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STOCHASTIC ANALYSIS OF A THREE UNIT COMPLEX SYSTEM WITH ACTIVE AND PASSIVE REDUNDANCIES AND CORRELATED FAILURES AND REPAIR TIMES

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Abstract: The paper deals with the cost-benefit and reliability analysis of a system model consisting of three non-identical units- A, B and C. The system functions satisfactorily if at least one unit is operative but if at least two units are good then they work in parallel. The priority in operation is being given to unit-A and unit-B over unit-C. So that initially unit A and B work in parallel and unit-C is kept into cold standby. A single repairman is always available to repair a failed unit and priority in repair is being given to unit-A and B over unit-C. However, the repair discipline is FCFS for unit-A and unit-B. The failure and repair times of each unit are correlated having their joint distribution as Bivariate Exponentials. By using regenerative point technique various important measures of system effectiveness have been obtained.

Keywords: Transition probabilities, Mean sojourn time, MTSF, Availability, Expected busy period of repairman and Net expected profit.

2010 AMS Subject Classification: 90B25.

1. INTRODUCTION

The study of complex reparable system models is of great interest for reliability engineers and industry managers due to their wide applicability. Gupta et al. [2] analysed a complex system with two physical conditions of repairman. The system consists of two sub-systems A and B arranged in series configuration. Subsystem A comprises two identical units in passive redundancy. The repair time distributions of the units are taken Inverse-Gaussian, Gupta et al. [3] further analysed a complex system model of three non-identical units namely A, B and C. The units are arranged in such a way that the

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system failure occurs if either unit-A or both the units B and C fail totally. The joint distribution of life times of units B and C working in parallel is taken as Bivariate Exponential.

The purpose of present paper is to analyse a three unit complex system model using the concept of correlation between the failure and repair times of the units. The three non-identical units are named as unit-A, B and C and for the successful operation of the system at least one of the units is needed to be operative. Initially, unit A and B work in parallel configuration and unit-C is kept into cold standby. Moreso, if any two-units are in good condition, they work in parallel configuration and the priority in operation is given to units A and B over the unit-C. A single repairman is always available to repair a failed unit and joint distributions of failure and repair times of units-A, B and C are taken Bivariate Exponential with different parameters. Using regenerative point technique, the following important measures of system effectiveness have been obtained:

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and mean time to system failure.
iii. Pointwise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. The probability that the repairman will be busy at epoch t and in steady-state.
v. Expected busy period of repairman during time interval.
vi. Net expected profit earned by the system during time interval (0, t) and in steady-state.

2. ASSUMPTIONS AND SYSTEM DESCRIPTION

i The system comprises of three non-identical units which are named as unit-A, B and C. Initially unit-A and B work in parallel configuration and unit-C is kept as cold standby.

ii Each unit has two modes—good and failed.

iii If either of the units (A or B) working in parallel configuration fails, the standby unit-C takes its place instantaneously with the help of a perfect and instantaneous switching device and the failed unit goes into repair.

iv A single repairman is always available with the system and the priority in repair is being given to unit-A and unit-B over the unit-C. However, the repair discipline is FCFS for unit-A and unit-B.

v The priority in operation is being given to unit-A and unit-B over the unit-C i.e. if unit-A/ unit-B is repaired while the unit B/unit-A and unit-C are operative in parallel then unit-A and unit-B become operative in parallel and the unit-C goes into cold standby. All this process is automatic and instantaneous.

vi Whenever the unit-A or unit-B fails during the repair of unit-C, then the later failed unit is taken up for repair discontinuing the repair of unit-C. The re-started repair of unit-C is of pre-emptive repeat type i.e. the time already spent in the repair of unit-C goes to waste.

vii The failure and repair times of each unit are assumed to be correlated random variables having their joint distribution as Bivariate Exponential having the density function of the type

$$f(x, y) = \lambda\mu(1-r)e^{-\lambda x-\mu y}\sum_{j=0}^{\infty}\frac{(\lambda\mu rxy)^j}{(j!)^2}; \quad x, y, \lambda, \mu > 0; \; 0 \le r < 1$$

viii Each repaired unit works as good as new.

3. NOTATIONS AND STATES OF THE SYSTEM

For defining the states of the system we assume the following notations—

A_0, B_0, C_0	:	Unit A, B and C are good and operative.
C_s	:	Unit C is in cold standby.
A_w, B_w, C_w	:	Unit A, B and C are failed and waiting for repair.
A_r, B_r, C_r	:	Unit A, B and C are failed and under repair.

Considering the above symbols for three units and taking in view the assumptions stated earlier, the possible states of the system model along with transition times is shown in Fig. 1. In the figure the states S_0 to S_7 are the up states whereas the states S_8 and S_9 are failed states. We observe that the epochs of entrance into the states S_3 and S_4 from S_1; S_5 and S_6 from S_2; S_8 from S_3, S_4 and S_9 from S_6, S_5 are non-regenerative. Further, we define the following notations—

E	:	Set of regenerative states $\equiv \{S_0 \text{ to } S_7\}$.
$\bar{E}$	:	Set of non-regenerative states $\equiv \{S_3 \text{ to } S_6, S_8, S_9\}$.
$X_1 (i=1, 2, 3)$	:	Random variables representing the failure time of unit A, B and C for i = 1, 2, 3.
$Y_1 (i=1, 2, 3)$	:	Random variables representing the repair time of unit A, B and C for i = 1, 2, 3.
$p_{ij}, p_{ij\|x}$	:	Direct steady state unconditional probability of transition from state S_i to S_j and conditional probability of transition from state S_i to S_j for given that the unit under repair in state S_i has failed after working for a duration of time x.

$f_i(x, y)$: Joint probability density function of (X_i, Y_i); $i = 1, 2, 3$

$$= \lambda_i \mu_i (1-r_i) e^{-\lambda_i x - \mu_i y} \sum_{j=0}^{\infty} \frac{(\lambda_i \mu_i r_i x y)^2}{(j!)^2}$$

$g_i(\cdot)$: Marginal p.d.f. of $X_i = \lambda_i (1-r_i) e^{-\lambda_i (1-r_i) x}$

$h_i(y|x)$: Conditional p.d.f. of Y_i given $X_i = x$

$$= \mu_i e^{-\mu_i y - \lambda_i r_i x -} \sum_{j=0}^{\infty} \frac{(\lambda_i \mu_i r_i x y)^j}{(j!)^2}$$

$H_i(y|x)$: Conditional c.d.f. of Y_i given $X_i = x$

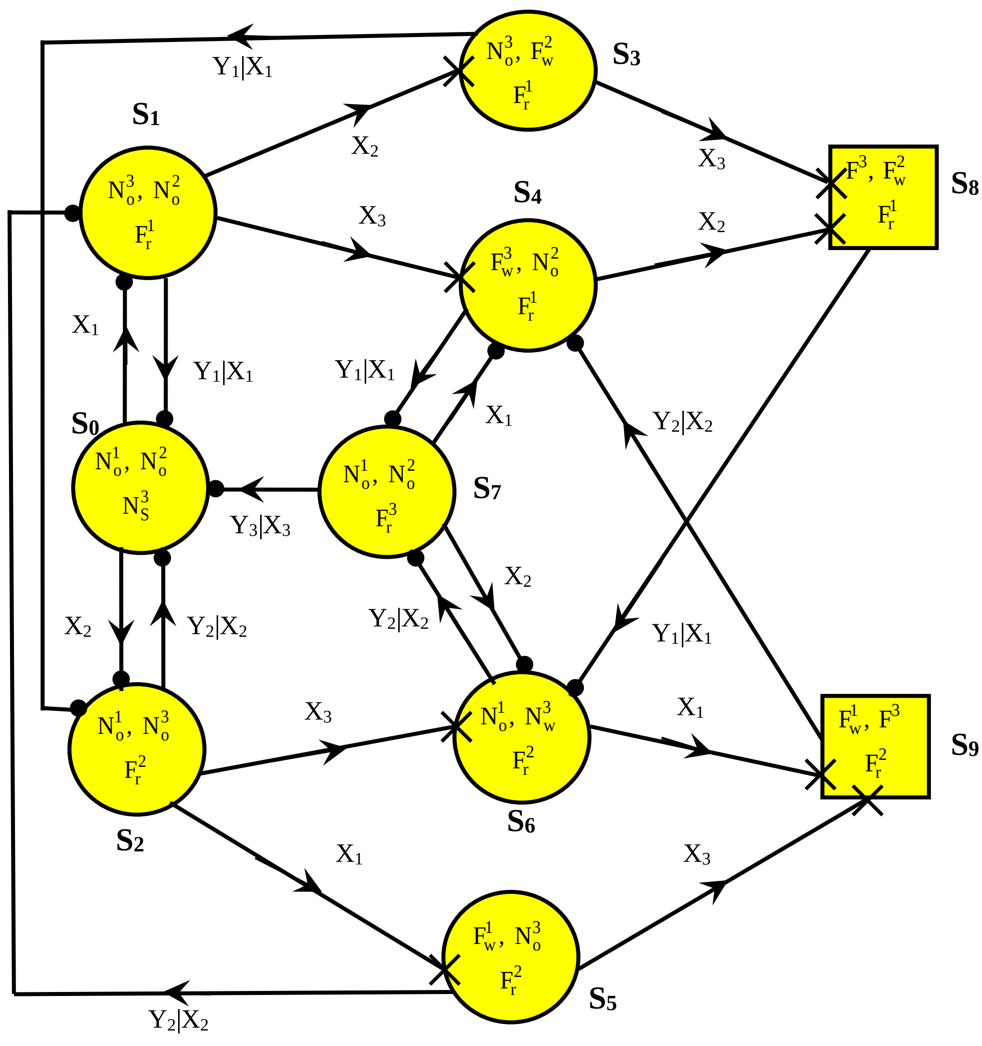

Fig. 1

Regenerative Point

Non-regenerative Point

Up State

Failed State

4. TRANSITION PROBABILITIES

(a) First we obtain the direct conditional and unconditional steady-state transition probabilities as follows:

$$p_{01} = \frac{a}{a+b}\ ;\ p_{02} = \frac{b}{a+b}\ ;\ \text{where } a = \lambda_1(1-r_1),\ b = \lambda_2(1-r_2)$$

$$p_{10|x} = \mu_1' e^{-\lambda_1 r_1 x(1-\mu_1')}\ ;\ \text{where } \mu_1' = \mu_1/(\mu_1+b+c),\ c = \lambda_3(1\text{-}r_3)$$

$$p_{20|x} = \mu_2' e^{-\lambda_2 r_2 x(1-\mu_2')}\ ;\ \text{where } \mu_2' = \mu_2/(\mu_2+a+c)$$

$$p_{47|x} = \mu_1'' e^{-\lambda_1 r_1 x(1-\mu_1'')}\ ;\ \text{where } \mu_1'' = \mu_1/(\mu_1+b)$$

$$p_{67|x} = \mu_2'' e^{-\lambda_2 r_2 x(1-\mu_2'')}\ ;\ \text{where } \mu_2'' = \mu_2/(\mu_2+a)$$

$$p_{70|x} = \mu_3' e^{-\lambda_3 r_3 x(1-\mu_3')}\ ;\ \text{where } \mu_3' = \mu_3/(\mu_3+a+b)$$

$$p_{74|x} = \frac{a}{a+b}\left[1-\mu_3' e^{-\lambda_3 r_3 (1-\mu_3')x}\right]$$

$$p_{76|x} = \frac{b}{a+b}\left[1-\mu_3' e^{-\lambda_3 r_3 (1-\mu_3')x}\right] \qquad (4.1\text{-}4.9)$$

(b) Now the indirect conditional steady-state transition probabilities via one or more non-regenerative states are given by

$$p_{16|x}^{(3,8)} = 1-\mu_1''' e^{-\lambda_1 r_1 x(1-\mu_1''')} - \frac{c}{b+c}\left\{1-\mu_1' e^{-\lambda_1 r_1 x(1-\mu_1')}\right\}$$

$$\text{where } \mu_1''' = \mu_1/(\mu_1+c)$$

$$p_{16|x}^{(4,8)} = 1-\mu_1'' e^{-\lambda_1 r_1 x(1-\mu_1'')} - \frac{b}{b+c}\left\{1-\mu_1' e^{-\lambda_1 r_1 x(1-\mu_1')}\right\}$$

$$p_{12|x}^{(3)} = \mu_1''' e^{-\lambda_1 r_1 x(1-\mu_1''')} - \mu_1' e^{-\lambda_1 r_1 x(1-\mu_1')}$$

$$p_{17|x}^{(4)} = \mu_1'' e^{-\lambda_1 r_1 x(1-\mu_1'')} - \mu_1' e^{-\lambda_1 r_1 x(1-\mu_1')}$$

$$p_{24|x}^{(5,9)} = 1-\mu_2''' e^{-\lambda_2 r_2 x(1-\mu_2''')} - \frac{c}{a+c}\left\{1-\mu_2' e^{-\lambda_2 r_2 x(1-\mu_2')}\right\}$$

$$\text{where } \mu_2'''' = \mu_2/(\mu_2 + c)$$

$$p_{24|x}^{(6,9)} = 1 - \mu_2'' e^{-\lambda_2 r_2 x(1-\mu_2'')} - \frac{a}{a+c}\left\{1 - \mu_2' e^{-\lambda_2 r_2 x(1-\mu_2')}\right\}$$

$$p_{21|x}^{(5)} = \mu_2'''' e^{-\lambda_2 r_2 x(1-\mu_2'''')} - \mu_2' e^{-\lambda_2 r_2 x(1-\mu_2')}$$

$$p_{27|x}^{(6)} = \mu_2'' e^{-\lambda_2 r_2 x(1-\mu_2'')} - \mu_2' e^{-\lambda_2 r_2 x(1-\mu_2')}$$

$$p_{46|x}^{(8)} = 1 - \mu_1'' e^{-\lambda_1 r_1 x(1-\mu_1'')}$$

$$p_{64|x}^{(9)} = 1 - \mu_2'' e^{-\lambda_2 r_2 x(1-\mu_2'')} \qquad (4.10\text{- }4.19)$$

If can be easily verified that,

$$p_{01} + p_{02} = 1$$
$$p_{10|x} + p_{12|x}^{(3)} + p_{16|x}^{(3,8)} + p_{16|x}^{(4,8)} + p_{17|x}^{(4)} = 1$$
$$p_{20|x} + p_{21|x}^{(5)} + p_{24|x}^{(5,9)} + p_{24}^{(6,9)} + p_{27|x}^{(6)} = 1 \qquad (4.20 - 4.25)$$
$$p_{46|x}^{(8)} + p_{47|x} = 1 \text{ , } \quad p_{64|x}^{(9)} + p_{67|x} = 1$$
$$p_{70|x} + p_{74|x} + p_{76|x} = 1$$

(c) From the conditional steady-state transition probabilities, the unconditional steady-state transition probabilities can be obtained by using the result—

$$p_{ij} = \int p_{ij|x} \cdot g(x)dx$$

Thus

$$p_{10} = \int p_{10|x} \cdot g_1(x)dx = \mu_1'(1-r_1)/(1-r_1\mu_1')$$

Similarly,

$$p_{20} = \mu_2'(1-r_2)/(1-r_2\mu_2')$$
$$p_{47} = \mu_1''(1-r_1)/(1-r_1\mu_1'')$$

$$p_{67} = \mu_2''(1-r_2)/(1-r_2\mu_2'')$$
$$p_{70} = \mu_3'(1-r_3)/(1-r_3\mu_3')$$
$$p_{74} = \frac{a}{a+b}\left[1-\mu_3'(1-r_3)/(1-r_3\mu_3')\right]$$
$$p_{76} = \frac{b}{a+b}\left[1-\mu_3'(1-r_3)/(1-r_3\mu_3')\right]$$

$$p_{16}^{(3,8)} = 1-\frac{\mu_1'''(1-r_1)}{(1-r_1\mu_1''')}-\frac{c}{b+c}\left\{1-\frac{\mu_1'(1-r_1)}{(1-r_1\mu_1')}\right\}$$

$$p_{16}^{(4,8)} = 1-\frac{\mu_1''(1-r_1)}{(1-r_1\mu_1'')}-\frac{b}{b+c}\left\{1-\frac{\mu_1'(1-r_1)}{(1-r_1\mu_1')}\right\}$$

$$p_{12}^{(3)} = \frac{\mu_1'''(1-r_1)}{(1-r_1\mu_1''')}-\frac{\mu_1'(1-r_1)}{(1-r_1\mu_1')}$$

$$p_{17}^{(4)} = \frac{\mu_1''(1-r_1)}{(1-r_1\mu_1'')}-\frac{\mu_1'(1-r_1)}{(1-r_1\mu_1')}$$

$$p_{24}^{(5,9)} = 1-\frac{\mu_2'''(1-r_2)}{(1-r_2\mu_2''')}-\frac{c}{a+c}\left\{1-\frac{\mu_2'(1-r_2)}{(1-r_2\mu_2')}\right\}$$

$$p_{24}^{(6,9)} = 1-\frac{\mu_2''(1-r_2)}{(1-r_2\mu_2'')}-\frac{a}{a+c}\left\{1-\frac{\mu_2'(1-r_2)}{(1-r_2\mu_2')}\right\}$$

$$p_{21}^{(5)} = \frac{\mu_2'''(1-r_2)}{(1-r_2\mu_2''')}-\frac{\mu_2'(1-r_2)}{(1-r_2\mu_2')}$$

$$p_{27}^{(6)} = \frac{\mu_2''(1-r_2)}{(1-r_2\mu_2'')}-\frac{\mu_2'(1-r_2)}{(1-r_2\mu_2')}$$

$$p_{46}^{(8)} = 1-\frac{\mu_1''(1-r_1)}{(1-r_1\mu_1'')}$$

$$p_{64}^{(9)} = 1-\frac{\mu_2''(1-r_2)}{(1-r_2\mu_2'')} \qquad (4.26 - 4.42)$$

Thus we observe

$$p_{10}+p_{12}^{(3)}+p_{16}^{(3,8)}+p_{16}^{(4,8)}+p_{17}^{(4)} = 1$$
$$p_{20}+p_{21}^{(5)}+p_{24}^{(5,9)}+p_{24}^{(6,9)}+p_{27}^{(6)} = 1$$
$$p_{46}^{(8)}+p_{47} = 1\ ,\quad p_{64}^{(9)}+p_{67} = 1$$
$$p_{70}+p_{74}+p_{76} = 1 \qquad (4.42 - 4.46)$$

5. MEAN SOJOURN TIMES

Let T_i be the sojourn time in state S_i, then mean sojourn time in state S_i is given by

$$\psi_i = \int P(U_i > t)\,dt$$

Using it, the mean sojourn times in various states are as follows:

$$\psi_0 = \int e^{-\lambda_1(1-r_1)t} e^{-\lambda_2(1-r_2)t}\, dt = 1/(a+b) \tag{5.1}$$

Similarly,

$$\psi_{1|x} = \int e^{-(b+c)t} \left\{ \int_t^{\infty} \mu_1 e^{-\mu_1 y - \lambda_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 \mu_1 r_1 xy)^j}{(j!)^2} dy \right\} dt$$

$$= \frac{1-\mu_1^{'} e^{-\lambda_1 r_1 x(1-\mu_1^{'})}}{b+c}$$

So that

$$\psi_1 = \int \psi_{1|x} \cdot g_1(x)dx = \left[1 - \frac{\mu_1^{'}(1-r_1)}{1-r_1\mu_1^{'}} \right] /(b+c) \tag{5.2}$$

$$\psi_{2|x} = \int e^{-(a+c)t} \left\{ \int_t^{\infty} \mu_2 e^{-\mu_2 y - \lambda_2 r_2 x} \sum_{j=0}^{\infty} \frac{(\lambda_2 \mu_2 r_2 xy)^j}{(j!)^2} dy \right\} dt$$

$$= \frac{1-\mu_2^{'} e^{-\lambda_2 r_2 x(1-\mu_2^{'})}}{a+c}$$

So that

$$\psi_2 = \left[1 - \frac{\mu_2^{'}(1-r_2)}{1-r_2\mu_2^{'}} \right] /(a+c) \tag{5.3}$$

$$\psi_{4|x} = \int e^{-bt} \left\{ \int_t^{\infty} \mu_1 e^{-\mu_1 y - \lambda_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 \mu_1 r_1 xy)^j}{(j!)^2} dy \right\} dt$$

$$= \frac{1-\mu_1^{''} e^{-\lambda_1 r_1 x(1-\mu_1^{''})}}{b}$$

So that

$$\psi_4 = \left[1 - \frac{\mu_1^{''}(1-r_1)}{1-r_1\mu_1^{''}} \right] / b \tag{5.4}$$

$$\psi_{6|x} = \int e^{-at} \left\{ \int_t^{\infty} \mu_2 e^{-\mu_2 y - \lambda_2 r_2 x} \sum_{j=0}^{\infty} \frac{(\lambda_2 \mu_2 r_2 xy)^j}{(j!)^2} dy \right\} dt$$

$$= \frac{1-\mu_2^{''} e^{-\lambda_2 r_2 x(1-\mu_2^{''})}}{a}$$

So that

$$\psi_6 = \left[1 - \frac{\mu_2^{''}(1-r_2)}{1-r_2\mu_2^{''}} \right] / a \tag{5.5}$$

$$\psi_{7|x} = \int e^{-(a+b)t}\left\{\int_t^{\infty} \mu_3 e^{-\mu_3 y-\lambda_3 r_3 x}\sum_{j=0}^{\infty}\frac{(\lambda_3\mu_3 r_3 xy)^j}{(j!)^2}dy\right\}dt$$

$$= \frac{1-\mu_3' e^{-\lambda_3 r_3 x(1-\mu_3')}}{a+b}$$

So that

$$\psi_7 = \left[1-\frac{\mu_3'(1-r_3)}{1-r_3\mu_3'}\right]/(a+b) \tag{5.6}$$

6. ANALYSIS OF CHARACTERISTICS

(a) Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system does not enter into any of the failed state S_8 or S_9 during time interval (0, t) when system initially starts from up state $S_i \in E$. To obtain it we assume these failed states as absorbing. Now by using the additive law of probability we have the following set of integral equations for $R_i(t)$; i = 0, 1, 2, 4, 6, 7.

$$R_0(t) = Z_0(t)+q_{01}(t)©R_1(t)+q_{02}(t)©R_2(t)$$

$$R_1(t) = Z_1(t)+q_{10}(t)©R_0(t)+q_{12}^{(3)}(t)©R_2(t)+q_{17}^{(4)}(t)©R_7(t)$$

$$R_2(t) = Z_2(t)+q_{20}(t)©R_0(t)+q_{21}^{(5)}(t)©R_1(t)+q_{27}^{(6)}(t)©R_7(t)$$

$$R_4(t) = Z_4(t)+q_{47}(t)©R_7(t)$$

$$R_6(t) = Z_6(t)+q_{67}(t)©R_7(t)$$

$$R_7(t) = Z_7(t)+q_{70}(t)©R_0(t)+q_{74}(t)©R_4(t)+q_{76}(t)©R_6(t) \tag{6.1-6.6}$$

where

$$Z_0(t) = e^{-(a+b)t}$$

$$Z_1(t) = e^{-(b+c)t}\int \bar{H}_1(t|x)g_1(x)dx$$

$$Z_2(t) = e^{-(a+c)t}\int \bar{H}_2(t|x)g_2(x)dx$$

$$Z_4(t) = e^{-bt}\int \bar{H}_1(t|x)g_1(x)dx$$

$$Z_6(t) = e^{-at}\int \bar{H}_2(t|x)g_2(x)dx$$

$$Z_7(t) = e^{-(a+b)t}\int \bar{H}_3(t|x)g_3(x)dx$$

Taking Laplace Transforms of equations (6.1-6.6) we get simple algebraic equations in $R_i^*(s)$; $i = 0,1,2,4,6,7.$ Upon solving these algebraic equations for $R_0^*(s)$, one gets

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{6.7}$$

where

$$\begin{aligned} N_1(s) = {} & [(1-q_{12}^{(3)*}q_{21}^{(5)*})Z_0^* + (q_{01}^* + q_{02}^* q_{21}^{(5)*})Z_1^* + (q_{02}^* + q_{01}^* q_{12}^{(3)*})Z_2^*] \\ & \times(1-q_{47}^* q_{74}^* - q_{67}^* q_{76}^*) + (q_{74}^* Z_4^* + q_{76}^* Z_6^* + Z_7^*)[q_{01}^*(q_{12}^{(3)*}q_{27}^{(6)*} \\ & + q_{17}^{(4)*}) + q_{02}^*(q_{21}^{(5)*}q_{17}^{(4)*} + q_{27}^{(6)*})] \end{aligned}$$

and

$$\begin{aligned} D_1(s) = {} & [1-q_{12}^{(3)*}q_{21}^{(5)*} - (q_{01}^* + q_{02}^* q_{21}^{(5)*})q_{10}^* - (q_{02}^* + q_{01}^* q_{12}^{(3)*})q_{20}^*](1-q_{47}^* q_{74}^* - q_{67}^* q_{76}^*) \\ & - q_{70}^*[q_{01}^*(q_{12}^{(3)*}q_{27}^{(6)*} + q_{17}^{(4)*}) + q_{02}^*(q_{21}^{(5)*}q_{17}^{(4)*} + q_{27}^{(6)*})] \end{aligned}$$

For brevity, we have omitted the argument's' from $q_{ij}^*(s)$, $Z_i^*(s)$ and $R_i^*(s)$. Taking the inverse Laplace Transform of (6.7), we can get the reliability of the system when initially system starts from S_0.

The mean time to system failure (MTSF) can be obtained using the well known formula

$$E(T_0) = \int R_0(t)\,dt = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} \tag{6.8}$$

where

$$\begin{aligned} N_1(0) = {} & [(1-p_{12}^{(3)}p_{21}^{(5)})\psi_0 + (p_{01} + p_{02}p_{21}^{(5)})\psi_1 + (p_{02} + p_{01}p_{12}^{(3)})\psi_2] \\ & \times(1-p_{47}p_{74} - p_{67}p_{76}) + (p_{74}\psi_4 + p_{76}\psi_6 + \psi_7)[p_{01}(p_{12}^{(3)}p_{27}^{(6)} \\ & + p_{17}^{(4)}) + p_{02}(p_{21}^{(5)}p_{17}^{(4)} + p_{27}^{(6)})] \end{aligned}$$

and

$$\begin{aligned} D_1(0) = {} & [1-p_{12}^{(3)}p_{21}^{(5)} - (p_{01} + p_{02}p_{21}^{(5)})p_{10} - (p_{02} + p_{01}p_{12}^{(3)})p_{20}]\,(1-p_{47}p_{74} - p_{67}p_{76}) \\ & - p_{70}[p_{01}(p_{12}^{(3)}p_{27}^{(6)} + p_{17}^{(4)}) + p_{02}(p_{21}^{(5)}p_{17}^{(4)} + p_{27}^{(6)})] \end{aligned}$$

(b) Availability Analysis

Let us define $A_i(t)$ as the probabilities that the system is up (operative) at epoch t when initially the system starts from state $S_i \in E$. By using simple probabilistic arguments in renewal theoretic approach as in case of reliability analysis, one can obtain the value of $A_0(t)$ in terms of its Laplace transform i.e. $A_0^*(s)$.

The steady-state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = N_2 / D_2 \tag{6.9}$$

where

$$N_2 = C_0\psi_0 + C_1W_1 + C_2W_2 + C_4\psi_4 + C_6\psi_6 + C_7\psi_7$$

and

$$D_2 = C_0\psi_0 + C_1n_1 + C_2n_2 + C_4n_1 + C_6n_2 + C_7\psi_7$$

Also,

$$n_1 = \frac{1}{\mu_1(1-r_1)},\ \ n_2 = \frac{1}{\mu_2(1-r_2)}$$

$$W_1 = \psi_4 + \frac{1}{c}\left[1 - \frac{\mu_1'''(1-r_1)}{1-r_1\mu_1'''}\right] - \psi_1$$

$$W_2 = \psi_6 + \frac{1}{c}\left[1 - \frac{\mu_2'''(1-r_2)}{1-r_2\mu_2'''}\right] - \psi_2$$

$$C_0 = (1-p_{12}^{(3)}p_{21}^{(5)})\{(1-p_{67}p_{76}) - p_{64}^{(9)}(p_{46}^{(8)} + p_{47}p_{76}) - p_{74}(p_{46}^{(8)}p_{67} + p_{47})\}$$

$$C_1 = (p_{01} + p_{02}p_{21}^{(5)})\{(1-p_{67}p_{76}) - p_{64}^{(9)}(p_{46}^{(8)} + p_{47}p_{76}) - p_{74}(p_{46}^{(8)}p_{67} + p_{47})\}$$

$$C_2 = (p_{02} + p_{01}p_{12}^{(3)})\{(1-p_{67}p_{76}) - p_{64}^{(9)}(p_{46}^{(8)} + p_{47}p_{76}) - p_{74}(p_{46}^{(8)}p_{67} + p_{47})\}$$

$$C_4 = (p_{01}p_{12}^{(3)} + p_{02})\{(1-p_{67}p_{76})p_{24} - p_{27}^{(6)}(p_{76}p_{64}^{(9)} + p_{74})\}$$
$$+ (p_{02}p_{21}^{(5)} + p_{02})\{(p_{16} + p_{17}^{(4)}p_{76})p_{64}^{(9)} + p_{74}(p_{16}p_{67} + p_{17}^{(4)})\}$$

$$C_6 = (p_{02}p_{21}^{(5)} + p_{01})\{(1-p_{47}p_{74})p_{16} - p_{17}^{(4)}(p_{74}p_{46}^{(8)} + p_{76})\}$$
$$+ (p_{01}p_{12}^{(3)} + p_{02})\{(p_{24} + p_{27}^{(6)}p_{74})p_{46}^{(8)} + p_{76}(p_{24}p_{47} + p_{27}^{(6)})\}$$

$$C_7 = (1-p_{46}^{(8)}p_{64}^{(9)})(p_{01}p_{12}^{(3)} + p_{02}p_{21}^{(5)})$$

$$p_{16} = (p_{16}^{(3,8)} + p_{16}^{(4,8)}),\ p_{24} = (p_{24}^{(5,9)} + p_{24}^{(6,9)})$$

(c) Busy Period Analysis of Repairman

Let $B_i(t)$ be the probability that the repair facility is busy at epoch t when system initially starts from state $S_i \in E$. By using the same probabilistic arguments as in case of reliability and availability analysis one can develop the recurrence relations in $B_i(t)$ for i = 0, 1, 2, 4, 6, 7. On taking the Laplace Transforms and solving the resulting set of algebraic equations, the value of $B_0(t)$ in terms of its L.T. i.e. $B_0^*(s)$ can be obtained.

In the long-run, the probability that the repair facility will be busy in the repair of a failed unit is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = N_3 / D_2 \qquad (6.10)$$

Where

$$N_3 = (C_1 + C_4)n_1 + (C_2 + C_6)n_2 + C_7\psi_7$$

(d) Profit Function Analysis

The expected profit incurred by the system during time interval (0, t) is given by

P(t) = Expected total revenue in (0, t) – Expected total repair cost in (0, t)

$$= K_0\mu_{up}(t) - K_1\mu_b(t) - K_2 t \qquad (6.11)$$

where K_0 is the revenue per-unit time by the system when it is operative, K_1 is the per-unit time cost incurred in repairing a failed unit (excluding repairman charges) and K_2 is the per-unit time repairman charges during which he is busy or idle as it is assumed that the repairman is full time employed. Also, $\mu_{up}(t)$ and $\mu_b(t)$ are defined as follows—

$\mu_{up}(t)$ = Expected up (operative) time of the system during (0, t)

$$= \int_0^t A_0(u)\,du$$

So that $$\mu_{up}^*(s) = A_0^*(s)/s \qquad (6.12)$$

and

μ_b (t) = Expected busy period of repairman during (0, t)

$$= \int_0^t B_0(u)\,du$$

So that $$\mu_b^*(s) = B_0^*(s)/s \qquad (6.13)$$

Now, the expected total profit per-unit time in steady-state is given by

$$P = \lim_{t\to\infty} \frac{P(t)}{t} = \lim_{s\to 0} s^2P^*(s) = K_0A_0 - K_1B_0 - K_2 \qquad (6.14)$$

8. GRAPHICAL REPRESENTATION

The curves for MTSF and profit function are drawn in respect of different parameters of the joint p.d.f. of failure and repair times of unit-1. Fig. 2 depicts the variations in MTSF with respect to λ_1 for three different values of μ_1 (0.1, 0.3 and 0.5) and two values of r_1 (0.1 and 0.3) when the values of other parameters are kept fix as $\lambda_2 = 0.1$, $\lambda_3 = 0.3$, $\mu_2 = 0.1$, $\mu_3 = 0.4$, $r_2 = 0.4$ and $r_3 = 0.6$. We may clearly observe from this figure that the MTSF decreases uniformly as λ_1 increases. More so, MTSF increases with the increase in the values of μ_1 and r_1.

Similarly, Fig. 3 reveals the variations in steady state Profit (P) with respect to λ_1 for three different values of μ_1 (0.25, 0.30 and 0.35) and two values of r_1 (0.1 and 0.3) when the values of λ_2, λ_3, μ_2, μ_3, r_2 and r_3 are taken same as in case of MTSF and the values of $K_0 = 60$, $K_1 = 40$ and $K_3 = 20$ are taken respectively. From this figure the same trends in profit variation in respect of λ_1, μ_1 and r_1 have been observed as reported in case of MTSF. Further, it is important to note from dotted curves that system goes in loss if λ_1 exceeds from 0.019, 0.0202 and 0.0212 respectively for $r_1 = 0.3$.Similarly, from smooth curves it is obvious that the system goes in loss if λ_1 exceeds from 0.016, 0.017 and 0.0177 respectively for $r_1 = 0.1$. Thus, the higher correlation between failure and repair times of unit-1 provides the better system performances. Similar conclusions may be drawn about the correlation coefficients r_2 and r_3 for unit-2 and unit-3.

Behavior of MTSF with respect to λ_1, μ_1 and r_1.

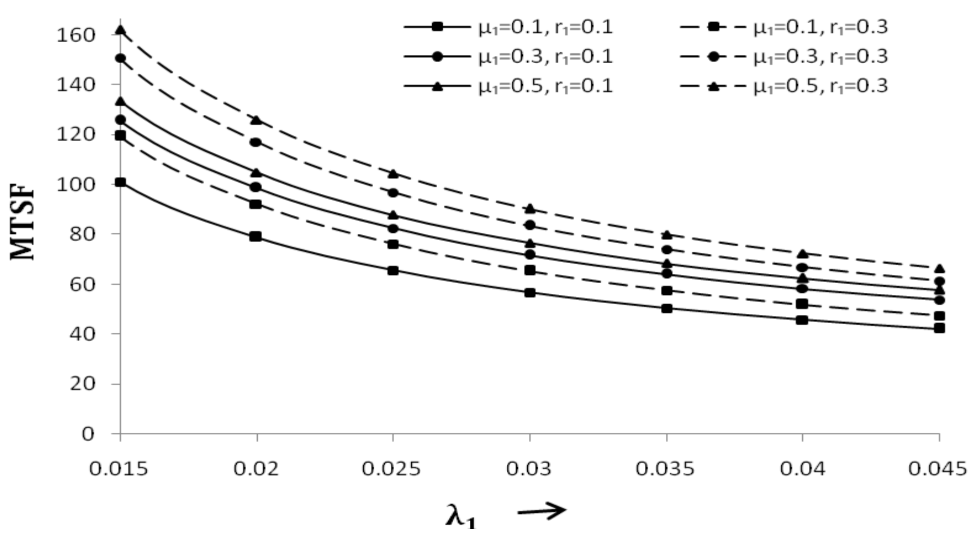

Fig. 2

Behavior of profit functions with respect to λ_1, μ_1 and r_1

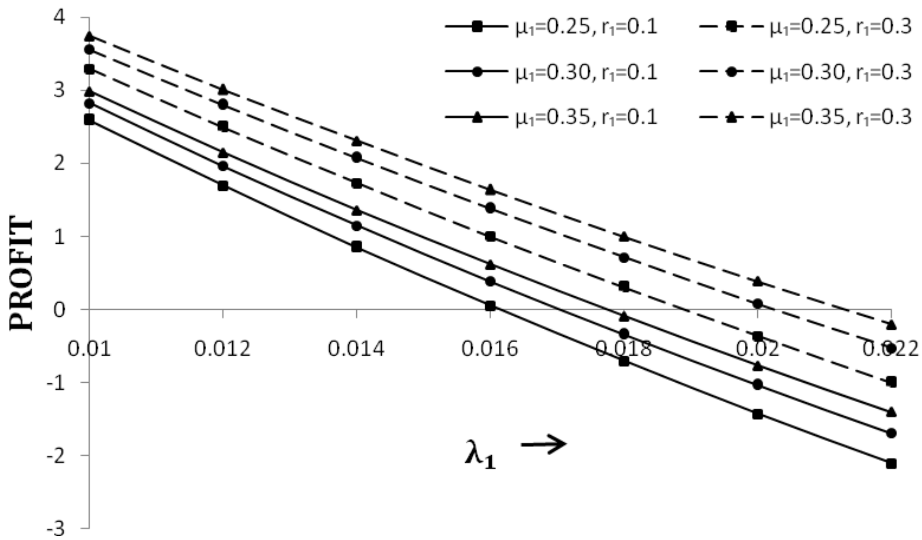

Fig. 3

Conflict of Interests

The authors declare that there is no conflict of interests.

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