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ON APPLICATIONS OF RAMANUJAN'S SUM

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Abstract. In the present paper, we have obtained some new transformations of Ramanujan's ${}_1\psi_1$ sum. As an application of our results, we have deduced some identities of q-gamma and eta-functions.

Keywords: Ramanujan's Sum; q-series; η-function; q-gamma function.

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1. Introduction

Ramanujan's most famous ${}_1\psi_1$ summation formula was recorded by him in his Notebook [10]. It was brought to the attention of mathematical community by Hardy [8] and he described it as "a remarkable formula with many parameters". In modern notation the ${}_1\psi_1$ sum can be stated as

$$ {}_1\psi_1(a;b;q,z) = \sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = \frac{(q,b/a,az,q/az)_\infty}{(b,q/a,z,b/az)_\infty}, \tag{1.1} $$

where $|b/a| < |z| < 1$.

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The first proof [7] of (1.1) was given by Hahn in 1949 and subsequently a number of alternative proofs have been given.

In what follows, we have used the following notations and definitions:

$$(a;q^k)_n = (1-a)(1-aq^k)(1-aq^{2k}).....(1-aq^{k(n-1)}).$$

For k=1, we write

$$(a;q)_n = (a)_n,$$

$$(a)_\infty = (a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n),$$

where $|q| < 1$. The generalized basic hypergeometric series is defined by

$${}_{r+1}\phi_r\left(\begin{array}{cccc} a_1, & a_2 & .. & a_{r+1}; \quad q;z \\ b_1 & b_2 & .. & b_r \end{array}\right) = \sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n..(a_{r+1})_n}{(q)_n(b_1)_n..(b_r)_n}z^n,$$

where $|z| < 1$, $|q| < 1$. The bilateral basic hypergeometric series is defined by

$${}_r\psi_r\left(\begin{array}{cccc} a_1, & a_2 & .. & a_r; \quad q;z \\ b_1 & b_2 & .. & b_r \end{array}\right) = \sum_{n=-\infty}^{\infty}\frac{(a_1)_n(a_2)_n..(a_r)_n}{(q)_n(b_1)_n..(b_r)_n}z^n,$$

where $|\frac{b_1...b_r}{a_1...a_r}| < |z| < 1$, $|q| < 1$.

The Dedekind eta-function is given by

$$\eta(\tau) = e^{\pi\iota\tau/12}\prod_{n=1}^{\infty}(1-e^{2n\pi\iota\tau}) = q^{1/24}(q;q)_\infty,$$

where $q = e^{2\pi\iota\tau}$ and $Im\tau > 0$.

The q-analogue of gamma function due to Jackson [9] is

$$\Gamma_q(x) = \frac{(q)_\infty}{(q^x)_\infty}(1-q)^{1-x},\ 0 < q < 1.$$

We shall also use the Heine's transformations [6]

$$\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(q)_n(c)_n}z^n = \frac{(b)_\infty(az)_\infty}{(c)_\infty(z)_\infty}\sum_{n=0}^{\infty}\frac{(c/b)_n(z)_n}{(q)_n(az)_n}b^n, \tag{1.2}$$

$$= \frac{(c/b)_\infty (bz)_\infty}{(c)_\infty (z)_\infty} \sum_{n=0}^{\infty} \frac{(abz/c)_n (b)_n}{(q)_n (bz)_n} (c/b)^n, \tag{1.3}$$

$$= \frac{(abz/c)_\infty}{(z)_\infty} \sum_{n=0}^{\infty} \frac{(c/a)_n (c/b)_n}{(q)_n (c)_n} (abz/c)^n, \tag{1.4}$$

and Jackson's transformation[6]

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(q)_n (c)_n} z^n = \frac{(abz/c)_\infty}{(bz/c)_\infty} \sum_{n=0}^{\infty} \frac{(c/b)_n (a)_n}{(q)_n (c)_n (cq/bz)_n} q^n,$$

$$+ \frac{(a, bz, c/b)_\infty}{(c, z, c/bz)_\infty} \sum_{n=0}^{\infty} \frac{(z)_n (abz/c)_n}{(q)_n (bz)_n (bzq/c)_n} q^n. \tag{1.5}$$

2. Main results

Theorem 2.1. *If* $| q/z |<| z | < 1$ *and* $| q |< 1$, *then*

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = -1 + \frac{(q, az)_\infty}{(b, z)_\infty} \sum_{n=0}^{\infty} \frac{(b/q)_n (z)_n}{(q)_n (az)_n} q^n + \frac{(q/z)_\infty}{(b/az)_\infty} \sum_{n=0}^{\infty} \frac{(1/a)_n (b/a)_n}{(q)_n (q/a)_n} (q/z)^n. \tag{2.1}$$

Proof. Note that

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} z^n + \sum_{n=1}^{\infty} \frac{(q/b)_n}{(q/a)_n} (b/az)^n,$$

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = -1 + \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} z^n + \sum_{n=0}^{\infty} \frac{(q/b)_n}{(q/a)_n} (b/az)^n. \tag{2.2}$$

Taking $b = q, c = b$ in (1.2), we get

$$\sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} z^n = \frac{(q, az)_\infty}{(b, z)_\infty} \sum_{n=0}^{\infty} \frac{(b/q)_n (z)_n}{(q)_n (az)_n} q^n \tag{2.3}$$

and $a = q, b = q/b, c = q/a, z = b/az$ in (1.4), we obtain

$$\sum_{n=0}^{\infty} \frac{(q/b)_n}{(q/a)_n} (b/az)^n = \frac{(q/z)_\infty}{(b/az)_\infty} \sum_{n=0}^{\infty} \frac{(1/a)_n (b/a)_n}{(q)_n (q/a)_n} (q/z)^n. \tag{2.4}$$

Substituting (2.3) and (2.4) in (2.2) we obtain (2.1).

Theorem 2.2. *If* $\mid q/b \mid < \mid z \mid < 1$ *and* $\mid qaz/b \mid < 1$, *then*

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = -1 + \frac{(qaz/b)_\infty}{(z)_\infty} \sum_{n=0}^{\infty} \frac{(b/q)_n (b/a)_n}{(q)_n (b)_n} (qaz/b)^n$$

$$+ \frac{(q/b, qb/az)_\infty}{(q/a, b/az)_\infty} \sum_{n=0}^{\infty} \frac{(b/a)_n (b/az)_n}{(q)_n (bq/az)_n} (q/b)^n. \tag{2.5}$$

Proof. Putting $a = q, b = a, c = b$ in (1.4), we obtain

$$\sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} z^n = \frac{(qaz/b)_\infty}{(z)_\infty} \sum_{n=0}^{\infty} \frac{(b/q)_n (b/a)_n}{(q)_n (b)_n} (qaz/b)^n. \tag{2.6}$$

Taking $a = q, b = q/b, c = q/a, z = b/az$ in (1.2), we obtain

$$\sum_{n=0}^{\infty} \frac{(q/b)_n}{(q/a)_n} (b/az)^n = \frac{(q/b, qb/az)_\infty}{(q/a, b/az)_\infty} \sum_{n=0}^{\infty} \frac{(b/a)_n (b/az)_n}{(q)_n (qb/az)_n} (q/b)^n. \tag{2.7}$$

Substituting (2.6) and (2.7) in (2.2), we obtain (2.5).

Theorem 2.3. *If* $\mid q \mid < \mid z \mid < 1$, *then*

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = -1 + \frac{(q, az)_\infty}{(b, z)_\infty} \sum_{n=0}^{\infty} \frac{(b/q)_n (z)_n}{(q)_n (az)_n} q^n$$

$$+ \frac{(q/z)_\infty}{(1/z)_\infty} \sum_{n=0}^{\infty} \frac{(b/a)_n}{(qz)_n (q/a)_n} q^n + \frac{(q, q/az, b/a)_\infty}{(q/a, b/az, z)_\infty} \sum_{n=0}^{\infty} \frac{(b/az)_n}{(q)_n (q/az)_n} q^n. \tag{2.8}$$

Proof. Putting $a = q, b = q/b, c = q/a, z = b/az$ in (1.5), we obtain

$$\sum_{n=0}^{\infty} \frac{(q/b)_n}{(q/a)_n} (b/az)^n = \frac{(q/z)_\infty}{(1/z)_\infty} \sum_{n=0}^{\infty} \frac{(b/a)_n}{(qz)_n (q/a)_n} q^n$$

$$+ \frac{(q, q/az, b/a)_\infty}{(q/a, b/az, z)_\infty} \sum_{n=0}^{\infty} \frac{(b/az)_n}{(q)_n (q/az)_n} q^n. \tag{2.9}$$

Substituting (2.3) and (2.9) in (2.2), we obtain (2.8).

Theorem 2.4. *If* $\mid q/b \mid < \mid z \mid < 1$*and* $\mid q \mid < 1$, *then*

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = -1 + \frac{(q, az)_\infty}{(b, z)_\infty} \sum_{n=0}^{\infty} \frac{(b/q)_n (z)_n}{(q)_n (az)_n} q^n$$

$$+ \frac{(q/b, qb/az)_\infty}{(q/a, b/az)_\infty} \sum_{n=0}^{\infty} \frac{(b/a)_n (b/az)_n}{(q)_n (qb/az)_n} (q/b)^n. \tag{2.10}$$

Proof. Putting $a=q,b=q/b,c=q/a,z=b/az$ in (1.2),we obtain

$$\sum_{n=0}^{\infty}\frac{(q/b)_n}{(q/a)_n}(b/az)^n=\frac{(q/b,qb/az)_\infty}{(q/a,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(b/a)_n(b/az)_n}{(q)_n(qb/az)_n}(q/b)^n. \tag{2.11}$$

Substituting (2.3) and (2.11) in (2.2), we obtain (2.10).

Theorem 2.5. *If* $|q|<|z|<1$*, then*

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=-1+\frac{(qaz/b)_\infty}{(az/b)_\infty}\sum_{n=0}^{\infty}\frac{(b/a)_n}{(b)_n(bq/az)_n}q^n+\frac{(q,az,b/a)_\infty}{(b,az,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(z)_n}{(q)_n(az)_n}q^n$$

$$+\frac{(q/z)_\infty}{(1/z)_\infty}\sum_{n=0}^{\infty}\frac{(b/a)_n}{(qz)_n(q/a)_n}q^n+\frac{(q,q/az,b/a)_\infty}{(q/a,b/az,z)_\infty}\sum_{n=0}^{\infty}\frac{(b/az)_n}{(q)_n(q/az)_n}q^n. \tag{2.12}$$

Proof. Putting $a=q,b=a,c=b$ in (1.5), we get

$$\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}z^n=\frac{(qaz/b)_\infty}{(az/b)_\infty}\sum_{n=0}^{\infty}\frac{(b/a)_n}{(b)_n(bq/az)_n}q^n$$

$$+\frac{(q,az,b/a)_\infty}{(b,z,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(z)_n}{(q)_n(az)_n}q^n. \tag{2.13}$$

Substituting (2.9) and (2.13) in (2.2), we obtain (2.12).

Theorem 2.6. *If* $|b/a|<|z|<1$ *and* $|q|<1$*, then*

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=-1+\frac{(b/a,az)_\infty}{(b,z)_\infty}\sum_{n=0}^{\infty}\frac{(qza/b)_n(a)_n}{(q)_n(az)_n}(b/a)^n+\frac{(q/z)_\infty}{(1/z)_\infty}\sum_{n=0}^{\infty}\frac{(b/a)_n}{(qz)_n(q/a)_n}q^n$$

$$+\frac{(q,q/az,b/a)_\infty}{(q/a,b/az,z)_\infty}\sum_{n=0}^{\infty}\frac{(b/az)_n}{(q)_n(q/az)_n}q^n. \tag{2.14}$$

Proof. Putting $a=q,b=a,c=b$ in (1.3),we obtain

$$\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}z^n=\frac{(b/a,az)_\infty}{(b,z)_\infty}\sum_{n=0}^{\infty}\frac{(qza/b)_n(a)_n}{(q)_n(az)_n}(b/a)^n. \tag{2.15}$$

Substituting (2.15) and (2.9) in (2.2), we obtain (2.14).

Theorem 2.7. *If* $|b/a|<|z|<1$ *and* $|q|<1$*, then*

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=-1+\frac{(qaz/b)_\infty}{(az/b)_\infty}\sum_{n=0}^{\infty}\frac{(b/a)_n}{(b)_n(bq/az)_n}q^n+\frac{(q,az,b/a)_\infty}{(b,z,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(z)_n}{(q)_n(az)_n}q^n$$

$$+\frac{(b/a,q/az)_\infty}{(q/a,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(q/z)_n(q/b)_n}{(q)_n(q/az)_n}(b/a)^n. \tag{2.16}$$

Proof. Putting $a=q, b=q/b, c=q/a, z=b/az$ in (1.3),we obtain

$$\sum_{n=0}^{\infty}\frac{(q/b)_n}{(q/a)_n}(b/az)^n=\frac{(b/a,q/az)_\infty}{(q/a,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(q/z)_n(q/b)_n}{(q)_n(q/az)_n}(b/a)^n. \tag{2.17}$$

Substituting (2.13) and (2.17) in (2.2), we obtain (2.16).

Theorem 2.8. *If* $|\, b/a \,|<|\, z \,|<1$ *and* $|\, q \,|<|\, z \,|<1$, *then*

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=-1+\frac{(b/a,az)_\infty}{(b,z)_\infty}\sum_{n=0}^{\infty}\frac{(qza/b)_n(a)_n}{(q)_n(az)_n}(b/a)^n$$

$$+\frac{(q/z)_\infty}{(b/az)_\infty}\sum_{n=0}^{\infty}\frac{(1/a)_n(b/a)_n}{(q)_n(q/a)_n}(q/z)^n. \tag{2.18}$$

Proof. Putting $a=q, b=a, c=b$ in (1.3), we obtain

$$\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}z^n=\frac{(b/a,az)_\infty}{(b,z)_\infty}\sum_{n=0}^{\infty}\frac{(qza/b)_n(a)_n}{(q)_n(az)_n}(b/a)^n. \tag{2.19}$$

Putting $a=q, b=q/b, c=q/a$ and $z=b/az$ in (1.4), we obtain

$$\sum_{n=0}^{\infty}\frac{(q/b)_n}{(q/a)_n}(b/az)^n=\frac{(q/z)_\infty}{(b/az)_\infty}\sum_{n=0}^{\infty}\frac{(1/a)_n(b/a)_n}{(q)_n(q/a)_n}(q/z)^n. \tag{2.20}$$

Substituting (2.19) and (2.20) in (2.2), we obtain (2.18).

Theorem 2.9. *If* $|\, b/a \,|<|\, z \,|<1$ *and* $|\, qaz/b \,|<|\, z \,|<1$, *then*

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=-1+\frac{(qaz/b)_\infty}{(z)_\infty}\sum_{n=0}^{\infty}\frac{(b/q)_n(b/a)_n}{(q)_n(b)_n}(qaz/b)^n$$

$$+\frac{(b/a,q/az)_\infty}{(q/a,b/az)_\infty}\sum_{n=0}^{\infty}\frac{(q/z)_n(q/b)_n}{(q)_n(q/az)_n}(b/a)^n. \tag{2.21}$$

Proof. Substituting (2.6) and (2.17) in (2.2), we obtain (2.21).

Theorem 2.10. *If* $|\, q/z \,|<|\, z \,|<1$ *and* $|\, qaz/b \,|<|\, z \,|<1$, *then*

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=-1+\frac{(qaz/b)_\infty}{(z)_\infty}\sum_{n=0}^{\infty}\frac{(b/q)_n(b/a)_n}{(q)_n(b)_n}(qaz/b)^n$$

$$+\frac{(q/z)_\infty}{(b/az)_\infty}\sum_{n=0}^{\infty}\frac{(1/a)_n(b/a)_n}{(q)_n(q/a)_n}(q/z)^n. \tag{2.22}$$

Proof. Substituting (2.6) and (2.4) in (2.2) we obtain (2.22).

3. Applications

Applying Ramanujan's ${}_1\psi_1$ sum in (2.1) and then putting $a = 1/q$, $b = q^{5/2}, z = q^{3/2}$ and changing *q to* q^2, we obtain

$$\frac{\eta(\tau)}{\eta(2\tau)} = \frac{-q^{-1/24}(1-q^5)(q^4;q^2)_\infty}{(1-q)(1-q^2)} + \frac{q^{-1/24}(q^4;q^2)_\infty}{(1-q)(q^7;q^2)_\infty}\sum_{n=0}^{\infty}\frac{(q^3;q^2)_n}{(q^2;q^2)_n}q^{2n} + \frac{q^{-1/24}(1-q^5)(q^{-1};q^2)_\infty}{(1-q)(1-q^2)}\sum_{n=0}^{\infty}\frac{(q^7;q^2)_n}{(q^4;q^2)_n}q^{-n}. \tag{3.1}$$

Applying Ramanujan's ${}_1\psi_1$ sum in (2.18) and then putting $a = 1/q$, $b = q^{1/2}, z = q^{1/2}$ and changing *q to* q^2, we obtain

$$\frac{\eta(\tau)}{\eta^2(2\tau)} = q^{-1/8} - \frac{(q^{7/8})(q;q^2)_\infty}{(1+q)(q^2;q^2)_\infty}\sum_{n=0}^{\infty}\frac{(q^3;q^2)_n}{(q^4;q^2)_n}q^n. \tag{3.2}$$

Applying Ramanujan's ${}_1\psi_1$ sum in (2.18) and then putting $a = 1/q$, $b = q^{1/2}, z = q^{1/2}$ and changing *q to* q^2, we can also obtain

$$\frac{\eta(\tau)}{\eta(2\tau)} = q^{-1/24}(1-q^2)(q^4;q^2)_\infty - \frac{(q^{23/24})(q;q^2)_\infty}{(1+q)}\sum_{n=0}^{\infty}\frac{(q^3;q^2)_n q^n}{(q^4;q^2)_n}. \tag{3.3}$$

Applying Ramanujan's ${}_1\psi_1$ sum in (2.14) and then putting $a = 1/q$, $b = q^{1/2}, z = q^{1/2}$ and changing *q to* q^2, we obtain

$$\frac{\eta(\tau)}{\eta^2(2\tau)} = q^{-1/8} - \frac{(q^{15/8})}{(1-q^2)}\sum_{n=0}^{\infty}\frac{q^{2n}}{(q^4;q^2)_n} - \frac{(q^{7/8})(q^3;q^2)_\infty}{(1-q^2)(q^4;q^2)_\infty}\sum_{n=0}^{\infty}\frac{q^{2n}}{(q^3;q^2)_n}. \tag{3.4}$$

Now, changing a to q^a, b to q^b, and z to q^z in (2.1), after some simple manipulations, we obtain

$$\frac{\Gamma_q(b)\Gamma_q(1-a)\Gamma_q(z)\Gamma_q(b-a-z)}{\Gamma_q(b-a)\Gamma_q(a+z)\Gamma_q(1-a-z)} = -(1-q)^{a+1-b} + \frac{\Gamma_q(b)\Gamma_q(z)}{\Gamma_q(a+z)}\sum_{n=0}^{\infty}\frac{(q^{b-1})_n(q^z)_n}{(q)_n(q^{a+z})_n}q^n + \frac{\Gamma_q(b-a-z)}{\Gamma_q(1-z)}\sum_{n=0}^{\infty}\frac{(q^{-a})_n(q^{b-a})_n}{(q)_n(q^{1-a})_n}(q^{1-z})^n. \tag{3.5}$$

which is valid for $0 < z < b-a < 1$ and $b < 1$.

Changing a to q^a, b to q^b and z to q^z in (2.8), we obtain

$$\frac{\Gamma_q(b)\Gamma_q(1-a)\Gamma_q(z)\Gamma_q(b-a-z)}{\Gamma_q(b-a)\Gamma_q(a+z)\Gamma_q(1-a-z)} = -(1-q)^{a+1-b} + \frac{\Gamma_q(b)\Gamma_q(z)}{\Gamma_q(a+z)}\sum_{n=0}^{\infty}\frac{(q^{b-1})_n(q^z)_n}{(q)_n(q^{a+z})_n}q^n$$

$$+(1-q)^{a-b}\frac{\Gamma_q(-z)}{\Gamma_q(1-z)}\sum_{n=0}^{\infty}\frac{(q^{b-1})_n(q^z)_n}{(q)_n(q^{a+z})_n}q^n$$

$$+(1-q)^{a-b+z}\frac{\Gamma_q(1-a)\Gamma_q(b-a-z)\Gamma_q(z)}{\Gamma_q(1-a-z)\Gamma_q(b-a)}\sum_{n=0}^{\infty}\frac{(q^{b-a-z})_n}{(q)_n(q^{1-a-z})_n}q^n \tag{3.6}$$

which for $q \to 1$ gives

$$\frac{\Gamma(1-a)\Gamma(b-a-z)}{\Gamma(b-a)\Gamma(1-a-z)} = \sum_{n=0}^{\infty}\frac{(b-1)_n(z)_n}{n!(a+z)_n}. \tag{3.7}$$

Taking $a = 0$ in (3.7) and then $1-z = x$ and $b-1 = y$, we obtain,

$$\frac{1}{B(x,y)} = \sum_{n=0}^{\infty}\frac{\prod_{k=0}^{n-1}(k+y)y}{n!}. \tag{3.8}$$

Lastly, changing a to q^a, b to q^b, and z to q^z in (2.18), we have

$$\frac{\Gamma_q(b)\Gamma_q(1-a)\Gamma_q(z)\Gamma_q(b-a-z)}{\Gamma_q(b-a)\Gamma_q(a+z)\Gamma_q(1-a-z)}$$

$$= -(1-q)^{a+1-b} + (1-q)^{a+1-b}\frac{\Gamma_q(b)\Gamma_q(z)}{\Gamma_q(a+z)\Gamma_q(b-a)}\sum_{n=0}^{\infty}\frac{(q^{1+a+z-b})_n(q^a)_n}{(q)_n(q^{a+z})_n}(q^{b-a})^n$$

$$+\frac{\Gamma_q(b-a-z)}{\Gamma_q(1-z)}\sum_{n=0}^{\infty}\frac{(q^{-a})_n(q^{b-a})_n}{(q)_n(q^{1-a})_n}(q^{1-z})^n. \tag{3.9}$$

If we take $q \to 1$ and then put $b = 1$, $a = \frac{1}{2}$ and $z = \frac{1}{4}$, we get the following identity

$$\sum_{n=0}^{\infty}\frac{(1/2)_n}{n!(1-2n)} = 1.$$

Conflict of Interests

The authors declare that there is no conflict of interests.

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