $x_m = y_m$ if $m \neq j$ and the jth coordinate of x, y are x_j and y_j, respectively. Then $x \neq y$. Suppose for $x_j, y_j \in X_j$, $x_j \neq y_j$ and $A_j = (\mu_{A_j}, \upsilon_{A_j}) \in t_j$ such that $A_j(x_j) \neq A_j(y_j)$. Let $A_m = (1^\sim, 0^\sim)$, for $m \neq j$, then $A = \prod A_m \in t$ and $A(x) \neq A(y)$ where $A = (\mu_A, \nu_A)$. Therefore, since (X, t) is IF$-$R$_1$(i), then $\exists\, B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ such that $\mu_B(y) = 1$, $\mu_C(y) = 1$ and $B \cap C = 0_\sim$. Now, $\mu_B(x) = 1 \Longrightarrow \inf_{m \in J} \mu_{B_m}(x_m) = 1 \Longrightarrow \mu_{B_m}(x_m) = 1$ and $\mu_C(y) = 1 \Longrightarrow \inf_{m \in J} \mu_{C_m}(y_m) = 1 \Longrightarrow \mu_{C_m}(y_m) = 1$, for all $m \in J$. Hence we have $\mu_{B_j}(x_j) = 1$ and $\mu_{C_j}(y_j) = 1$ and $B_j \cap C_j = 0_\sim$. Thus (X_j, t_j) is IF$-$R$_1$(i). Therefore $\{(X_m, t_m) : m \in J\}$ is IF$-$R$_1$(i).

For n = ii, iii, iv, we can prove that if suppose $\{(X_m, t_m) : m \in J\}$be a finite family of intuitionistic fuzzy topological spaces and let (X, t) be their product IFTS. Then each IFTS (X_m, t_m) is IF−R_1(n) if the product IFTS $(\prod X_m, \prod t_m)$ is IF−R_1(n).

Theorem: 2.12. Let $\{(X_m, t_m) : m \in J\}$ is a finite family of intuitionistic fuzzy topological spaces. Let (X, t) be their product. Then each IFTS (X_m, t_m) is α −IF−R_1(n) if the product IFTS $(\prod X_m, \prod t_m)$ is α −IF−R_1(n). n = i, ii, iii.

Proof: The above theorem can be proved in the similar way.

Conflict of Interests

The authors declare that there is no conflict of interests.

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