Available online at http://scik.org
J. Math. Comput. Sci. 5 (2015), No. 5, 708-715
ISSN: 1927-5307

ROOT SQUARE MEAN GRAPHS OF ORDER ≤ 5

S.S. SANDHYA[1] S. SOMASUNDARAM[2] AND S. ANUSA[3,*]
[1]Department of Mathematics, Sree Ayyappa College for women, Chunkankadai-629003, India
[2]Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012, India
[3]Department of Mathematics, Arunachala College of Engineering for Women, Vellichanthai-629203, India

Abstract. The concept of Root Square Mean labeling was introduced in [5]. In this paper we prove the Root Square Mean labeling of $C_n \hat{o} K_{1,m}$, $C_n \tilde{o} K_{1,m}$, $K_n - e$ and graphs of order ≤ 5.

Keywords: Root Square Mean; graphs.
2010 AMS Mathematics Subject Classification: 05C78.

1. Introduction

All graphs in this paper are finite, simple and undirected graph $G = (V, E)$ with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. Now we provide the definitions and theorems which are useful for the present study.

Definition 1.1: A graph $G = (V, E)$ with p vertices and q edges is called a Root square mean graph if it is possible to label the vertices $x \epsilon V$ with distinct elements $f(x)$ from $1, 2, \dots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case f is called a Root square Mean labeling of G.

Definition 1.2: The graph $G - e$ is obtained from G by deleting the edge e from G.

Definition 1.3: The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with

*Corresponding author
Received May 4, 2015

$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Definition 1.4: A graph $C_n \hat{o} K_{1,m}$ is obtained from C_n and $K_{1,m}$ by identifying any vertex of C_n and the central vertex of $K_{1,m}$.

Definition 1.5: The graph $C_n \tilde{o} K_{1,m}$ is obtained from C_n and $K_{1,m}$ by identifying any vertex of C_n and a pendent vertex of $K_{1,m}$ (ie. non-central vertex of $K_{1,m}$).

Theorem 1.6: Any path is a Root Square Mean graph.

Theorem 1.7: Any cycle is a Root Square Mean graph.

Theorem 1.8: The complete graph K_n is a Root Square mean graph if and only if $n \leq 4$.

Theorem 1.9: The graph $K_{1,n}$ is a Root Square mean graph if $n \leq 6$.

Theorem 1.10: Dragon $C_n @ P_m$ is a Root Square Mean graph.

Theorem 1.11: The graph $C_m \cup P_n$ is a Root Square Mean graph.

Remark 1.12: If $p > q + 1$, then the graph G is not a Root Square Mean graph, since there is no sufficient labels from $1, 2, \ldots, q + 1$ for all the vertices.

2. Main Results

Theorem 2.1: The graph $C_n \hat{o} K_{1,m}$ is a Root Square Mean graph if $m \leq 4$.

Proof:

Case (i): If $1 \leq m \leq 4$

Let $u_1, u_2, \ldots, u_n$ be the vertices of C_n and $u, v_1, v_2, \ldots, v_m$ be the vertices of $K_{1,m}$. Let u be the central vertex of $K_{1,m}$. Identify u_n with u.

Define a function $f: V(C_n \hat{o} K_{1,m}) \to \{1, 2, \ldots, q + 1\}$ by

$f(u_i) = i \, , 1 \leq i \leq n - 1$

$f(u_n) = f(u) = n + 1$

$f(v_i) = n + i + 1 \, , 1 \leq i \leq m.$

Then the edge labels are distinct. Hence $C_n \hat{o} K_{1,m}$ is a Root Square mean graph.

The labeling pattern of $C_6 \hat{o} K_{1,4}$ is shown below.

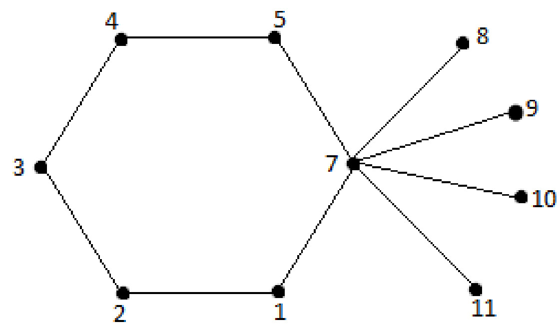


Figure 1

Case (ii): If $m > 4$, then we have the repetition of edge labels. The labeling pattern of $C_5\hat{o}K_{1,5}$ is shown below.

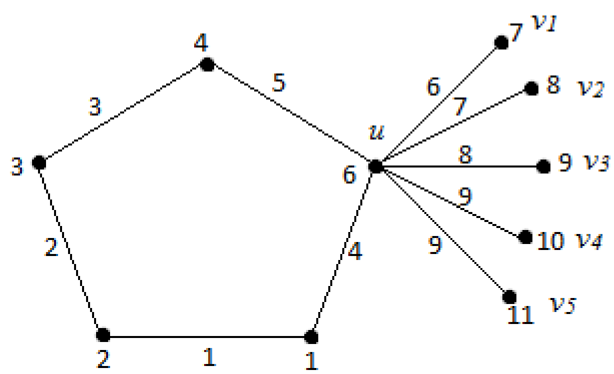


Figure 2

Here the edge labels of (u, v_4) and (u, v_5) are repeated. The same condition repeats when we take $u = n + 1$ and $m > 4$. Hence $C_n\hat{o}K_{1,m}$ is not a Root Square mean graph for m $>$ 4 .

Theorem 2.2: The graph $C_n\tilde{o}K_{1,m}$ is a Root Square Mean graph if $m \leq 5$.

Proof:

Case (i) : If $1 \leq m \leq 5$

Since $C_n\tilde{o}K_{1,1} = C_n@P_2$ and $C_n\tilde{o}K_{1,2} = C_n@P_3$, by theorem 1.10, $C_n\tilde{o}K_{1,1}$ and $C_n\tilde{o}K_{1,2}$ are Root Square mean graphs.

Let $u_1, u_2, \dots, u_n$ be the vertices of C_n and $u, v_1, v_2, \dots, v_m$ be the vertices of $K_{1,m}$. Let u be the central vertex of $K_{1,m}$. Identify v_m with u_n .

Define a function $f: V\left(C_n \tilde{o} K_{1,m}\right) \to \{1,2, \dots, q+1\}$ by

$f(u_i) = i\,, 1 \le i \le n-1$

$f(u_n) = f(v_m) = n+1$

$f(u) = n+2$

$f(v_i) = n+i+2\,, 1 \le i \le m-1.$

Then the edge labels are distinct. Hence $C_n \tilde{o} K_{1,m}$ is a Root Square mean graph.

The labeling pattern of $C_5 \tilde{o} K_{1,5}$ is shown below.

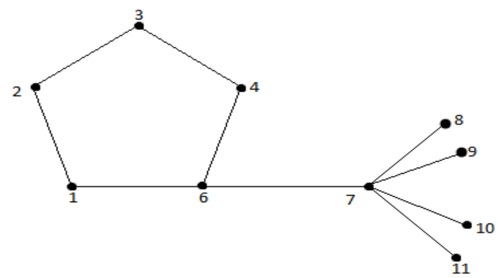


Figure 3

Case (ii): If m >5 , then we have the repetition of edge labels.

The labeling pattern of $C_4 \tilde{o} K_{1,6}$ is given below.

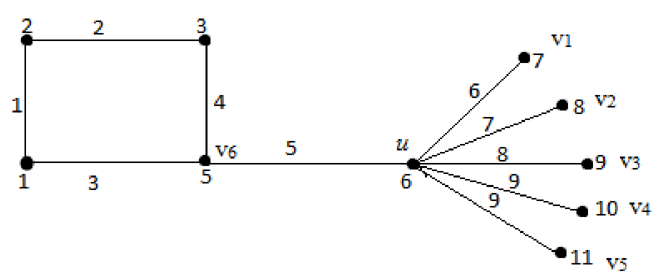


Figure 4

Here the edge labels of (u, v_4) and (u, v_5) are same. The same condition repeats when we take $u = n+1$ and $m > 5$. Hence $C_n \tilde{o} K_{1,m}$ is not a Root Square Mean graph if $m > 5$.

Remark 2.3:

If n $= 2$, $K_2 - e$ is a set of two isolated vertices and here $p = 2$ and $q+1 = 0+1 = 1$. Since $p > q+1$, by remark 1.12, $K_2 - e$ is not a Root Square mean graph.

Remark 2.4: If $\text{n} = 3$, $K_3 - e = P_2$, which is a path on two vertices, by theorem 1.6, $K_3 - e$ is a Root Square mean graph. The labeling pattern is shown below.

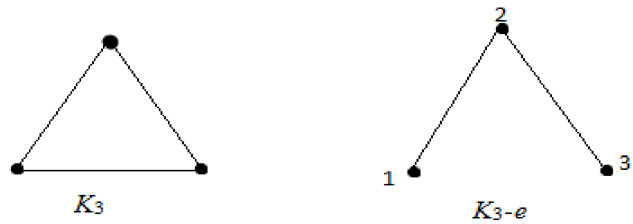


Figure 5

Remark 2.5 : If $n = 4$, $K_4 - e$ is a Root Square mean graph and the labeling pattern is shown below.

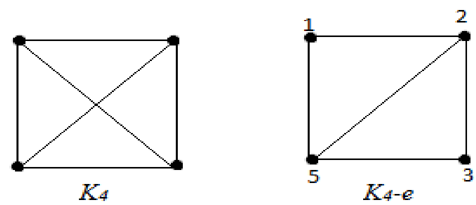


Figure 6

Remark 2.6: If $\text{n} > 4$, $K_n - e$ is not a Root Square mean graph and the labeling pattern of $K_5 - e$ is shown below.

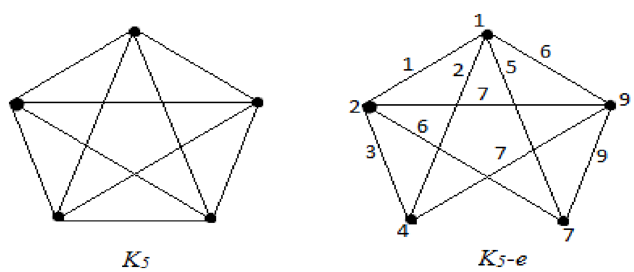
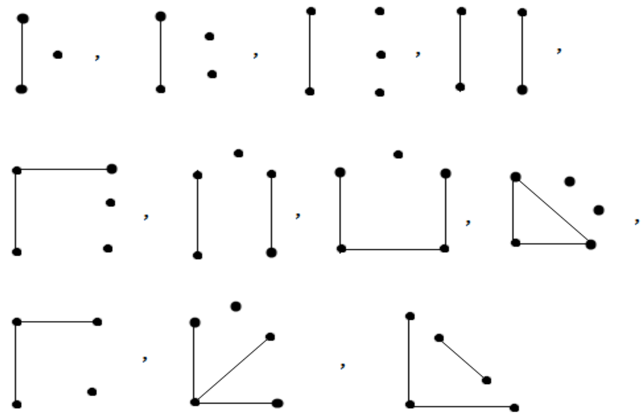


Figure 7

Here the labeling of the edges (2,9) and (4,9) are repeated. So $K_n - e$ is not a Root Square mean graph for $n > 4$.

Theorem 2.7: The following graphs of order ≤ 5 are not Root square mean graphs.

i) K_2^c , K_3^c , K_4^c , K_5^c



ii) K_5

iii) $K_5 - e$

Proof:

- The graphs in case(i) are not Root Square mean graphs by remark 1.12.
- By theorem 1.8 , the graph K_5 is not a Root Square mean graph.
- By remark 2.6 , $K_5 - e$ is not a Root Square mean graph.

Theorem 2.8: The following graphs of order ≤ 5 are Root square mean graphs.

i) P_1 , P_2 , P_3 , P_4 , P_5

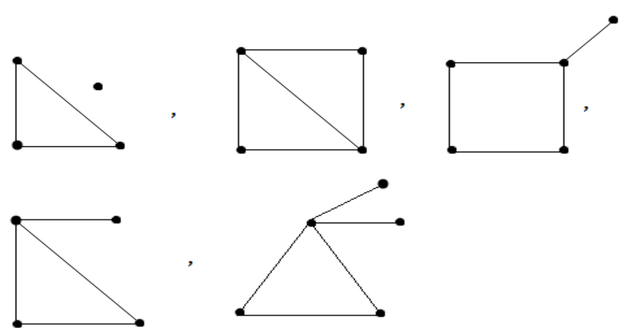
ii) C_3 , C_4 , C_5

iii) K_4

iv) $K_{1,3}$, $K_{1,4}$

v) $C_3 \cup P_2$

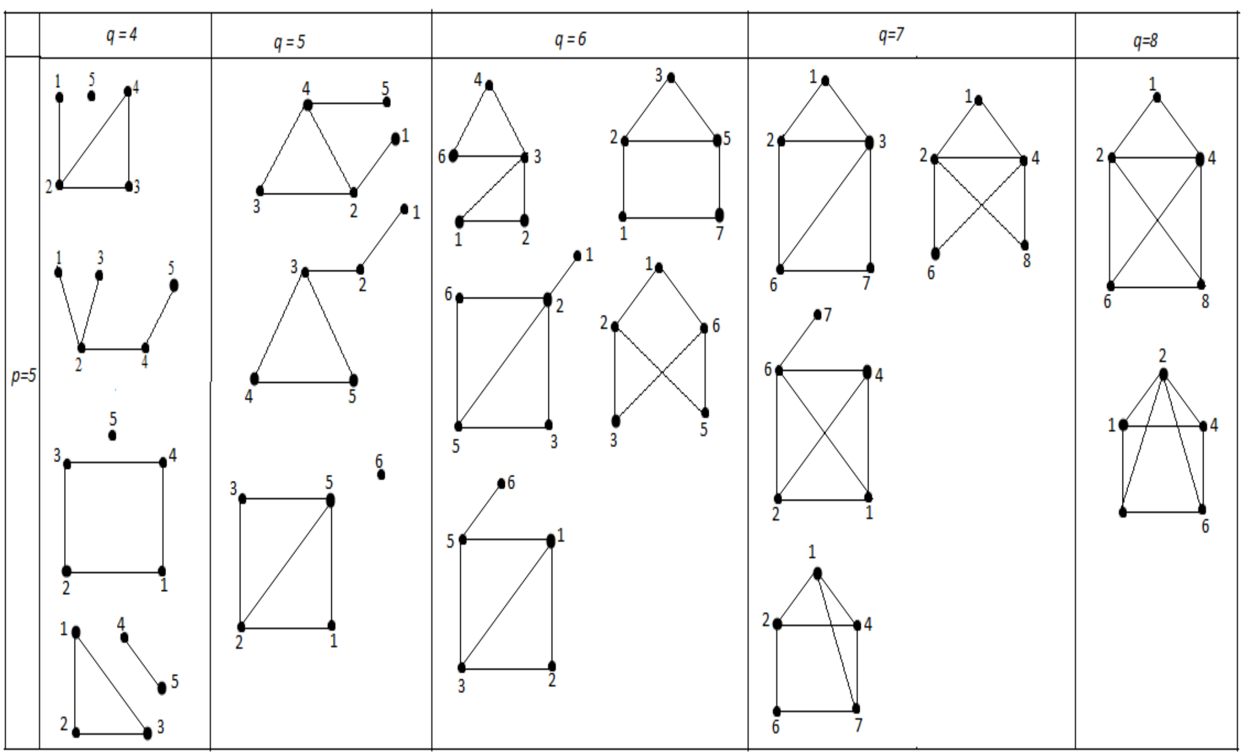
vi)



Proof:

- Since all the paths are Root Square mean graphs, by theorem 1.6, the graphs in case (i) are Root Square mean graphs.
- By theorem 1.7, the graphs in case (ii) are Root Square mean graphs.
- By theorem 1.8, K_4 is a Root Square mean graph.
- By theorem 1.9, $K_{1,3}$, $K_{1,4}$ are Root square mean graphs.
- The graphs in case (v) are Root Square mean graphs by theorem 1.11.
- By theorem 2.1 and remark 2.5, the grahs in case (vi) are Root Square mean graphs.

The remaining graphs of order ≤ 5 are Root Square mean graphs by giving specific labeling assigned to the vertices of each such graphs. These graphs are classified according their size of q and drawn with a specific Root Square mean labeling in the following table.



Remark:

The following table gives the number of graphs of order ≤ 5 which are Root Square mean graph and not Root square mean graph.

Order	Root Square mean	Not Root Square mean
1	1	0
2	1	1
3	2	2
4	7	4
5	24	10

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCE

[1] Gallian J.A., A Dynamic Survey of graph labeling. The electronic Journal of Combinatories, 17(2010), #DS6.

[2] Harary.F., Graph theory, Narosa Publishing House Reading, New Delhi,1988.

[3] Somasundaram S. and Ponraj R., On Mean Graphs of Order ≤ 5 , Journal of Decision and Mathematical Sciences, 9 (2004), 48 - 58.

[4] Sandhya S.S., Somasundaram.S and Ponraj.R, Some More Results on Harmonic Mean Graphs, Journal of Mathematics Research, 4(2012), 21-29.

[5] Sandhya S.S., Somasundaram.S and Anusa.S, Root Square Mean Labeling of Graphs, International Journal of Contemporary Mathematical Sciences, 9(2014), no.14, 667-676.