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ON THE SOFT UNIFORMITY AND ITS SOME PROPERTIES

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Abstract. In the present paper, we introduce the notion of a soft uniformity by using the soft set theory. The concepts of a soft uniformity, a soft uniformity base and a soft uniform continuity are presented and their related properties are studied. Also, it is obtained a soft uniformity's relationship with a soft metric and a soft topology. Moreover, it is shown that a soft uniformly continuous mapping is a soft continuous. This research form the theoretical basis for further applications of a soft uniformity.

Keywords: Soft set; Soft topology; Soft uniformity.

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1. Introduction

There are diverse uncertainties for most of complex problems in economics, engineering, environmental science and social science. Several set theories can be regarded as mathematical tools for dealing with these uncertainties, for example the theory of fuzzy sets [28], the theory of intuitionistic fuzzy sets [3,4], the theory of vague sets [13], the theory of interval mathematics [4,15] and the theory of rough sets [25]. However, these theories have their inherent difficulties

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because of the inadequacy of the parameterization tool of the theories as cited by Molodtsov [23].

In 1999, Molodtsov [23] initiated the concept of a soft set theory as a new approach for coping with uncertainties and also presented the basic results of the new theory. This new theory does not require the specification of a parameter. We can utilize any parametrization with the aid of words, sentences, real numbers and so on. This implies that the problem of setting the membership function does not arise. Hence, soft set theory has compelling applications in several diverse fields, most of these applications was shown by Molodtsov [23].

Maji et al. [22] gave the first practical application of soft sets in decision making problems. Chen et al. [8] presented a novel concept of parameterization reduction in soft sets. Kong et al. [18] introduced the notion of a normal parameter reduction and presented an algorithm for normal parameter reduction. Then, Ma et al. [20] proposed a simpler and more easily comprehensible algorithm. Pei and Miao [26] showed that soft sets are a class of special information systems. Maji et al. [21] studied on soft set theory in detail. Ali et al. [2] presented some new algebraic operations on soft sets. Aktaş and Çağman [1] introduced the soft group and also compared soft sets to fuzzy set and rough set. Feng et al. [12] worked on soft semirings, soft ideals and idealistic soft semirings. Das and Samanta [10] introduced the notions of soft real sets and soft real numbers and studied their properties. Shabir and Naz [27] initiated the study of soft topological spaces. Aygünoğlu and Aygün [5] introduced soft product topology, soft compactness and generalized Tychonoff theorem to the soft topological spaces. Studies on the soft topological spaces have been accelerated [6, 11, 14, 16, 19, 24, 29].

Uniformity is a suitable tool for an investigation of topology. Also, there exist its remarkable analogies with metrics, but the scope of its applicability is much wider. Therefore, it can be considered as a bridge between metric and topology. Çetkin and Aygün [9] studied an extension of uniform structures to the soft sets. On the base of the axioms suggested by Bourbaki [7], they defined the concept of a soft uniformity consisting of soft sets on $X \times X$ with the set E of parameters and studied its basic properties.

In this work, as distinct from Çetkin and Aygün's definition, we define the notion of a soft uniformity consisting of soft sets on $SP(X) \times SP(X)$ with the set E of parameters, where $SP(X)$

is the family of all soft points over X. We study some basic properties of a soft uniformity and give several examples. Also, we compare soft uniformities to soft metric and soft topology. Moreover, we establish that every soft uniformly continuous mapping is a soft continuous. Hence, this study form the theoretical basis for further applications of a soft uniformity.

2. Preliminaries

In this section, we recollect some basic notions regarding soft sets. Throughout this work, let X be an initial universe, $P(X)$ be the power set of X and E be a set of parameters for X, where parameters are usually the characteristics or properties of objects in X.

Definition 2.1. [23] A soft set F on the universe X is defined by the set of ordered pairs

$$F = \{(e, F(e)) : e \in E, F(e) \in P(X)\}$$

where F is a mapping given by $F : E \to P(X)$.

Throughout this paper, the family of all soft sets over X is denoted by $S(X, E)$ [5].

Example 2.2. *Let $X = \mathbb{N}$, where $\mathbb{N}$ is a set of all natural numbers and let $E = \{very\,young, young, middle, old, very\,old\}$. Then, we define a soft set on X as following:*

$F = \{(very\,young, \{a \in X : a \leq 20\}), (young, \{a \in X : 20 < a \leq 40\}), (middle, \{a \in X : 40 < a \leq 60\}), (old, \{a \in X : 60 < a \leq 80\}), (very\,old, \{a \in X : 80 < a \leq 100\})\}$.

Definition 2.3. [2, 21, 26] Let $F, G \in S(X, E)$. Then,

(i) The soft set F is called null soft set, denoted by Φ, if $F(e) = \emptyset$ for every $e \in E$.

(ii) If $F(e) = X$ for all $e \in E$, then F is called absolute soft set, denoted by $\widetilde{X}$.

(iii) F is a soft subset of G if $F(e) \subseteq G(e)$ for every $e \in E$. It is denoted by $F \sqsubseteq G$.

(iv) F and G are equal if $F \sqsubseteq G$ and $G \sqsubseteq F$. It is denoted by $F = G$.

(v) The complement of F is denoted by F^c, where $F^c : E \to P(X)$ is a mapping defined by $F^c(e) = X - F(e)$ for all $e \in E$. Clearly, $(F^c)^c =$ F.

(vi) The union of F and G is a soft set H defined by $H(e) = F(e) \cup G(e)$ for all $e \in E$. H is denoted by $F \sqcup G$.

(vii) The intersection of F and G is a soft set H defined by $H(e) = F(e) \cap G(e)$ for all $e \in E$. H is denoted by $F \sqcap G$.

Definition 2.4. [11, 19, 24] A soft set P over X is said to be a soft point if there exists $e \in E$ such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \emptyset$ for all $e' \in E \setminus \{e\}$. The soft point denoted as x^e.

From now on, let $SP(X)$ be the family of all soft points over X.

Definition 2.5. [11, 24] A soft point x^e is said to belongs to a soft set F, denoted by $x^e \widetilde{\in} F$, if $x \in F(e)$.

Definition 2.6. [11] Two soft points ${x_1}^{e_1}, {x_2}^{e_2}$ are said to be equal if $e_1 = e_2$ and $x_1 = x_2$. Thus, ${x_1}^{e_1} \neq {x_2}^{e_2} \Leftrightarrow x_1 \neq x_2$ or $e_1 \neq e_2$.

Proposition 2.7. [11] *The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as a union of all soft points belonging to it; i.e.,* $F = \bigsqcup_{x^e \widetilde{\in} F} x^e$.

Definition 2.8. [17] Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings. Then, the mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi : S(X,E) \to S(Y,K)$.

(1) Let $F \in S(X,E)$. Then $\varphi_\psi(F)$ is the soft set over Y defined as follows:

$$\varphi_\psi(F)(k) = \begin{cases} \bigcup_{e \in \psi^{-1}(k)} \varphi(F(e)), & \text{if } \psi^{-1}(k) \neq \emptyset; \\ \emptyset, & \text{otherwise.} \end{cases}$$

for all $k \in K$.

$\varphi_\psi(F)$ is called a soft image of a soft set F.

(2) Let $G \in S(Y,K)$. Then $\varphi_\psi^{-1}(G)$ is the soft set over X defined as follows:

$$\varphi_\psi^{-1}(G)(e) = \varphi^{-1}(G(\psi(e)))$$

for all $e \in E$.

$\varphi_\psi^{-1}(G)$ is called a soft inverse image of a soft set G.

The soft mapping φ_ψ is called injective, if φ and ψ are injective. The soft mapping φ_ψ is called surjective, if φ and ψ are surjective [5, 29].

The following propositions follow easily from the definition:

Proposition 2.9. *Let $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping and $x^e \in SP(X)$. Then $\varphi_\psi(x^e) = \varphi(x)^{\psi(e)} \in SP(Y)$.*

Proposition 2.10. *Let $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping and $y^k \in SP(Y)$. If φ_ψ is bijective, then $\varphi_\psi^{-1}(y^k) = \varphi^{-1}(y)^{\psi^{-1}(k)} \in SP(X)$.*

Theorem 2.11. [17] *Let $F_i \in S(X,E)$ and $G_i \in S(Y,K)$ for all $i \in J$ where J is an index set. Then, for a soft mapping $\varphi_\psi : S(X,E) \to S(Y,K)$, the following conditions are satisfied.*

(1) If $F_1 \sqsubseteq F_2$, then $\varphi_\psi(F_1) \sqsubseteq \varphi_\psi(F_2)$.

(2) If $G_1 \sqsubseteq G_2$, then $\varphi_\psi^{-1}(G_1) \sqsubseteq \varphi_\psi^{-1}(G_2)$.

(3) $\varphi_\psi(\bigsqcup_{i\in J} F_i) = \bigsqcup_{i\in J} \varphi_\psi(F_i)$.

(4) $\varphi_\psi(\bigsqcap_{i\in J} F_i) \sqsubseteq \bigsqcap_{i\in J} \varphi_\psi(F_i)$.

(5) $\varphi_\psi^{-1}(\bigsqcup_{i\in J} G_i) = \bigsqcup_{i\in J} \varphi_\psi^{-1}(G_i)$.

(6) $\varphi_\psi^{-1}(\bigsqcap_{i\in J} G_i) = \bigsqcap_{i\in J} \varphi_\psi^{-1}(G_i)$.

(7) $\varphi_\psi^{-1}(\widetilde{Y}) = \widetilde{X}$, $\varphi_\psi^{-1}(\Phi) = \Phi$ and $\varphi_\psi(\Phi) = \Phi$.

Theorem 2.12. [5, 29] *Let $F, F_i \in S(X,E)$ for all $i \in J$ where J is an index set and let $G \in S(Y,K)$. Then, for a soft mapping $\varphi_\psi : S(X,E) \to S(Y,K)$, the following conditions are satisfied.*

(1) $F \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(F))$, the equality holds if φ_ψ is injective.

(2) $\varphi_\psi(\varphi_\psi^{-1}(G)) \sqsubseteq G$, the equality holds if φ_ψ is surjective.

Definition 2.13. [10] Let X be a non-empty set and E be a non-empty parameter set. Then a function $\varepsilon : E \to X$ is said to be a soft element of X. A soft element ε of X is said to belong to a soft set F of X, denoted by $\varepsilon \widetilde{\in} F$, if $\varepsilon(e) \in F(e)$ for every $e \in E$. Thus a soft set F can be expressed as $F(e) = \{\varepsilon(e) : \varepsilon \widetilde{\in} F\}$ for every $e \in E$.

Note 2.14. [10] It is to be noted that every singleton soft set (that is, for every $e \in E$, $F(e)$ is a singleton set) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall e \in E$.

Definition 2.15. [10] Let $\mathbb{R}$ be the set of real numbers, $\mathfrak{B}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be a set of parameters. Then, $F = \{(e, F(e)) : e \in E, F(e) \in \mathfrak{B}(\mathbb{R})\}$ is called a soft real set.

If specifically F is a singleton soft set, then after identifying F with the corresponding soft element, it will be called a soft real number.

Definition 2.16. [10] Let F, G be soft real numbers. Then,

(i) The sum is defined by $(F+G)(e) = F(e)+G(e)$, for every $e \in E$.

(ii) The difference is defined by $(F-G)(e) = F(e)-G(e)$, for every $e \in E$.

(iii) The product is defined by $(F.G)(e) = F(e).G(e)$, for every $e \in E$.

(iv) The modulus of F is denoted by $|F|$ and is defined by $|F|(e) = |F(e)|$, for every $e \in E$.

From the above definition of soft real numbers it follows that $F+G, F-G, F.G$ and $|F|$ are soft real numbers.

Definition 2.17. [10] Let F be a soft real number. Then, F is said to be a non-negative soft real number if $F(e)$ is a non-negative real number for every $e \in E$.

Let $\mathbb{R}(E)^*$ denote the set of all non-negative soft real numbers.

We use notations $\tilde{r}, \tilde{s}, \tilde{t}$ to denote soft real numbers whereas $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(e) = r$, for all $e \in E$ etc. For example $\overline{0}$ is the soft real number where $\bar{0}(e) = 0$, for all $e \in E$ [11].

Definition 2.18. [11] Let $\tilde{r}$ and $\tilde{s}$ be two soft real numbers. Then,

(i) $\tilde{r} \mathrel{\widetilde{\leq}} \tilde{s}$ if $\tilde{r}(e) \leq \tilde{s}(e)$, for every $e \in E$.

(ii) $\tilde{r} \mathrel{\widetilde{\geq}} \tilde{s}$ if $\tilde{r}(e) \geq \tilde{s}(e)$, for every $e \in E$.

(iii) $\tilde{r} \mathrel{\widetilde{<}} \tilde{s}$ if $\tilde{r}(e) < \tilde{s}(e)$, for every $e \in E$.

(iv) $\tilde{r} \mathrel{\widetilde{>}} \tilde{s}$ if $\tilde{r}(e) > \tilde{s}(e)$, for every $e \in E$.

Definition 2.19. [11] A mapping $d : SP(X) \times SP(X) \to \mathbb{R}(E)^*$ is called a soft metric on X if it satisfies the following axioms:

(sm_1) $d(x_1^{e_1}, x_2^{e_2}) \mathrel{\widetilde{\geq}} \overline{0}$, for all $x_1^{e_1}, x_2^{e_2} \mathrel{\widetilde{\in}} \widetilde{X}$.

(sm_2) $d(x_1^{e_1}, x_2^{e_2}) = \overline{0}$ if and only if $x_1^{e_1} = x_2^{e_2}$.

(sm_3) $d(x_1^{e_1}, x_2^{e_2}) = d(x_2^{e_2}, x_1^{e_1})$ for all $x_1^{e_1}, x_2^{e_2} \mathrel{\widetilde{\in}} \widetilde{X}$.

(sm_4) $d(x_1^{e_1}, x_2^{e_2}) \widetilde{\leq} d(x_1^{e_1}, x_3^{e_3}) + d(x_3^{e_3}, x_2^{e_2})$, for all $x_1^{e_1}, x_2^{e_2}, x_3^{e_3} \widetilde{\in} \widetilde{X}$.

A soft metric space is the pair (X, d) consisting of a set X and a metric d on X.

Example 2.20. [11] *Let $X = E = \mathbb{R}$. Then, $d : SP(X) \times SP(X) \to \mathbb{R}(E)^*$, where*

$$d(x_1^{e_1}, x_2^{e_2}) = |\overline{x_1} - \overline{x_2}| + |\overline{e_1} - \overline{e_2}| \;\; for\ every\ x_1^{e_1}, x_2^{e_2} \widetilde{\in} \widetilde{X}$$

is a soft metric on X.

3. Soft topological spaces

In this section, we recall some fundamental properties concerning soft topological spaces and give a new definition such as soft continuity. In the next section, these concepts will be used.

Definition 3.1. [27] Let τ be a collection of soft sets over X, then τ is said to be a soft topology on X if

(st_1) $\Phi, \widetilde{X}$ belong to τ.

(st_2) the union of any number of soft sets in τ belongs to τ.

(st_3) the intersection of any two soft sets in τ belongs to τ

(X, τ) is called a soft topological space. The members of τ are called soft open sets in X. A soft set F over X is called a soft closed in X if $F^c \in \tau$.

Definition 3.2. [5, 24] Let (X, τ) be a soft topological space. A subcollection $\mathcal{B}$ of τ is called a base for τ if every member of τ can be expressed as a union of some members of $\mathcal{B}$.

Definition 3.3. [29] Let (X, τ) be a soft topological space and $F \in S(X, E)$. The soft interior of F is the soft set $F^o = \bigsqcup\{G : G\ is\ soft\ open\ set\ and\ G \sqsubseteq F\}$.

Definition 3.4. [27] Let (X, τ) be a soft topological space and $F \in S(X, E)$. The soft closure of F is the soft set $\overline{F} = \bigsqcap\{G : G\ is\ soft\ closed\ set\ and\ F \sqsubseteq G\}$.

Definition 3.5. [24] A soft set F in a soft topological space (X, τ) is called a soft neighborhood of the soft point x^e if there exists a soft open set G such that $x^e \widetilde{\in} G \sqsubseteq F$.

The soft neighborhood system of a soft point x^e, denoted by $\mathcal{N}_\tau(x^e)$, is the family of all its soft neighborhoods.

Proposition 3.6. [24] *Let (X,τ) be a soft topological space and $\{\mathcal{N}_\tau(x^e) : x^e \widetilde{\in} \widetilde{X}\}$ be the system of soft neighborhoods. Then,*

(sn_1) *$\mathcal{N}_\tau(x^e) \neq \emptyset$, for all $x^e \widetilde{\in} \widetilde{X}$,*

(sn_2) *If $F \in \mathcal{N}_\tau(x^e)$, then $x^e \widetilde{\in} F$,*

(sn_3) *If $F \in \mathcal{N}_\tau(x^e)$ and $F \sqsubseteq G$, then $G \in \mathcal{N}_\tau(x^e)$,*

(sn_4) *If $F,G \in \mathcal{N}_\tau(x^e)$, then $F \sqcap G \in \mathcal{N}_\tau(x^e)$,*

(sn_5) *If $F \in \mathcal{N}_\tau(x^e)$, then there exists a $G \in \mathcal{N}_\tau(x^e)$ such that $F \in \mathcal{N}_\tau(y^\alpha)$, for every $y^\alpha \widetilde{\in} G$.*

Conversely, let there be assigned to every soft point $x^e \in SP(X)$ a collection $\mathcal{N}(x^e)$ of soft subsets of $S(X,E)$ satisfying all these axioms. Then, there exists a soft topology τ on X such that, for every $x^e \widetilde{\in} \widetilde{X}$, $\mathcal{N}(x^e)$ is the τ-soft neighborhood system of x^e.

Definition 3.7. [24] A soft set F over X is soft open iff F is a soft neighborhood of each of its soft points.

Theorem 3.8. [19] *Let (X,τ) be a soft topological space. A soft point $x^e \widetilde{\in} \overline{F}$ iff each soft neighborhood of x^e intersects F.*

Definition 3.9. Let (X,τ_1) and (Y,τ_2) be two soft topological spaces and let $\varphi_\psi : (X,\tau_1) \to (Y,\tau_2)$. Then φ_ψ is called soft continuous at $x^e \widetilde{\in} \widetilde{X}$ if for every soft neighborhood G of $\varphi_\psi(x^e)$ in Y, there exists a soft neighborhood F of x^e in X such that $\varphi_\psi(F) \sqsubseteq G$.

A soft mapping φ_ψ is called soft continuous on X if it is soft continuous at each $x^e \widetilde{\in} \widetilde{X}$.

Theorem 3.10. *Let (X,τ_1) and (Y,τ_2) be two soft topological spaces and $\varphi_\psi : (X,\tau_1) \to (Y,\tau_2)$ be a soft mapping. Then the following conditions are equivalent:*

(i) *φ_ψ is soft continuous.*

(ii) *For every soft open set G in (Y,τ_2), $\varphi_\psi^{-1}(G)$ is soft open in (X,τ_1).*

(iii) *For every soft closed set F in (Y,τ_2), $\varphi_\psi^{-1}(F)$ is soft closed in (X,τ_1).*

(iv) *For every $F \in S(X,E)$, $\varphi_\psi(\overline{F}) \sqsubseteq \overline{\varphi_\psi(F)}$.*

Proof. $(i) \Rightarrow (ii)$: Let $G \in \tau_2$ and $x^e \widetilde{\in} \varphi_\psi^{-1}(G)$. Then, G is a soft neighborhood of $\varphi_\psi(x^e)$. Hence, by soft continuity of φ_ψ, there exists a soft neighborhood F of x^e such that $\varphi_\psi(F) \sqsubseteq G$; that is, $F \sqsubseteq \varphi_\psi^{-1}(G)$. Thus, $\varphi_\psi^{-1}(G)$ is a soft neighborhood of each of its soft points and therefore is soft open.

$(ii) \Rightarrow (iii)$: If F is soft closed in (Y,τ_2), then $\varphi_\psi^{-1}(\widetilde{Y}-F)$ is soft open in (X,τ_1), by part (ii). Hence, since $\varphi_\psi^{-1}(\widetilde{Y}-F)=\widetilde{X}-\varphi_\psi^{-1}(F)$, $\varphi_\psi^{-1}(F)$ is soft closed in (X,τ_1).

$(iii) \Rightarrow (iv)$: Let H be any soft closed set in (Y,τ_2) containing $\varphi_\psi(F)$. By part (iii), $\varphi_\psi^{-1}(H)$ is soft closed set in (X,τ_1) containing F. Hence, $\overline{F}\sqsubseteq\varphi_\psi^{-1}(H)$, and it follows that $\varphi_\psi(\overline{F})\sqsubseteq H$. Since this is true for any soft closed set H containing $\varphi_\psi(F)$, we have $\varphi_\psi(\overline{F})\sqsubseteq\overline{\varphi_\psi(F)}$.

$(iv) \Rightarrow (i)$: Let $x^e\,\widetilde{\in}\,\widetilde{X}$ and let G be a soft neighborhood of $\varphi_\psi(x^e)$. Then, there exists a soft open set H in (Y,τ_2) such that $\varphi_\psi(x^e)\,\widetilde{\in}\,H\sqsubseteq G$. Set $B=\widetilde{X}-\varphi_\psi^{-1}(H)$ and let $F=\widetilde{X}-\overline{B}$. It is easy to verify that, since $\varphi_\psi(\overline{B})\sqsubseteq\overline{\varphi_\psi(B)}$, we have $x^e\,\widetilde{\in}\,F$. Also, it is clear that $\varphi_\psi(F)\sqsubseteq G$. Hence, φ_ψ is soft continuous at x^e.

Definition 3.11. [6] A soft topological space (X,τ) is a soft T_1-space if whenever $x_1^{e_1}$ and $x_2^{e_2}$ are distinct soft points in X, there exists a soft neighborhood of each not containing the other.

Example 3.12. *Let X be a non-empty set. Then, (X,τ), where*

$$\tau=\{F\in S(X,E): for\ every\ e\in E,\ X-F(e)\ is\ finite\}\cup\{\Phi\}$$

is a soft topological space. Also, we observe that the soft topological space (X,τ) is a soft T_1-space.

Theorem 3.13. *(X,τ) is a soft T_1-space iff every soft point in X is soft closed.*

Proof. Let (X,τ) be a soft T_1-space and $x^e\,\widetilde{\in}\,\widetilde{X}$. Then, for every $y^k\,\widetilde{\in}\,\widetilde{X}-x^e$, there exists a soft open set F such that $x^e\,\widetilde{\notin}\,F$ and $y^k\,\widetilde{\in}\,F$. Since $x^e\sqcap F=\Phi$, then we have $y^k\,\widetilde{\in}\,F\sqsubseteq\widetilde{X}-x^e$. So $\widetilde{X}-x^e$ is a soft open set and thus x^e is soft closed.

Conversely, let $x^e\neq y^k$. Then by hypothesis, x^e and y^k are soft closed sets. Thus, there exist soft open sets $\widetilde{X}-x^e$ and $\widetilde{X}-y^k$ such that $x^e\,\widetilde{\in}\,\widetilde{X}-y^k$, $y^k\,\widetilde{\notin}\,\widetilde{X}-y^k$ and $y^k\,\widetilde{\in}\,\widetilde{X}-x^e$, $x^e\,\widetilde{\notin}\,\widetilde{X}-x^e$.

Recall that a uniformity on a set X is a structure given by a set $\mathcal{U}$ of subsets of $X\times X$ which satisfies the following axioms (see, [7]):

(u_1) If $K\in\mathcal{U}$, then $\Delta\subseteq K$.

(u_2) If $K\in\mathcal{U}$, then there exists an $L\in\mathcal{U}$ such that $L\circ L\subseteq K$.

(u_3) If $K\in\mathcal{U}$, then there exists a $K\in\mathcal{U}$ such that $L^{-1}\subseteq K$.

(u_4) If $K,L\in\mathcal{U}$, then $K\cap L\in\mathcal{U}$.

(u_5) If $K \in \mathcal{U}$ and $K \subseteq L$, then $L \in \mathcal{U}$.

The sets of $\mathcal{U}$ are called entourages of the uniformity defined on X by $\mathcal{U}$. A set endowed with a uniformity is called uniform space.

4. Soft uniformity

In this section, we introduce a soft uniform space and obtain its some properties. Also, we present its relation with soft metric and soft topology.

We use $S(SP(X)\times SP(X),E)$ to represent the collection of all soft sets on $SP(X)\times SP(X)$ with the set E of parameters.

Definition 4.1. (a) The soft set $\Delta \in S(SP(X)\times SP(X),E)$ is said to be diagonal soft set which is defined by $\Delta(e) = \{(x^\alpha, x^\alpha) : x^\alpha \in SP(X)\}$, for every $e \in E$.

(b) Let $U \in S(SP(X)\times SP(X),E)$. Then,

$$U^{-1}(e) = \{(x_1^{e_1}, x_2^{e_2}) : (x_2^{e_2}, x_1^{e_1}) \in U(e)\}$$

for every $e \in E$. If $U = U^{-1}$, then U is said to be symmetric.

(c) Let $U, V \in S(SP(X)\times SP(X),E)$. Then,

$$U \circ V(e) = \{(x_1^{e_1}, x_2^{e_2}) : for\ some\ z^\alpha \in SP(X), (x_1^{e_1}, z^\alpha) \in V(e)\ and\ (z^\alpha, x_2^{e_2}) \in U(e)\}$$

for every $e \in E$.

Remark 4.2. Let $U, V, W \in S(SP(X)\times SP(X),E)$. Then, the following statements are satisfied:

(i) If $U \sqsubseteq V$, then $U^{-1} \sqsubseteq V^{-1}$ and $U \circ W \sqsubseteq V \circ W$.

(ii) $(U \circ V)^{-1} = V^{-1} \circ U^{-1}$.

(iii) $(U \circ V) \circ W = U \circ (V \circ W)$.

Definition 4.3. The non-empty family $\mathcal{U} \subseteq S(SP(X)\times SP(X),E)$ is called a soft uniformity for X if the following axioms are satisfied:

(su_1) If $U \in \mathcal{U}$, then $\Delta \sqsubseteq U$.

(su_2) If $U \in \mathcal{U}$, then there exists a $V \in \mathcal{U}$ such that $V \circ V \sqsubseteq U$.

(su_3) If $U \in \mathcal{U}$, then there exists a $V \in \mathcal{U}$ such that $V^{-1} \sqsubseteq U$.

(su_4) If $U, V \in \mathcal{U}$, then $U \sqcap V \in \mathcal{U}$.

(su_5) If $U \in \mathcal{U}$ and $U \sqsubseteq V$, then $V \in \mathcal{U}$.

A soft uniform space is the pair $(X, \mathcal{U})$ consisting of a set X and a soft uniformity $\mathcal{U}$ on X.

Example 4.4. *Let $X = \{x\}, E = \{e_1, e_2\}$ and let $\Delta, U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8, U_9, U_{10}, U_{11}, U_{12}, U_{13}, U_{14}, U_{15} \in S(SP(X) \times SP(X), E)$ where*

$\Delta(e_1) = \{(x^{e_1}, x^{e_1}), (x^{e_2}, x^{e_2})\}$, $\Delta(e_2) = \{(x^{e_1}, x^{e_1}), (x^{e_2}, x^{e_2})\}$,

$U_1(e_1) = \Delta(e_1)$, $U_1(e_2) = \Delta(e_2) \cup \{(x^{e_1}, x^{e_2})\}$,

$U_2(e_1) = \Delta(e_1)$, $U_2(e_2) = \Delta(e_2) \cup \{(x^{e_2}, x^{e_1})\}$,

$U_3(e_1) = \Delta(e_1)$, $U_3(e_2) = SP(X) \times SP(X)$,

$U_4(e_1) = \Delta(e_1) \cup \{(x^{e_1}, x^{e_2})\}$, $U_4(e_2) = \Delta(e_2)$,

$U_5(e_1) = \Delta(e_1) \cup \{(x^{e_1}, x^{e_2})\}$, $U_5(e_2) = \Delta(e_2) \cup \{(x^{e_1}, x^{e_2})\}$,

$U_6(e_1) = \Delta(e_1) \cup \{(x^{e_1}, x^{e_2})\}$, $U_6(e_2) = \Delta(e_2) \cup \{(x^{e_2}, x^{e_1})\}$,

$U_7(e_1) = \Delta(e_1) \cup \{(x^{e_1}, x^{e_2})\}$, $U_7(e_2) = SP(X) \times SP(X)$,

$U_8(e_1) = \Delta(e_1) \cup \{(x^{e_2}, x^{e_1})\}$, $U_8(e_2) = \Delta(e_2)$,

$U_9(e_1) = \Delta(e_1) \cup \{(x^{e_2}, x^{e_1})\}$, $U_9(e_2) = \Delta(e_2) \cup \{(x^{e_1}, x^{e_2})\}$,

$U_{10}(e_1) = \Delta(e_1) \cup \{(x^{e_2}, x^{e_1})\}$, $U_{10}(e_2) = \Delta(e_2) \cup \{(x^{e_2}, x^{e_1})\}$,

$U_{11}(e_1) = \Delta(e_1) \cup \{(x^{e_2}, x^{e_1})\}$, $U_{11}(e_2) = SP(X) \times SP(X)$,

$U_{12}(e_1) = SP(X) \times SP(X)$, $U_{12}(e_2) = \Delta(e_2)$,

$U_{13}(e_1) = SP(X) \times SP(X)$, $U_{13}(e_2) = \Delta(e_2) \cup \{(x^{e_1}, x^{e_2})\}$,

$U_{14}(e_1) = SP(X) \times SP(X)$, $U_{14}(e_2) = \Delta(e_2) \cup \{(x^{e_2}, x^{e_1})\}$,

$U_{15}(e_1) = SP(X) \times SP(X)$, $U_{15}(e_2) = SP(X) \times SP(X)$.

Then, $\mathcal{U} = \{\Delta, U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8, U_9, U_{10}, U_{11}, U_{12}, U_{13}, U_{14}, U_{15}\} \subseteq S(SP(X) \times SP(X), E)$ is a soft uniformity for X and therefore $(X, \mathcal{U})$ is a soft uniform space.

Additionally, with respect to the above example, Çetkin and Aygün's soft uniform structure on X is $\mathcal{U} = \{\Delta\}$, where Δ is a soft set on $X \times X$ which is defined by $\Delta(e_1) = \Delta(e_2) = \{(x, x)\}$. Therefore, it is seen that the authors' definition of soft uniformity is different from our work.

Definition 4.5. The non-empty subfamily $\mathcal{B}$ of a soft uniformity $\mathcal{U}$ is called a soft base of $\mathcal{U}$ if for any $U \in \mathcal{U}$ there exists a $V \in \mathcal{B}$ such that $V \sqsubseteq U$.

Theorem 4.6. *The non-empty family $\mathcal{B} \subseteq S(SP(X) \times SP(X), E)$ is a soft base of some soft uniformity on X if it satisfies the following axioms:*

(sb_1) *If $B \in \mathcal{B}$, then $\Delta \sqsubseteq B$.*

(sb_2) *If $B \in \mathcal{B}$, then there exists a $C \in \mathcal{B}$ such that $C \circ C \sqsubseteq B$.*

(sb_3) *If $B \in \mathcal{B}$, then there exists a $C \in \mathcal{B}$ such that $C^{-1} \sqsubseteq B$.*

(sb_4) *If $B_1, B_2 \in \mathcal{B}$, then there exists a $B_3 \in \mathcal{B}$ such that $B_3 \sqsubseteq B_1 \sqcap B_2$.*

Proof. It is easy to see that a non-empty family $\mathcal{B}$ satisfying $(sb_1) - (sb_4)$ generate a soft uniformity $\mathcal{U} = \{U \in S(SP(X) \times SP(X), E) : for\ some\ B \in \mathcal{B}, B \sqsubseteq U\}$.

Definition 4.7. The non-empty subfamily $\mathcal{S}$ of a soft uniformity $\mathcal{U}$ is called a soft subbase of $\mathcal{U}$ if all finite intersections of members of $\mathcal{S}$ form a soft base of $\mathcal{U}$.

Example 4.8. *(a) Let $X = E = \mathbb{R}$. Then, $\mathcal{B} = \{D_{x^e} : x^e \widetilde{\in} \widetilde{X}\}$ is a soft base for some soft uniformity on $\mathbb{R}$ such that*

$$D_{x^e}(\alpha) = \Delta(\alpha) \cup \{(x_1^{e_1}, x_2^{e_2}) : x_1, x_2 > x \text{ and } e_1, e_2 > e\}\ for\ every\ \alpha\ \in\ E.$$

Obviously, the family $\mathcal{B}$ satisfies the conditions (sb_1) and (sb_4). Since for every $x^e \widetilde{\in} \widetilde{X}$, we have $D_{x^e} \circ D_{x^e} \sqsubseteq D_{x^e}$ and $D_{x^e}^{-1} \sqsubseteq D_{x^e}$, it follows that the conditions (sb_2) and (sb_3) is also satisfied.

(b) The family $\mathcal{U} = \{U : \Delta \sqsubseteq U\}$ is a soft uniformity on a set X, which is called discrete soft uniformity. Also, the family $\mathcal{B} = \{\Delta\}$ is a soft base for $\mathcal{U}$.

Example 4.9. *Let $X = E = \mathbb{R}$. Then, $\mathcal{B} = \{D_{\tilde{\epsilon}} : \tilde{\epsilon} \widetilde{>} \overline{0}\}$ is a soft base for some soft uniformity on $\mathbb{R}$ such that*

$$D_{\tilde{\epsilon}}(e) = \{(x_1^{e_1}, x_2^{e_2}) : |\overline{x_1} - \overline{x_2}| + |\overline{e_1} - \overline{e_2}| \widetilde{<} \tilde{\epsilon}\}\ for\ every\ e\ \in\ E$$

where $|.|$ denotes the modulus of soft real numbers. Let us prove that $\mathcal{B}$ satisfies axioms $(sb_1) - (sb_4)$.

(sb_1) *Let $\tilde{\epsilon} \widetilde{>} \overline{0}$ and $e \in E$. For all $x^\alpha \widetilde{\in} \widetilde{X}$, since $|\overline{x} - \overline{x}| + |\overline{\alpha} - \overline{\alpha}| = \overline{0}$, $(x^\alpha, x^\alpha) \in D_{\tilde{\epsilon}}(e)$.*

(sb_2) *For every $\tilde{\epsilon} \widetilde{>} \overline{0}$, we have $D_{\tilde{\epsilon}/2} \circ D_{\tilde{\epsilon}/2} \sqsubseteq D_{\tilde{\epsilon}}$. Indeed, let $e \in E$ and $(x_1^{e_1}, x_2^{e_2}) \in D_{\tilde{\epsilon}/2} \circ D_{\tilde{\epsilon}/2}(e)$. Then, there exists a $z^\beta \widetilde{\in} \widetilde{X}$ such that $(x_1^{e_1}, z^\beta) \in D_{\tilde{\epsilon}/2}(e)$ and $(z^\beta, x_2^{e_2}) \in D_{\tilde{\epsilon}/2}(e)$. Hence,*

$$|\overline{x_1} - \overline{x_2}| + |\overline{e_1} - \overline{e_2}| \widetilde{\leq} |\overline{x_1} - \overline{z}| + |\overline{z} - \overline{x_2}| + |\overline{e_1} - \overline{\beta}| + |\overline{\beta} - \overline{e_2}| \widetilde{<} \tilde{\epsilon}/2 + \tilde{\epsilon}/2 = \tilde{\epsilon}.$$

So, $(x_1^{e_1}, x_2^{e_2}) \in D_{\tilde{\epsilon}}(e)$.

(sb_3) *It is clear because* $D_{\tilde{\epsilon}} = D_{\tilde{\epsilon}}^{-1}$ *for any* $\tilde{\epsilon} \,\widetilde{>}\, \overline{0}$.

(sb_4) *Let* $\tilde{\epsilon_1}, \tilde{\epsilon_2} \,\widetilde{>}\, \overline{0}$. *Now, let us take* $\tilde{\epsilon} \,\widetilde{>}\, \overline{0}$ *such that* $\tilde{\epsilon}(e) = min\{\tilde{\epsilon_1}(e), \tilde{\epsilon_2}(e)\}$ *for every* $e \in E$. *Therefore, it is easy to see that* $D_{\tilde{\epsilon}} \sqsubseteq D_{\tilde{\epsilon_1}} \sqcap D_{\tilde{\epsilon_2}}$.

Generally, we obtain the following theorem.

Theorem 4.10. *Let* (X, d) *be a soft metric space. Then,* $\mathcal{B} = \{D_{\tilde{\epsilon}} : \tilde{\epsilon} \,\widetilde{>}\, \overline{0}\}$ *is a soft base for some soft uniformity on X such that*

$$D_{\tilde{\epsilon}}(e) = \{(x_1^{e_1}, x_2^{e_2}) : d(x_1^{e_1}, x_2^{e_2}) \,\widetilde{<}\, \tilde{\epsilon}\} \text{ } for\ every\ e \in E.$$

Proof. We need to verify axioms $(sb_1)-(sb_4)$. It is clear from the definition of soft metric space and Example 4.9.

This soft uniformity is called the soft metric uniformity generated by the soft metric d. A soft uniform space $(X, \mathcal{U})$ is called a metrizable soft uniform space if its soft uniformity is induced by a soft metric on X.

Let (X, d) be a soft metric space. Then, the soft uniformities induced by the soft metrics d and 2d overlap. Therefore, we can say that different soft metrics generate the same soft uniformity. Thus, a soft uniformity represents less structure than a soft metric.

Remark 4.11. Let $(X, \mathcal{U})$ be a soft uniform space.

(a) If $U \in \mathcal{U}$ then $U^{-1} \in \mathcal{U}$.

(b) The conjunction of axioms (su_2) and (su_3) is equivalent to the following axiom:

$$For\ every\ U \in \mathcal{U}\ there\ exists\ V \in \mathcal{U}\ such\ that\ V \circ V^{-1} \sqsubseteq U.$$

(c) The family $\mathcal{B}$ consisting of the symmetric soft sets in $\mathcal{U}$ form a soft base for $\mathcal{U}$.

Proof. (a) and (c) are obvious.

(b) Let us assume that (su_2) and (su_3) hold and let $U \in \mathcal{U}$. Then, there exists a $V_1 \in \mathcal{U}$ such that $V_1 \circ V_1 \sqsubseteq U$ by (su_2) and there exists a $V_2 \in \mathcal{U}$ such that $V_2^{-1} \sqsubseteq U$ by (su_3). Take $V = V_1 \sqcap V_2 \in \mathcal{U}$. Thus, $V \circ V^{-1} \sqsubseteq U$.

Conversely, let the condition be satisfied. Then, for every $U \in \mathcal{U}$ there exists a $V \in \mathcal{U}$ such that $V \circ V^{-1} \sqsubseteq U$. Therefore, $V^{-1} \sqsubseteq U$ by (su_1). Let us choose $G = V \sqcap V^{-1}$. Thus, $G \in \mathcal{U}$ by (a) and (su_4) and we have $G \circ G \sqsubseteq U$.

Definition 4.12. Let $(X, \mathcal{U})$ be a soft uniform space and $x^e \widetilde{\in} \widetilde{X}$. Then, for every $U \in \mathcal{U}$

$$U[x^e] = \bigsqcup\{z^\beta \widetilde{\in} \widetilde{X} : (x^e, z^\beta) \in U(\alpha), \forall \alpha \in E\}$$

is a soft set on X. This is extended to the soft set F on X, denoted by

$$U[F] = \bigsqcup_{x^e \widetilde{\in} F} U[x^e] = \bigsqcup\{z^\beta \widetilde{\in} \widetilde{X} : for\ some\ x^e \widetilde{\in} F,\ (x^e, z^\beta) \in U(\alpha), \forall \alpha \in E\}.$$

Example 4.13. *Let $(X, \mathcal{U})$ be a soft uniform space which is defined in Example 4.4 and $x^{e_1} \widetilde{\in} \widetilde{X}$. Then, for $U_5, U_6 \in \mathcal{U}$*

$$U_5[x^{e_1}] = x^{e_1} \sqcup x^{e_2} = \widetilde{X} \in S(X,E) \ \ and \ \ U_6[x^{e_1}] = x^{e_1} \in S(X,E).$$

Theorem 4.14. *Let $(X, \mathcal{U})$ be a soft uniform space, and for every $x^e \widetilde{\in} \widetilde{X}$ let $\mathcal{N}_{x^e} = \{U[x^e] : U \in \mathcal{U}\}$. Then, there exists a soft topology on X such that, for every $x^e \widetilde{\in} \widetilde{X}$, the collection $\mathcal{N}_{x^e}$ is the soft neighborhood system of x^e in this soft topology.*

Proof. By Proposition 3.6, it suffices to show that the family $\mathcal{N}_{x^e}$ has properties $(sn_1) - (sn_5)$.

(sn_1) is obvious.

(sn_2) Let $U[x^e] \in \mathcal{N}_{x^e}$. Since by (su_1) we have $\Delta \sqsubseteq U$, then $(x^e, x^e) \in U(\alpha)$ for every $\alpha \in E$. Thus $x^e \widetilde{\in} U[x^e]$.

(sn_3) Let $U[x^e] \in \mathcal{N}_{x^e}$ and $U[x^e] \sqsubseteq V$. Let us define $W(\alpha) = U(\alpha) \cup \{(x^e, z^\beta) : z^\beta \widetilde{\in} V\}$ for every $\alpha \in E$. Then, it is easy to verify that $W \in \mathcal{U}$ and $V = W[x^e]$. Therefore, we obtain $V \in \mathcal{N}_{x^e}$.

(sn_4) Let $U_1[x^e], U_2[x^e] \in \mathcal{N}_{x^e}$. Since $U_1, U_2 \in \mathcal{U}$, we have $U_1 \sqcap U_2 \in \mathcal{U}$. Now, we shall show that $U_1[x^e] \sqcap U_2[x^e] = (U_1 \sqcap U_2)[x^e]$.

$$\begin{aligned} z^\beta \widetilde{\in} U_1[x^e] \sqcap U_2[x^e] &\iff (x^e, z^\beta) \in U_1(\alpha) \text{ and } (x^e, z^\beta) \in U_2(\alpha),\ \ \forall \alpha \in E. \\ &\iff (x^e, z^\beta) \in U_1(\alpha) \cap U_2(\alpha) = (U_1 \sqcap U_2)(\alpha),\ \ \forall \alpha \in E. \\ &\iff z^\beta \widetilde{\in} (U_1 \sqcap U_2)[x^e]. \end{aligned}$$

Thus, $U_1[x^e] \sqcap U_2[x^e] \in \mathcal{N}_{x^e}$.

(sn_5) Let $U[x^e] \in \mathcal{N}_{x^e}$. Because $U \in \mathcal{U}$, there exists $V \in \mathcal{U}$ such that $V \circ V \sqsubseteq U$. Let $z^\beta \widetilde{\in} V[x^e]$, that is, $(x^e, z^\beta) \in V(\alpha)$ for every $\alpha \in E$. Now, we shall show that $V[z^\beta] \sqsubseteq U[x^e]$. If $t^\lambda \widetilde{\in} V[z^\beta]$, that is, $(z^{\beta}, t^{\lambda}) \in V(\alpha)$ for every $\alpha \in E$, then $(x^e, t^{\lambda}) \in (V \circ V)(\alpha)$ for every $\alpha \in E$. Therefore $(x^e, t^{\lambda}) \in U(\alpha)$ for every $\alpha \in E$ and $t^{\lambda} \widetilde{\in} U[x^e]$. Since $V[z^{\beta}] \in \mathcal{N}_{z^{\beta}}$, by (sn_3), we have $U[x^e] \in \mathcal{N}_{z^{\beta}}$.

Definition 4.15. The soft topology defined in Theorem 4.14 is called the soft uniform topology generated by the soft uniformity $\mathcal{U}$, which is denoted by $\tau_{\mathcal{U}}$. A soft topological space (X, τ) is called a uniformizable soft topological space if its soft topology is induced by a soft uniformity on X.

Example 4.16. *(a) On any set X, the soft topology induced by the discrete soft uniformity is the discrete soft topology.*

(b) Let us consider the soft uniformity on $\mathbb{R}$ whose soft base is $\{D_{x^e} : x^e \widetilde{\in} \widetilde{\mathbb{R}}\}$ such that

$$D_{x^e}(\alpha) = \Delta(\alpha) \cup \{(x_1^{e_1}, x_2^{e_2}) : x_1, x_2 > x \text{ and } e_1, e_2 > e\} \text{ for every } \alpha \in E$$

which is defined in Example 4.8 (a). The soft topology induced by this soft uniformity is also the discrete soft topology. Because, for every $z^{\beta} \widetilde{\in} \widetilde{\mathbb{R}}$, $D_{x^e}[z^{\beta}] = z^{\beta}$ whenever $x \geq z$ or $e \geq \beta$.

This example shows that different soft uniformities can generate the same soft topology. Hence, a soft topology represents less structure than a soft uniformity.

Theorem 4.17. *(a) Let $(X, \mathcal{U})$ be a soft uniform space and $F \in S(X, E)$. Then,*

$$\overline{F} = \sqcap_{U \in \mathcal{U}} U[F].$$

(b) $(X, \tau_{\mathcal{U}})$ is a soft T_1-space if and only if for any $x^e \widetilde{\in} \widetilde{X}$ we have $\sqcap_{U \in \mathcal{U}} U[x^e] = x^e$.

Proof. (a) $x^e \widetilde{\in} \overline{F} \iff U[x^e] \sqcap F \neq \Phi$, for every $U \in \mathcal{U}$.

$\iff$ There exists a $z^{\beta} \widetilde{\in} \widetilde{X}$ such that $z^{\beta} \widetilde{\in} U[x^e]$ and $z^{\beta} \widetilde{\in} F$, $\forall U \in \mathcal{U}$.

$\iff \exists z^{\beta} \widetilde{\in} F : (x^e, z^{\beta}) \in U(\alpha)\ (\forall \alpha \in E)$, for every $U \in \mathcal{U}$.

$\iff \exists z^{\beta} \widetilde{\in} F : (z^{\beta}, x^e) \in U^{-1}(\alpha)\ (\forall \alpha \in E)$, for every $U \in \mathcal{U}$.

$\iff x^e \widetilde{\in} U^{-1}[F]$, for every $U \in \mathcal{U}$.

$\iff x^e \widetilde{\in} U[F]$, for every $U \in \mathcal{U}$.

(b) If $(X, \tau_{\mathcal{U}})$ is a soft T_1-space, then from 3.13, $\overline{x^e} = x^e$ for any $x^e \widetilde{\in} \widetilde{X}$. Thus, by (a), we have $\sqcap_{U \in \mathcal{U}} U[x^e] = \overline{x^e} = x^e$.

For the converse, if $\sqcap_{U \in \mathcal{U}} U[x^e] = x^e$ for any $x^e \widetilde{\in} \widetilde{X}$, then by (a) $\overline{x^e} = x^e$, that is, $(X, \tau_{\mathcal{U}})$ is a soft T_1-space.

Definition 4.18. A soft uniformity $\mathcal{U}$ is called soft separate if $\sqcap_{U \in \mathcal{U}} U = \Delta$.

One can easily verify that the discrete soft uniformity is a soft separate.

Theorem 4.19. *Let $E = \{e\}$. Then, $\mathcal{U}$ is soft separate if and only if $\tau_{\mathcal{U}}$ is a soft T_1- topology.*

Proof. Let $\mathcal{U}$ be soft separate and let us assume that $z^e \widetilde{\in} \bigsqcap_{U\in\mathcal{U}} U[x^e]$, where $x \neq z$. Then, $z^e \widetilde{\in} U[x^e]$ for every $U \in \mathcal{U}$, that is, $(x^e, z^e) \in U(e)$ for every $U \in \mathcal{U}$. Thus, $(x^e, z^e) \in \bigcap_{U\in\mathcal{U}} U(e) = \Delta(e)$, which is a contradiction.

Conversely, let $\tau_{\mathcal{U}}$ be soft T_1-topology. Assume that $(x^e, z^e) \in \bigcap_{U\in\mathcal{U}} U(e)$ such that $x \neq z$. Then, $(x^e, z^e) \in U(e)$ for every $U \in \mathcal{U}$, i.e., $z^e \widetilde{\in} U[x^e]$ for every $U \in \mathcal{U}$. This implies that $z^e \widetilde{\in} \bigsqcap_{U\in\mathcal{U}} U[x^e] = x^e$, and we have a contradiction.

Definition 4.20. Let $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be two soft uniform spaces. A soft mapping $\varphi_\psi : (X, \mathcal{U}, E) \to (Y, \mathcal{V}, K)$ is soft uniformly continuous if for every $V \in \mathcal{V}$, there exists a $U \in \mathcal{U}$ such that $(x^\alpha, z^\beta) \in U(e)$ for every $e \in E$ implies $(\varphi_\psi(x^\alpha), \varphi_\psi(z^\beta)) \in V(k)$ for every $k \in K$.

Example 4.21. *Every soft mapping of a discrete soft uniform space into a soft uniform space is soft uniformly continuous.*

Theorem 4.22. *Every soft uniformly continuous mapping is soft continuous.*

Proof. Suppose $\varphi_\psi : (X, \mathcal{U}, E) \to (Y, \mathcal{V}, K)$ is soft uniformly continuous. Let $x^\alpha \widetilde{\in} \widetilde{X}$ and $V[\varphi_\psi(x^\alpha)]$ be a soft neighborhood of $\varphi_\psi(x^\alpha)$ with respect to $\tau_{\mathcal{V}}$. Then $V \in \mathcal{V}$ and by soft uniform continuity, there exists a $U \in \mathcal{U}$ such that $(x^\alpha, z^\beta) \in U(e)$ for every $e \in E$ implies $(\varphi_\psi(x^\alpha), \varphi_\psi(z^\beta)) \in V(k)$ for every $k \in K$. It is easy to see that $\varphi_\psi(U[x^\alpha]) \sqsubseteq V[\varphi_\psi(x^\alpha)]$. Thus, φ_ψ is soft continuous at x^α.

5. Conclusion

In the present work, we mainly introduce a soft uniformity, a soft uniformity base and a soft uniformly continuous. We have established their fundamental properties. We have compared soft uniformities to soft metric and soft topology with the help of examples. These findings will reinforce the foundations of the theory of soft uniform spaces. Also, we believe that these notions will help the researchers to advance and promote the further study on soft uniformity to carry out a general framework for their applications in practical life.

Conflict of Interests

The authors declare that there is no conflict of interests.

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