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VALIDATION OF A FACTOR MODEL FOR FACTORS INFLUENCING MATHEMATICS LEARNING AND PERFORMANCE IN TANZANIAN SECONDARY SCHOOLS

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Abstract. There are several ways of validating a factor model. Two of such ways are split-half method and a method that involves collecting new data. In this paper a sample of 520 secondary school students was randomly split into two equal halves using the split-half validation method. The two subsamples were subjected to factor analysis/principal component method. Communalities of individual variables and factors were determined. The analysis showed that the communalities of individual variables of the two subsamples were similar. Also the factor structures were alike. Thus, the results of the validation suggest that the results/findings of the study to why the secondary school students in Arusha and Kilimanjaro regions are performing poorly in mathematics can be generalized in Tanzanian secondary schools.

Keywords: factor model, split-half method, validation, communalities, factor, factor structure, subsamples.
2000 AMS Subject Classification: 97B20.

1. Introduction

The students' performance in mathematics in Tanzania secondary schools is poor [1]. A study was conducted in Arusha and Kilimanjaro regions to identify factors causing poor performance in mathematics. Thereafter, a factor model was developed and the purpose of this paper is to validate the factor model that describes the factors influencing mathematics learning and

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performance in Tanzanian secondary schools developed by Kisakali and Kuznetsov [2]. The factors were first recognized by first administering structured student questionnaires to 520 secondary school students with designed variables therein. Factor analysis/principal component analysis was used to identify factors. Lack of interest while studying mathematics, triviality and lack of practice by students, lack of drive and enthusiasm for teachers and students, perception and attitude towards the subject terming it to be difficult and lack of qualified mathematics teachers were identified as factors influencing mathematics learning and performance for the sampled schools. Factor analysis modeling was used to describe factors affecting students' performance in mathematics for secondary schools in Tanzania. The factor model was built and it explained 50.5% of the total variation in students' mathematics performance. The factor model comprised of five factors with eighteen (18) equations. Factor 1 (F_1), lack of interest while studying mathematics, included the variables $x_1,\ x_2,\ x_3\ and\ x_4$. Factor 2 ($F_2$), triviality and lack of practice by students included the variables $x_5,\ x_6,\ x_7,\ x_8\ and\ x_9$. Factor 3 ($F_3$), lack of drive and enthusiasm for teachers and students included the variables $x_{10},\ x_{11}\ and\ x_{12}$. Factor 4 ($F_4$), perception and attitude towards the subject terming it to be difficult included the variables $x_{13},\ x_{14}\ and\ x_{15}$. Factor 5($F_5$), lack of qualified mathematics teachers included variables $x_{16},\ x_{17}\ and\ x_{18}$. Table 1 illustrates the factor model developed while Table 2 is the structure matrix and it highlights correlations between variables and component/factors after rotation (N=520). The Cronbach Alpha Coefficient and Split-half reliability of the instrument were 0.71 and 0.65 respectively. The factor model appropriately fitted analysis of factors that affect the students' mathematics performance in Tanzanian secondary schools. The Split-half validation technique was used for the aim of generating and confirming the factor structure [3-5].

2. Methodology

The factor model was built using the factors extracted. The model comprised of eighteen equations and it explains 50.5% of the total variation in mathematics performance. This paper sought to validate the factor model developed by Kisakali and Kuznetsov [2]. A random split-half validation method was employed to validate the factor model and this is usually done in exploratory factor analysis [6]. The reason for using split-half method was to check the factor structure of each subsample and compare with the factor structure of the full sample[7, 8]. Cost and time made it infeasible to test the model through recollecting the data in the same population.

The original sample (N = 520) was randomly split into two equal halves [8, 9]. The first subsample comprised of 260 students (116 boys and 144 girls) and the second subsample comprised of 260 students (141 boys and 119 girls). Statistical Package for Social Scientists (SPSS) version 21.0, Stata 09 and R-software were used to analyse the subsamples and all gave the same results. The determinant of each subsample was determined. The determinant should be above 0.00001 to indicate the absence of multicollinearity [10]. The Kaiser–Meyer–Olkin Measure of Sampling Adequacy **(**KMO) and Bartlett's test of sphericity values were checked to determine if the subsamples were suitable for factor analysis. The KMO value should be at least 0.5 for factor analysis to be conducted [11]. The Bartlett's test of sphericity should be significant, that is, the probability, p, should be less than 0.005.

The factor analysis/principal component was conducted in each subsample and the results of the two subsamples were compared. An oblique rotation with direct Oblimin was employed to extract factors and allow the correlation between factors [12]. The results from the two subsamples were compared to the factor model formulated in terms of factor structures.

2.1 Factor analysis/principal component analysis of subsample n_1

The determinant and the KMO value of subsample n_1 were found to be 0.003 and 0.770 respectively which are above the acceptable limit. The Bartlett's test of sphericity was found to be, $\chi^2(231) = 1490.357,\ p = 0.000$, and which is highly significant. Using principal component analysis, five factors were extracted which explained 50.7% of the total variation in mathematics performance. The communalities of each individual variable and factor structures are as indicated in Tables 3a and 3b respectively, and the results were compared to subsample n_2.

2.2 Factor analysis/principal component analysis of subsample n_2

The determinant and KMO of subsample n_2 was found to be 0.003 and 0.769 respectively, which are above the acceptable limit. The Bartlett's test of sphericity was found to be, $\chi^2(231) = 1496.096,\ p = 0.000$, which is highly significant. Using principal component analysis, five factors were extracted which explained 50.8% of the total variation in the mathematics performance. The communalities of each individual variable and factor structures are as indicated in Tables 4a and 4b respectively, and the results were compared to subsample n_1.

3 Results and Discussion

The results of the analysis showed that the application of factor analysis/principal component method to each subsample produced five factors which explain 51% (approximately) of the total variation in mathematics performance. The communalities of each individual variable in the two subsamples were similar. Also, the factor structures of the two subsamples were alike. Furthermore, the results of analysis of the two subsamples were compared to the previously developed solution, that is the factor model formulated in terms factor structures. The two solutions were alike, that is the factor structures were similar to that reported in the developed factor model in [2]. Thus, the factor structures are similar when sample was split into two equal halves.

4. Conclusions

Split-half method was applied to divide the data into two subsamples. The result of validation showed that both subsamples have similar communalities and factor structures. Thus, the factor structure was stable when assessed in different samples. In both subsamples the following factors were extracted: lack of interest while studying mathematics, triviality and lack of practice by students, lack of drive and enthusiasm for teachers and students, perception and attitude towards the subject terming it to be difficult and lack of qualified mathematics teachers were identified as factors influencing mathematics learning and performance. Thus, the finding of this study, that the causes of poor performance in mathematics are the aforementioned factors can be generalized to the population of Tanzanian secondary schools [2]. Further studies need to be done to explore the attitude of students towards studying mathematics.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] Hamilton, M., et al., *A Needs Assessment Study of Tanzania Science Education*, The economic and social research foundation. 2010.

[2] Kisakali, J. and D. Kuznetsov, *Modeling factors influencing mathematics learning and performance in Tanzanian secondary schools.* Asian Journal of Mathematics and Applications, 2015 (2015): Article ID ama0244, 11 pages.

[3] LaBrie, J.W., et al., *Measuring college students' motives behind prepartying drinking: Development and validation of the prepartying motivations inventory.* Addictive behaviors, 37(2012): 962-969.

[4] Sivakumar, K. and M. Srinivasan, *Factor Analytic Approach in the Study of Industrial Production Data.* International Journal on Information Sciences and Computing,. 2(2011), No. 1.

[5] Stedmon, C.A. and R. Bro, *Characterizing dissolved organic matter fluorescence with parallel factor analysis: a tutorial.* Limnology and Oceanography: Methods, 6(2008): 572-579.

[6] McCarthy, P.J., *The use of balanced half-sample replication in cross-validation studies.* Journal of the American Statistical Association, 71(1976): 596-604.

[7] Cudeck, R. and M.W. Browne, *Cross-validation of covariance structures.* Multivariate Behavioral Research, 18(1983): 147-167.

[8] Loehlin, J.C., *Latent Variable Models: An Introduction to Factor, Path, and Structural Equation Analysis (Fourth Edition)*, London: Lawrence Erlbaum Associates, 2004.

[9] Palacios, A., V. Arias, and B. Arias, *Attitudes towards mathematics: Construction and validation of a measurement instrument.* Revista de Psicodidáctica, 19(2014): 67-91.

[10] Yong, A.G. and S. Pearce, *A beginner's guide to factor analysis: Focusing on exploratory factor analysis.* Tutorials in Quantitative Methods for Psychology, 9(2013): 79-94.

[11] Field, A., *Discovering Statistics Using SPSS (3rd edition)*, Los Angeles, CA: SAGE Publictions Ltd. 2009.

[12] Brown, J.D., *Statistics Corner, questions and answers about language testing statistics: Choosing the Right Type of Rotation in PCA and EFA.* Shiken: JALT Testing & Evaluation SIG Newsletter, 13(2009): 20-25.

Appendices

Table1 Factor model

$$x_1 = 0.83F_1 + 0.05F_2 + 0.06F_3 + 0.32F_4 + 0.22F_5 + 0.30$$

$$x_2 = 0.81F_1 + 0.04F_2 + 0.12F_3 + 0.26F_4 + 0.33F_5 + 0.32$$

$$x_3 = 0.80F_1 + 0.12F_2 + 0.10F_3 + 0.24F_4 + 0.12F_5 + 0.34$$

$$x_4 = 0.78F_1 + 0.16F_2 + 0.08F_3 + 0.37F_4 + 0.21F_5 + 0.37$$

$$x_5 = 0.11F_1 + 0.74F_2 + 0.04F_3 - 0.11F_4 - 0.01F_5 + 0.38$$

$$x_6 = 0.13F_1 + 0.62F_2 - 0.05F_3 + 0.04F_4 + 0.24F_5 + 0.52$$

$$x_7 = -0.10F_1 + 0.53F_2 - 0.06F_3 + 0.08F_4 - 0.13F_5 + 0.67$$

$$x_8 = 0.13F_1 + 0.52F_2 + 0.20F_3 + 0.27F_4 + 0.03F_5 + 0.68$$

$$x_9 = 0.18F_1 + 0.51F_2 + 0.21F_3 + 0.28F_4 + 0.18F_5 + 0.67$$

$$x_{10} = 0.08F_1 + 0.02F_2 + 0.73F_3 - 0.06F_4 + 0.11F_5 + 0.44$$

$$x_{11} = 0.08F_1 + 0.11F_2 + 0.70F_3 + 0.13F_4 + 0.11F_5 + 0.50$$

$$x_{12} = 0.05F_1 + 0.02F_2 + 0.58F_3 - 0.11F_4 + 0.22F_5 + 0.61$$

$$x_{13} = 0.43F_1 + 0.12F_2 - 0.02F_3 + 0.89F_4 + 0.23F_5 + 0.15$$

$$x_{14} = 0.26F_1 + 0.10F_2 + 0.03F_3 + 0.86F_4 + 0.15F_5 + 0.26$$

$$x_{15} = -0.36F_1 - 0.14F_2 + 0.06F_3 - 0.80F_4 - 0.17F_5 + 0.32$$

$$x_{16} = 0.30F_1 + 0.11F_2 - 0.04F_3 + 0.13F_4 + 0.75F_5 + 0.39$$

$$x_{17} = 0.39F_1 - 0.07F_2 + 0.13F_3 + 0.21F_4 + 0.72F_5 + 0.41$$

$$x_{18} = 0.18F_1 + 0.36F_2 + 0.34F_3 + 0.23F_4 + 0.56F_5 + 0.51$$

Table 2:A structure matrix showing correlations between variables and component/factors after rotation (N = 520)

Variable		Component				
		1	2	3	4	5
x_1	Mathematics lessons are boring.	**.829**	.054	.062	.319	.220
x_2	I do not understand my mathematics teacher when he/she is teaching.	**.805**	.036	.120	.260	.327
x_3	I do not do mathematics homework.	**.803**	.118	.096	.236	.121
x_4	I do not like mathematics.	**.776**	.155	.083	.365	.214
x_5	Students do not practice to solve mathematics questions thus they perform poorly in mathematics.	.112	**.738**	.038	-.107	-.006
x_6	The students are not serious in studying mathematics hence they perform poorly in mathematics.	.127	**.618**	-.049	.039	.240
x_7	Students are misbehaving in mathematics class and thus they do not understand fully the mathematics concepts which are being taught leading to poor performance in mathematics.	-.100	**.531**	-.058	.081	-.126
x_8	Poor background of students in mathematics is the most important factor of poor performance in mathematics.	.133	**.524**	.197	.273	.025
x_9	The mathematics language (for example, estimate, reminder) is not understood by the students, hence causes the students to perform poorly in mathematics.	.181	**.505**	.211	.278	.180
a	The language of instruction (English) is not understood by the students leading to poor understanding of mathematics concepts and poor performance in mathematics.	.083	.497	.256	.255	.065
b	The tendency of students to escape mathematics class (truancy) causes them to have partial knowledge, hence resulting in poor performance in mathematics.	-.129	.390	.339	-.011	-.216
x_{10}	Lack of motivation to mathematics teachers discourages teachers' commitment to work.	.077	.023	**.726**	-.055	.113
x_{11}	Lack of motivation to students performing well in mathematics discourages students' commitment to study mathematics.	.083	.111	**.701**	.134	.114
x_{12}	The teaching method or style (the teacher is demonstrating without allowing students to participate due to a large number of students in a class).	.050	.018	**.576**	-.112	.222
c	Lack of teaching and learning of mathematics material at your school lead to poor performance in mathematics.	.077	.201	.276	.210	.027
x_{13}	Mathematics is a difficult subject.	.432	.119	-.024	**.894**	.226
x_{14}	Sometimes I do not attend mathematics lessons.	.258	.099	.031	**.858**	.148
x_{15}	Mathematics is a simple subject.	-.364	-.139	.063	**-.800**	-.167
x_{16}	Lack of qualified mathematics teachers in your region to teach the subject lead to poor performance in mathematics.	.302	.109	-.035	.134	**.745**
x_{17}	Negative attitude towards mathematics causes students to perform poorly in mathematics.	.388	-.069	.127	.207	**.724**
x_{18}	Poor performance in mathematics could be explained by poor background in elementary mathematics.	.180	.363	.336	.234	**.557**
d	Poor parents/guardians economic status causes students to perform poorly in mathematics.	-.113	.003	.252	.084	.352

Extraction Method: Principal Component Analysis.

Rotation Method: Oblimin with Kaiser Normalization.

Note: Factor loading over 0.5 appears in bold has been used in writing the factor model.

Table 3a: Communalities before and after extracting factors (N=260)

Variable	Initial	Extraction
Poor parents/guardians economic status causes students to perform poorly in mathematics.	1.000	.531
Lack of motivation to students performing well in mathematics discourages students' commitment to study mathematics.	1.000	.555
Lack of motivation to mathematics teachers discourages teachers' commitment to work.	1.000	.452
The teaching method or style (the teacher is demonstrating without allowing students to participate due to large number of students in a class).	1.000	.455
The tendency of students to escape mathematics class (truancy) causes them to have partial knowledge, hence resulting in poor performance in mathematics.	1.000	.345
Poor performance in mathematics could be explained by poor background in elementary mathematics.	1.000	.471
Poor background of student in mathematics is the most important factor of poor performance in mathematics.	1.000	.370
Lack of qualified mathematics teachers in your region to teach the subject lead to poor performance in mathematics.	1.000	.514
Lack of teaching and learning of mathematics material at your school lead to poor performance in mathematics.	1.000	.170
Negative attitude towards mathematics causes students to perform poorly in mathematics.	1.000	.431
Students are misbehaving in mathematics class and thus they do not understand fully the mathematics concepts which are being taught leading to poor performance in mathematics.	1.000	.266
Students do not practice to solve mathematics questions thus they perform poorly in mathematics.	1.000	.646
The students are not serious in studying mathematics hence they perform poorly in mathematics.	1.000	.529
The language of instruction (English) is not understood by the students leading to poor understanding of mathematics concepts and poor performance in mathematics.	1.000	.358
The mathematics language (for example, estimate, reminder) is not understood by the students, hence causes the students to perform poorly in mathematics	1.000	.361
I do not like mathematics.	1.000	.548
Mathematics lessons are boring.	1.000	.640
I do not understand my mathematics teacher when he/she is teaching.	1.000	.653
I do not do mathematics homework.	1.000	.632
Mathematics is a simple subject.	1.000	.710
Sometimes I do not attend mathematics lessons.	1.000	.701
Mathematics is a difficult subject.	1.000	.827

Extraction Method: Principal Component Analysis.

Table 3b: Structure matrix (5 factors after rotation) N=260

Variable	Component				
	1	2	3	4	5
I do not understand my mathematics teacher when he/she is teaching.	**.800**	.054	.121	.211	.259
Mathematics lessons are boring.	**.792**	.091	.042	.306	.170
I do not do mathematics homework.	**.773**	.088	.182	.216	.031
I do not like mathematics.	**.710**	.137	.104	.370	.143
Students do not practice to solve mathematics questions thus they perform poorly in mathematics.	.082	**.796**	.110	.013	.071
The students are not serious in studying mathematics hence they perform poorly in mathematics.	.111	**.703**	-.006	.094	-.023
The language of instruction (English) is not understood by the students leading to poor understanding of mathematics concepts and poor performance in mathematics.	.155	**.579**	.068	.028	.128
Poor background of student in mathematics is the most important factor of poor performance in mathematics.	.111	**.538**	.260	.262	.196
The mathematics language (for example, estimate, reminder) is not understood by the students, hence causes the students to perform poorly in mathematics	.103	**.494**	.213	.365	.156
Students are misbehaving in mathematics class and thus they do not understand fully the mathematics concepts which are being taught leading to poor performance in mathematics.	-.150	**.482**	.041	.049	.048
The tendency of students to escape mathematics class (truancy) causes them to have partial knowledge, hence resulting in poor performance in mathematics.	-.130	**.435**	.370	.020	-.132
Lack of teaching and learning of mathematics material at your school lead to poor performance in mathematics.	.142	.299	-.009	-.030	.272
Lack of motivation to students performing well in Mathematics discourages students' commitment to study mathematics.	.173	.139	**.731**	.146	.089
The teaching method or style (the teacher is demonstrating without allowing students to participate due to large number of students in a class).	-.001	.028	**.661**	-.068	-.025
Lack of motivation to mathematics teachers discourages teachers' commitment to work.	.181	.015	**.605**	-.155	.177
Mathematics is a difficult subject.	.385	.079	-.052	**.888**	.149
Mathematics is a simple subject.	-.275	-.080	.069	**-.832**	-.039
Sometimes I do not attend mathematics lessons.	.255	.021	-.001	**.832**	.051
Poor parents/guardians economic status causes students to perform poorly in mathematics.	-.134	.032	.050	-.088	**.669**
Lack of qualified mathematics teachers in your region to teach the subject lead to poor performance in mathematics.	.382	.163	.060	.214	**.661**
Negative attitude towards mathematics causes students to perform poorly in mathematics.	.450	-.074	.117	.290	**.493**
Poor performance in mathematics could be explained by poor background in elementary mathematics.	.112	.408	.414	.319	**.414**

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

Table 4a: Communalities before and after extracting factors (N=260)

Variable	Initial	Extraction
Poor parents/guardians economic status causes students to perform poorly in mathematics.	1.000	.528
Lack of motivation to students performing well in mathematics discourages students' commitment to study mathematics.	1.000	.553
Lack of motivation to mathematics teachers discourages teachers' commitment to work.	1.000	.442
The teaching method or style (the teacher is demonstrating without allowing students to participate due to large number of students in a class).	1.000	.479
The tendency of students to escape mathematics class (truancy) causes them to have partial knowledge, hence resulting in poor performance in mathematics.	1.000	.354
Poor performance in mathematics could be explained by poor background in elementary mathematics.	1.000	.473
Poor background of student in mathematics is the most important factor of poor performance in mathematics.	1.000	.368
Lack of qualified mathematics teachers in your region to teach the subject lead to poor performance in mathematics.	1.000	.521
Lack of teaching and learning of mathematics material at your school lead to poor performance in mathematics.	1.000	.157
Negative attitude towards mathematics causes students to perform poorly in mathematics.	1.000	.431
Students are misbehaving in mathematics class and thus they do not understand fully the mathematics concepts which are being taught leading to poor performance in mathematics.	1.000	.265
Students do not practice to solve mathematics questions thus they perform poorly in mathematics.	1.000	.645
The students are not serious in studying mathematics hence they perform poorly in mathematics.	1.000	.529
The language of instruction (English) is not understood by the students leading to poor understanding of mathematics concepts and poor performance in mathematics.	1.000	.361
The mathematics language (for example, estimate, reminder) is not understood by the students, hence causes the students to perform poorly in mathematics	1.000	.359
I do not like mathematics.	1.000	.547
Mathematics lessons are boring.	1.000	.643
I do not understand my mathematics teacher when he/she is teaching.	1.000	.653
I do not do mathematics homework.	1.000	.633
Mathematics is a simple subject.	1.000	.710
Sometimes I do not attend mathematics lessons.	1.000	.703
Mathematics is a difficult subject.	1.000	.827

Extraction Method: Principal Component Analysis.

Table 4b: Structure matrix (5 factors after rotation) N=260

Variable	Component				
	1	2	3	4	5
I do not understand my mathematics teacher when he/she is teaching.	**.800**	.057	.125	.211	.260
Mathematics lessons are boring.	**.793**	.095	.036	.305	.172
I do not do mathematics homework.	**.773**	.089	.185	.215	.032
I do not like mathematics.	**.709**	.139	.110	.371	.142
Students do not practice to solve mathematics questions thus they perform poorly in mathematics.	.078	**.796**	.121	.015	.064
The students are not serious in studying mathematics hence they perform poorly in mathematics.	.107	**.703**	-.003	.095	-.026
The language of instruction (English) is not understood by the students leading to poor understanding of mathematics concepts and poor performance in mathematics.	.154	**.580**	.070	.027	.125
Poor background of student in mathematics is the most important factor of poor performance in mathematics.	.112	**.541**	.255	.257	.197
The mathematics language (for example, estimate, reminder) is not understood by the students, hence causes the students to perform poorly in mathematics	.102	**.495**	.214	.363	.155
Students are misbehaving in mathematics class and thus they do not understand fully the mathematics concepts which are being taught leading to poor performance in mathematics.	-.153	**.481**	.051	.052	.035
The tendency of students to escape mathematics class (truancy) causes them to have partial knowledge, hence resulting in poor performance in mathematics.	-.131	**.431**	.385	.020	-.142
Lack of teaching and learning of mathematics material at your school lead to poor performance in mathematics.	.137	.297	.017	-.024	.256
Lack of motivation to students performing well in Mathematics discourages students' commitment to study mathematics.	.176	.137	**.729**	.142	.090
The teaching method or style (the teacher is demonstrating without allowing students to participate due to large number of students in a class).	.000	.032	**.679**	-.067	-.014
Lack of motivation to mathematics teachers discourages teachers' commitment to work.	.186	.016	**.595**	-.160	.182
Poor performance in mathematics could be explained by poor background in elementary mathematics.	.111	.408	**.421**	.317	.415
Mathematics is a difficult subject.	.383	.081	-.052	**.889**	.148
Sometimes I do not attend mathematics lessons.	.253	.022	.006	**.833**	.050
Mathematics is a simple subject.	-.274	-.081	.071	**-.833**	-.039
Poor parents/guardians economic status causes students to perform poorly in mathematics.	-.133	.035	.053	-.089	**.667**
Lack of qualified mathematics teachers in your region to teach the subject lead to poor performance in mathematics.	.382	.168	.056	.212	**.665**
Negative attitude towards mathematics causes students to perform poorly in mathematics.	.448	-.073	.128	.292	**.494**

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.