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ON GENERALIZED INTUITIONISTIC FUZZY SOFT SET

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Abstract: The purpose of this paper is to study some basic operations and results available in the literature of generalized intuitionistic fuzzy soft sets. We have extended the notion of generalized intuitionistic fuzzy soft sets initiated by Dinda with some modifications and some new results have been put forward in our work.

Keywords: Intuitionistic Fuzzy Set, Soft Set, Fuzzy Soft Set, Intuitionistic Fuzzy Soft Set, Generalized Intuitionistic Fuzzy Soft Set .

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1. Introduction

Most of the real life problems have various uncertainties. The Theory of Probability, Evidence Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. act as mathematical tools in solving such problems. In 1999, Molodtsov [3] introduced Soft Set Theory and established the fundamental results

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related to this theory. In comparison, this theory is free from the inadequacy of parameterisation tool. It is a general mathematical tool for dealing problems in the fields of social science, economics, medical sciences etc. In 2003, Maji [5] studied the theory of soft set by Molodtsov and proposed the theory of Fuzzy Soft Set [6] which is a combination of fuzzy set theory proposed by Zadeh in 1965 and Molodtsov's Soft Set Theory. These results were further revised and improved by Ahmed and Kharal [1]. Intuitionistic Fuzzy Soft Set Theory is a combination of soft sets and Intuitionistic Fuzzy Sets initiated by Atanassov [4]. Majumder and Samanta [8] generalized the concept of fuzzy soft set introduced by Maji [6] and applied the notion of similarity between two generalized fuzzy soft sets in certain decision making problems. Keeping in view, Dinda gives [2] the notion of Generalized Intuitionistic Fuzzy Soft Set initiated by Maji [6]. In this paper, we attempt to extend the work of Dinda [2] on Intuitionistic Generalised Fuzzy Soft Set.

2. Preliminaries

In this section, we recall some basic definitions which would be needed in our work.

Definition 2.1 [3]

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 2.2 [6]

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \widetilde{P}(U)$ is a mapping from *A* into $\widetilde{P}(U)$.

Definition 2.3 [7]

Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \to IF^U$.

Definition 2.4 [8]

Let $U = \{x_1, x_2, x_3, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, e_3, ...e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, i.e. $\mu : E \to I = [0,1]$, where I^U is the collection of all fuzzy subsets of U. Let F_μ be the mapping $F_\mu : E \to I^U \times I$ be a function defined as follows: $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^U$. Then F_μ is called generalized fuzzy soft sets over the soft universe (U, E). Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Definition 2.5 [2]

Let $F : A \to IF^U$ and α be a fuzzy subset of A. Then $F_\alpha : A \to IF^U \times [0,1]$ is a function defined as: $F_\alpha(\varepsilon) = (F(\varepsilon), \alpha(\varepsilon)) = (\{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U\}, \alpha(\varepsilon))$, where μ, ν denote the degree of membership and degree of non-membership respectively. Then F_α is called a generalized intuitionistic fuzzy soft set (GIFSS) over (U, E).

Definition 2.6 [2]

Let F_α and G_β be two generalized intuitionistic fuzzy soft set over (U, E). Now F_α is called a generalized fuzzy soft subset of G_β if

(i) α is a fuzzy subset of β

(ii) $A \subseteq B$,

(iii) $\forall \varepsilon \in A, F(\varepsilon)$ is an intuitionistic fuzzy subset of $G(\varepsilon)$ *i.e.* $\mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x)$, $\nu_{F(\varepsilon)}(x) \geq \nu_{G(\varepsilon)}(x)$, $\forall e \in E$ and we write $F_\alpha \tilde{\subseteq} G_\beta$

Definition 2.7 [2]

The intersection of two GIFSS F_α and G_β over (U, E) is denoted by $F_\alpha \tilde{\cap} G_\beta$ and defined by GIFSS $H_\delta : A \cap B \to IF^U \times [0,1]$ such that for each $e \in A \cap B$,

$$H_\delta(e) = \left(\{x, \mu_{H(e)}(x), \nu_{H(e)}(x) : x \in U\}, \delta(e)\right),$$

where $\mu_{H(e)}(x) = \mu_{F(e)}(x) * \mu_{G(e)}(x)$ and

$$\nu_{H(e)}(x) = \nu_{F(e)}(x) \Diamond \nu_{G(e)}(x), \quad \delta(e) = \alpha(e) * \beta(e)$$

Definition 2.8 [2]

The union of two GIFSS F_α and G_β over (U, E) is denoted by $F_\alpha \tilde{\cup} G_\beta$ and defined by GIFSS $H_\delta : A \cup B \to IF^U \times [0,1]$ such that for each $e \in A \cup B$,

$$H_\delta(e) = \begin{cases} \left(\{x, \mu_{F(e)}(x), \nu_{F(e)}(x) : x \in U\}, \delta(e)\right), \text{ if } e \in A - B \\ \left(\{x, \mu_{G(e)}(x), \nu_{G(e)}(x) : x \in U\}, \delta(e)\right), \text{ if } e \in A - B \\ \left(\{x, \mu_{H(e)}(x), \nu_{H(e)}(x) : x \in U\}, \delta(e)\right), \text{ if } e \in A - B \end{cases}$$

Where $\mu_{H(e)}(x) = \mu_{F(e)}(x) \Diamond \mu_{G(e)}(x)$

and $\nu_{H(e)}(x) = \nu_{F(e)}(x) * \nu_{G(e)}(x)$, $\delta(e) = \alpha(e) \Diamond \beta(e)$

3. A study on the operations of GIFSS

In this section, we shall try to make the notion of GIFSS initiated by Dinda et al. [2] more rational and study some basic operations in our way. We would refer to the set of parameters under consideration whenever we talk about GIFSS. In what follows, in definition 2.5, we would use the notation (F_α, A) instead of F_α and in definitions 2.6 and in 2.7, we would use (F_α, A) and (G_β, B) instead of F_α and G_β respectively. Thus the definitions 2.5, 2.6 and 2.7 would take the following forms in our way:

Definition 3.1

Let $F : A \to IF^U$ and α be a fuzzy subset of *A*. Then $F_\alpha : A \to IF^U \times [0,1]$ is a function defined as: $F_\alpha(\varepsilon) = (F(\varepsilon), \alpha(\varepsilon)) = (\{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U\}, \alpha(\varepsilon))$, where μ, ν denote the degree of membership and degree of non-membership respectively. Then (F_α, A) is called a generalized intuitionistic fuzzy soft set (GIFSS) over (*U*, *E*).

Definition 3.2

Let (F_α, A) and (G_β, B) be two GIFSS over (*U*, *E*). Now (F_α, A) is called a generalized fuzzy soft subset of (G_β, B) if

(iv) α is a fuzzy subset of β

(v) $A \subseteq B$,

(vi) $\forall \varepsilon \in A, F(\varepsilon)$ is an intuitionistic fuzzy subset of $G(\varepsilon)$ *i.e.* $\mu_{F(\varepsilon)}(x) \le \mu_{G(\varepsilon)}(x)$, $\nu_{F(\varepsilon)}(x) \ge \nu_{G(\varepsilon)}(x)$, $\forall e \in E$ and we write $(F_\alpha, A) \tilde{\subseteq} (G_\beta, B)$

Definition 3.3

The intersection of two GIFSS (F_α, A) and (G_β, B) over (U, E) is denoted by $(F_\alpha, A)\tilde{\cap}(G_\beta, B)$ and defined by GIFSS $H_\delta : C \to IF^U \times [0,1]$, where $C = A \cap B$ and for each $e \in C$, $H_\delta(e) = (\{x, \mu_{H(e)}(x), \nu_{H(e)}(x) : x \in U\}, \delta(e))$,

where $\mu_{H(e)}(x) = \min(\mu_{F(e)}(x), \mu_{G(e)}(x))$ and

$\nu_{H(e)}(x) = \max(\nu_{F(e)}(x), \nu_{G(e)}(x)), \delta(e) = \min(\alpha(e), \beta(e))$

Definition 3.4

The union of two GIFSS (F_α, A) and (G_β, B) over (U, E) is denoted by $(F_\alpha, A)\tilde{\cup}(G_\beta, B)$ and defined by GIFSS $H_\delta : C \to IF^U \times [0,1]$, where $C = A \cup B$ and for each $e \in C$,

$$H_\delta(e) = \begin{cases} (\{x, \mu_{F(e)}(x), \nu_{F(e)}(x) : x \in U\}, \delta(e)), \text{ if } e \in A - B \\ (\{x, \mu_{G(e)}(x), \nu_{G(e)}(x) : x \in U\}, \delta(e)), \text{ if } e \in A - B \\ (\{x, \mu_{H(e)}(x), \nu_{H(e)}(x) : x \in U\}, \delta(e)), \text{ if } e \in A - B \end{cases}$$

where $\mu_{H(e)}(x) = \max(\mu_{F(e)}(x), \mu_{G(e)}(x))$, $\nu_{H(e)}(x) = \min(\nu_{F(e)}(x), \nu_{G(e)}(x))$,

$\delta(e) = \max(\alpha(e), \beta(e))$

Next, we give some definitions related to Generalized Intuitionistic Fuzzy Soft Sets as follows:

Definition 3.5

A GIFSS is said to be a generalized intuitionistic fuzzy soft null set, denoted by (θ_ϕ, A), if $\theta_\phi : A \to IF^U \times [0,1]$ such that $\theta_\phi(e) = (F(e), \phi(e))$ where $F(e) = \{x, 0, 1 : x \in U\}$, $\phi(e) = 0$,

$\forall e \in A \subseteq E$

Definition 3.6

A GIFSS is said to be a generalized intuitionistic absolute fuzzy soft set, denoted by $\left(U_{\bar{1}}, A\right)$, if

$U_{\bar{1}} : A \to I^U \times [0,1]$ such that $U_{\bar{1}}(e) = \left(F(e), \bar{1}(e)\right)$ where $F(e) = \{x, 1, 0 : x \in U\}$, $\bar{1}(e) = 1, \quad \forall e \in A \subseteq E$.

It is evident from our definition that the generalized intuitionistic fuzzy soft null and absolute sets are not unique in our way, it would depend upon the set of parameters under consideration.

Definition 3.7

Let (F_α, A) be a GIFSS over (*U*, *E*). Then the complement of (F_α, A), denoted by $(F_\alpha, A)^c$ and is defined by $(F_\alpha, A)^c = \left(F_\alpha{}^c, A\right)$, where $\forall \varepsilon \in A$,

$$F_\alpha{}^c(\varepsilon) = \left(\{x, \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x) : x \in U\}, 1 - \alpha(e)\right)$$

Definition 3.8

If (F_α, A) and (G_β, B) be two GIFSS over (*U*, *E*), then "(F_α, A) AND (G_β, B)" is a GIFSS denoted by $(F_\alpha, A) \wedge (G_\beta, B)$ and is defined by $(F_\alpha, A) \wedge (G_\beta, B) = (H_\gamma, A \times B)$, where

$$H_\gamma(\varepsilon_1, \varepsilon_2) = F_\alpha(\varepsilon_1) \cap G_\beta(\varepsilon_2), \ \forall (\varepsilon_1, \varepsilon_2) \in A \times B$$

$$= \left(\left\{x, \min\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right), \max\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right)\right\}, \min\left(\alpha(\varepsilon_1), \beta(\varepsilon_2)\right)\right)$$

Definition 3.9

If (F_α, A) and (G_β, B) be two GIFSS over (*U*, *E*), then "(F_α, A) OR (G_β, B)" is a GIFSS denoted by $(F_\alpha, A) \vee (G_\beta, B)$ and is defined by $(F_\alpha, A) \vee$

$\left(G_\beta, B\right)=\left(H_\gamma, A\times B\right)$, where

$H_\gamma(\varepsilon_1,\varepsilon_2)=F_\alpha(\varepsilon_1)\cup G_\beta(\varepsilon_2),\ \forall(\varepsilon_1,\varepsilon_2)\in A\times B$

$=\left(\left\{x,\max\left(\mu_{F(\varepsilon_1)}(x),\mu_{G(\varepsilon_2)}(x)\right),\min\left(\nu_{F(\varepsilon_1)}(x),\nu_{G(\varepsilon_2)}(x)\right)\right\},\quad \max\left(\alpha(\varepsilon_1),\beta(\varepsilon_2)\right)\right)$

Proposition 3.1

1. $\left(\theta_\phi, A\right)^c=\left(U_{\bar{1}}, A\right)$

Proof

Let $\left(\theta_\phi, A\right)=\left(F_\alpha, A\right)$

Then $\forall\varepsilon\in A,$

$$F_\alpha(\varepsilon)=\left(\left\{\left(x,\mu_{F(\varepsilon)}(x),\nu_{F(\varepsilon)}(x)\right): x\in U\right\},\alpha(\varepsilon)\right)$$

$$=\left(\left\{\left(x,0,1\right): x\in U\right\},0\right)$$

$\left(\theta_\phi, A\right)^c=\left(F_\alpha, A\right)^c=\left(F_\alpha{}^c, A\right)$, where $\forall\varepsilon\in A,$

$F_\alpha{}^c(\varepsilon)=\left(\left\{\left(x,1,0\right): x\in U\right\},1\right)$

Thus $\left(\theta_\phi, A\right)^c=\left(U_{\bar{1}}, A\right)$

2. $\left(U_{\bar{1}}, A\right)^c=\left(\theta_\phi, A\right)$

Proof

Let $\left(U_{\bar{1}}, A\right)=\left(F_\alpha, A\right)$

Then $\forall\varepsilon\in A,$

$$F_\alpha(\varepsilon)=\left(\left\{\left(x,\mu_{F(\varepsilon)}(x),\nu_{F(\varepsilon)}(x)\right): x\in U\right\},\alpha(\varepsilon)\right)$$

$$=\left(\left\{\left(x,1,0\right): x\in U\right\},1\right)$$

$\left(U_{\bar{1}}, A\right)^c=\left(F_\alpha, A\right)^c=\left(F_\alpha{}^c, A\right)$, where $\forall\varepsilon\in A,$

$F_\alpha{}^c(\varepsilon) = (\{(x,0,1): x \in U\}, 0)$

Thus $(U_{\tilde{1}}, A)^c = (\theta_\phi, A)$

3. $((F_\alpha, A)^c)^c = (F_\alpha, A)$

Proof

For (F_α, A), $\forall \varepsilon \in A$,

$$F_\alpha(\varepsilon) = (\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)): x \in U\}, \alpha(\varepsilon))$$

$(F_\alpha, A)^c = (F_\alpha{}^c, A)$, where $\forall \varepsilon \in A$,

$$F_\alpha{}^c(\varepsilon) = (\{(x, \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)): x \in U\}, \alpha(\varepsilon))$$

$((F_\alpha, A)^c)^c = \left((F_\alpha{}^c)^c, A\right) = (I_\alpha, A)$, say, where $\forall \varepsilon \in A$,

$$I_\alpha(\varepsilon) = (\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)): x \in U\}, \alpha(\varepsilon))$$

Thus $((F_\alpha, A)^c)^c = (F_\alpha, A)$

4. $(F_\alpha, A) \tilde{\cup} (\theta_\phi, A) = (F_\alpha, A)$

Proof

We have for (F_α, A)

$$F_\alpha(\varepsilon) = (\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)): x \in U\}, \alpha(\varepsilon)) \; \forall \varepsilon \in A$$

And let $(\theta_\phi, A) = (G_\beta, A)$, then

$$G_\beta(\varepsilon) = (\{(x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)): x \in U\}, \beta(\varepsilon)) \; \forall \varepsilon \in A$$

$$= (\{(x,0,1): x \in U\}, 0) \; \forall \varepsilon \in A$$

Let $(F_\alpha, A) \tilde{\cup} (\theta_\phi, A) = (H_\gamma, A)$

$H_\gamma(\varepsilon) \quad = \left(\left\{\left(x, \max\left(\mu_{F(\varepsilon)}(x), 0\right) \min\left(\nu_{F(\varepsilon)}(x), 1\right)\right) : x \in U\right\}, \ \max\left(\alpha(\varepsilon), 0\right)\right) \ \forall \varepsilon \in A$

$= \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \ \alpha(\varepsilon)\right) \ \forall \varepsilon \in A$

Thus $(F_\alpha, A) \tilde{\cup} (\theta_\phi, A) = (F_\alpha, A)$

5.$(F_\alpha, A) \tilde{\cup} (U_{\bar{1}}, A) = (U_{\bar{1}}, A)$

Proof

We have for (F_α, A)

$F_\alpha(\varepsilon) \quad = \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right) \ \forall \varepsilon \in A$

And let $(U_{\bar{1}}, A) = (G_\beta, A)$, then

$G_\beta(\varepsilon) \quad = \left(\left\{\left(x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right) : x \in U\right\}, \beta(\varepsilon)\right) \ \forall \varepsilon \in A$

$= \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right) \ \forall \varepsilon \in A$

$(F_\alpha, A) \tilde{\cup} (U_{\bar{1}}, A) = (H_\gamma, A)$

$H_\gamma(\varepsilon) \quad = \left(\left\{\left(x, \max\left(\mu_{F(\varepsilon)}(x), 1\right) \min\left(\nu_{F(\varepsilon)}(x), 0\right)\right) : x \in U\right\}, \ \max\left(\alpha(\varepsilon), 1\right)\right) \ \forall \varepsilon \in A$

$= \left(\left\{(x, 1, 0) : x \in U\right\}, \ 1\right) \ \forall \varepsilon \in A$

Thus $(F_\alpha, A) \tilde{\cup} (U_{\bar{1}}, A) = (U_{\bar{1}}, A)$

6.$(F_\alpha, A) \tilde{\cap} (\theta_\phi, A) = (\theta_\phi, A)$

Proof

We have for (F_α, A)

$F_\alpha(\varepsilon) \quad = \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right) \ \forall \varepsilon \in A$

And let $(\theta_\phi, A) = (G_\beta, A)$, then

$G_\beta(\varepsilon) \quad = \left(\left\{\left(x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right) : x \in U\right\}, \beta(\varepsilon)\right) \ \forall \varepsilon \in A$

$$= \left(\{(x,0,1): x \in U\}, 0\right) \forall \varepsilon \in A$$

Let $(F_\alpha, A) \tilde{\cap} (\theta_\phi, A) = (H_\gamma, A)$

$$H_\gamma(\varepsilon) = \left(\{(x, \min(\mu_{F(\varepsilon)}(x), 0) \max(\nu_{F(\varepsilon)}(x), 1)): x \in U\}, \ \min(\alpha(\varepsilon), 0)\right) \forall \varepsilon \in A$$

$$= \left(\{(x,0,1): x \in U\}, \ 0\right) \forall \varepsilon \in A$$

Thus $(F_\alpha, A) \tilde{\cap} (\theta_\phi, A) = (\theta_\phi, A)$

7. $(F_\alpha, A) \tilde{\cap} (U_{\bar{1}}, A) = (F_\alpha, A)$

Proof

We have for (F_α, A)

$$F_\alpha(\varepsilon) = \left(\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)): x \in U\}, \alpha(\varepsilon)\right) \forall \varepsilon \in A$$

And let $(U_{\bar{1}}, A) = (G_\beta, A)$, then

$$G_\beta(\varepsilon) = \left(\{(x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)): x \in U\}, \beta(\varepsilon)\right) \forall \varepsilon \in A$$

$$= \left(\{(x,1,0): x \in U\}, 1\right) \forall \varepsilon \in A$$

$(F_\alpha, A) \tilde{\cap} (U_{\bar{1}}, A) = (H_\gamma, A)$

$$H_\gamma(\varepsilon) = \left(\{(x, \min(\mu_{F(\varepsilon)}(x), 1) \max(\nu_{F(\varepsilon)}(x), 0)): x \in U\}, \ \min(\alpha(\varepsilon), 1)\right) \forall \varepsilon \in A$$

$$= \left(\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)): x \in U\}, \ \alpha(\varepsilon)\right) \forall \varepsilon \in A$$

Thus $(F_\alpha, A) \tilde{\cap} (U_{\bar{1}}, A) = (F_\alpha, A)$

8. $(F_\alpha, A) \tilde{\cup} (\theta_\phi, B) = (F_\alpha, A)$ if and only if $B \subseteq A$

Proof

We have for (F_α, A)

$$F_\alpha(\varepsilon) = \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right) \quad \forall \varepsilon \in A$$

Also, let $(\theta_\phi, B) = (G_\beta, B)$, then

$$G_\beta(\varepsilon) = \left(\{(x,0,1) : x \in U\}, 0\right) \ \forall \varepsilon \in B$$

Let $(F_\alpha, A)\tilde{\cup}(\theta_\phi, B) = (F_\alpha, A)\tilde{\cup}(G_\beta, B) = (H_\gamma, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H_\gamma(\varepsilon) = \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\left\{\left(x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right) : x \in U\right\}, \beta(\varepsilon)\right), \text{if } \varepsilon \in B - A \\ \left(\left\{\left(x, \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \min\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right) : x \in U\right\}, \right. \\ \quad \left. \max\left(\alpha(\varepsilon), \beta(\varepsilon)\right)\right) \text{ if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\{(x,0,1) : x \in U\}, 0\right), \text{if } \varepsilon \in B - A \\ \left(\left\{\left(x, \max\left(\mu_{F(\varepsilon)}(x), 0\right), \min\left(\nu_{F(\varepsilon)}(x), 1\right)\right) : x \in U\right\}, \max\left(\alpha(\varepsilon), 0\right)\right) \\ \quad \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\{(x,0,1) : x \in U\}, 0\right), \text{if } \varepsilon \in B - A \\ \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right) \text{ if } \varepsilon \in A \cap B \end{cases},$$

Let $B \subseteq A$. Then

$$H_\gamma(\varepsilon) = \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= F_\alpha(\varepsilon) \ \forall \varepsilon \in A$$

Conversely, let $(F_\alpha, A)\tilde{\cup}(\theta_\phi, B) = (F_\alpha, A)$

Then $A = A \cup B \Rightarrow B \subseteq A$

9. $(F_\alpha, A)\tilde{\cup}(U_{\bar{1}}, B) = (U_{\bar{1}}, B)$ if and only if $A \subseteq B$

Proof

We have for (F_α, A)

$$F_\alpha(\varepsilon) = \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right) \quad \forall \varepsilon \in A$$

Also, let $(U_{\bar{1}}, B) = (G_\beta, B)$, then

$$G_\beta(\varepsilon) = \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right) \; \forall \varepsilon \in B$$

Let $(F_\alpha, A) \tilde{\cup} (U_{\bar{1}}, A) = (F_\alpha, A) \tilde{\cup} (G_\beta, B) = (H_\gamma, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H_\gamma(\varepsilon) = \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\left\{\left(x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right) : x \in U\right\}, \beta(\varepsilon)\right), \text{if } \varepsilon \in B - A \\ \left(\left\{\left(x, \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \min\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right) : x \in U\right\}, \right. \\ \quad \left. \max\left(\alpha(\varepsilon), \beta(\varepsilon)\right)\right) \text{ if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right), \text{if } \varepsilon \in B - A \\ \left(\left\{\left(x, \max\left(\mu_{F(\varepsilon)}(x), 1\right), \min\left(\nu_{F(\varepsilon)}(x), 0\right)\right) : x \in U\right\}, \max\left(\alpha(\varepsilon), 1\right)\right) \\ \quad \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{\left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right) : x \in U\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A - B \\ \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right), \text{if } \varepsilon \in B - A \\ \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right) \text{ if } \varepsilon \in A \cap B \end{cases},$$

Let $A \subseteq B$. Then

$$H_\gamma(\varepsilon) = \begin{cases} \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right), \text{if } \varepsilon \in B - A \\ \left(\left\{(x, 1, 0) : x \in U\right\}, 1\right), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= G_\beta(\varepsilon) \; \forall \varepsilon \in A$$

Conversely, let $(F_\alpha, A) \tilde{\cup} (U_{\bar{1}}, B) = (U_{\bar{1}}, B)$

Then $B = A \cup B \Rightarrow A \subseteq B$

10. $(F_\alpha, A) \tilde{\cap} (\theta_\phi, B) = (\theta_\phi, A \cap B)$

Proof

We have for (F_α, A)

$$F_\alpha(\varepsilon) \quad = \left(\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)) : x \in U\}, \alpha(\varepsilon)\right) \quad \forall \varepsilon \in A$$

Also, let $(\theta_\phi, B) = (G_\beta, B)$, Then

$$G_\beta(\varepsilon) \quad = \left(\{(x, 0, 1) : x \in U\}, 0\right) \ \forall \varepsilon \in B$$

Let $(F_\alpha, A) \tilde{\cap} (\theta_\phi, B) = (F_\alpha, A) \tilde{\cap} (G_\beta, B) = (H_\gamma, C)$, where $C = A \cap B$ and $\forall \varepsilon \in C$,

$H_\gamma(\varepsilon)$

$$= \left(\{(x, \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))) : x \in U\}, \min(\alpha(\varepsilon), \beta(\varepsilon))\right)$$

$$= \left(\{(x, \min(\mu_{F(\varepsilon)}(x), 0), \max(\nu_{F(\varepsilon)}(x), 1)) : x \in U\}, \min(\alpha(\varepsilon), 0)\right)$$

$$= \left(\{(x, 0, 1) : x \in U\}, 0\right)$$

Thus $(F_\alpha, A) \tilde{\cap} (\theta_\phi, B) = (\theta_\phi, A \cap B)$

11. $(F_\alpha, A) \tilde{\cap} (U_{\bar{1}}, B) = (F_\alpha, A \cap B)$

Proof

We have for (F_α, A)

$$F_\alpha(\varepsilon) \quad = \left(\{(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)) : x \in U\}, \alpha(\varepsilon)\right) \quad \forall \varepsilon \in A$$

Also, let $(U_{\bar{1}}, B) = (G_\beta, B)$, Then

$$G_\beta(\varepsilon) \quad = \left(\{(x, 1, 0) : x \in U\}, 1\right) \ \forall \varepsilon \in B$$

Let $(F_\alpha, A) \tilde{\cap} (U_{\bar{1}}, B) = (F_\alpha, A) \tilde{\cap} (G_\beta, B) = (H_\gamma, C)$, where $C = A \cap B$ and $\forall \varepsilon \in C$,

$H_\gamma(\varepsilon)$

$$= \left(\{(x, \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))) : x \in U\}, \min(\alpha(\varepsilon), \beta(\varepsilon))\right)$$

$= \left(\left\{ \left(x, \min\left(\mu_{F(\varepsilon)}(x), 1\right), \max\left(\nu_{F(\varepsilon)}(x), 0\right) \right) : x \in U \right\}, \min\left(\alpha(\varepsilon), 1\right) \right)$

$= \left(\left\{ \left(x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \right) : x \in U \right\}, \alpha(\varepsilon) \right)$

Thus $(F_\alpha, A) \tilde{\cap} (U_{\bar{1}}, B) = (F_\alpha, A \cap B)$

Proposition 3.2

For GIFSS (F_α, A), (G_β, B) and (H_γ, C) over (U, E), the following results are valid.

1.(i) $(F_\alpha, A) \tilde{\cup} (G_\beta, B) = (G_\beta, B) \tilde{\cup} (F_\alpha, A)$

(ii) $(F_\alpha, A) \tilde{\cap} (G_\beta, B) = (G_\beta, B) \tilde{\cap} (F_\alpha, A)$

2.(i) $(F_\alpha, A) \tilde{\cup} \left((G_\beta, B) \tilde{\cup} (H_\gamma, C) \right) = \left((F_\alpha, A) \tilde{\cup} (G_\beta, B) \right) \tilde{\cup} (H_\gamma, C)$

(ii) $(F_\alpha, A) \tilde{\cap} \left((G_\beta, B) \tilde{\cap} (H_\gamma, C) \right) = \left((F_\alpha, A) \tilde{\cap} (G_\beta, B) \right) \tilde{\cap} (H_\gamma, C)$

3.(i) $(F_\alpha, A) \tilde{\cup} \left((G_\beta, B) \tilde{\cap} (H_\gamma, C) \right) = \left((F_\alpha, A) \tilde{\cup} (G_\beta, B) \right) \tilde{\cap} \left((F_\alpha, A) \tilde{\cup} (H_\gamma, C) \right)$

(ii) $(F_\alpha, A) \tilde{\cap} \left((G_\beta, B) \tilde{\cup} (H_\gamma, C) \right) = \left((F_\alpha, A) \tilde{\cap} (G_\beta, B) \right) \tilde{\cup} \left((F_\alpha, A) \tilde{\cap} (H_\gamma, C) \right)$

4.(i) $(F_\alpha, A) \tilde{\cup} (F_\alpha, A) = (F_\alpha, A)$

(ii) $(F_\alpha, A) \tilde{\cap} (F_\alpha, A) = (F_\alpha, A)$

Proof. The proof is straight forward and follows from definition.

It can be verified that De Morgan Laws are not valid for GIFSS with different sets of parameters under our definition of union, intersection and complement. However, we have the following inclusions for GIFSS (F_α, A) and (G_β, B) over (U, E) with different sets A and B of parameters.

Proposition 3.3 (De Morgan Inclusions)

For GIFSS (F_α, A) and (G_β, B) over (*U*, *E*), we have the following -

1. $(F_\alpha, A)^c \,\tilde{\cap}\, (G_\beta, B)^c \,\tilde{\subseteq}\, ((F_\alpha, A) \,\tilde{\cup}\, (G_\beta, B))^c$

2. $((F_\alpha, A) \,\tilde{\cap}\, (G_\beta, B))^c \,\tilde{\subseteq}\, (F_\alpha, A)^c \,\tilde{\cup}\, (G_\beta, B)^c$

Proof

1. Let $(F_\alpha, A) \,\tilde{\cup}\, (G_\beta, B) = (H_\gamma, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C,$

$$H_\gamma(\varepsilon) = \begin{cases} F_\alpha(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G_\beta(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F_\alpha(\varepsilon) \cup G_\beta(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right\}, \alpha(\varepsilon)\right), \text{if } \varepsilon \in A\text{-}B \\ \left(\left\{x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right\}, \beta(\varepsilon)\right), \text{if } \varepsilon \in B\text{-}A \\ \left(\left\{x, \max\left(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)}\right), \min\left(\nu_{F(\varepsilon)}, \nu_{G(\varepsilon)}\right)\right\}, \max(\alpha(\varepsilon), \beta(\varepsilon))\right), \text{if } \varepsilon \in A \cap B \end{cases}$$

Thus $((F_\alpha, A) \,\tilde{\cup}\, (G_\beta, B))^c = (H_\gamma, C)^c = (H_\gamma{}^c, C)$, where $C = A \cup B$ and

$\forall \varepsilon \in C,$

$$H_\gamma{}^c(\varepsilon) = (H_\gamma(\varepsilon))^c$$

$$= \begin{cases} F_\alpha{}^c(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G_\beta{}^c(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F_\alpha{}^c(\varepsilon) \cup G_\beta{}^c(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{x, \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right\}, 1-\alpha(\varepsilon)\right), \text{if } \varepsilon \in A\text{-}B \\ \left(\left\{x, \nu_{G(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right\}, 1-\beta(\varepsilon)\right), \text{if } \varepsilon \in B\text{-}A \\ \left(\left\{x, \min\left(\nu_{F(\varepsilon)}, \nu_{G(\varepsilon)}\right), \max\left(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)}\right)\right\}, 1-\max(1-\alpha(\varepsilon), 1-\beta(\varepsilon))\right), \\ \text{if } \varepsilon \in A \cap B \end{cases}$$

$$=\begin{cases}\left(\left\{x,\nu_{F(\varepsilon)}(x),\mu_{F(\varepsilon)}(x)\right\},1-\alpha(\varepsilon)\right), \text{if } \varepsilon\in A\text{-}B\\ \left(\left\{x,\nu_{G(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right\},1-\beta(\varepsilon)\right), \text{if } \varepsilon\in B\text{-}A\\ \left(\left\{x,\min\left(\nu_{F(\varepsilon)},\nu_{G(\varepsilon)}\right),\max\left(\mu_{F(\varepsilon)},\mu_{G(\varepsilon)}\right)\right\},\min(1-\alpha(\varepsilon),1-\beta(\varepsilon))\right),\\ \text{if } \varepsilon\in A\cap B\end{cases}$$

Again, $(F_\alpha,A)^c\,\tilde{\cap}\,(G_\beta,B)^c$ = $\left(F_\alpha{}^c,A\right)\tilde{\cap}\left(G_\beta{}^c,B\right)$ = (I_δ,J) , say, where $J=A\cap B$ and $\forall\varepsilon\in J$,

$$I_\delta(\varepsilon) = F_\alpha{}^c(\varepsilon)\cap G_\beta{}^c(\varepsilon)$$

$$=\left(\left\{x,\min\left(\nu_{F(\varepsilon)}(x),\nu_{G(\varepsilon)}(x)\right),\max\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right)\right\},\min(1-\alpha(\varepsilon),1-\beta(\varepsilon))\right)$$

We see that $J\subseteq C$ and $\forall\varepsilon\in J, I_\delta(\varepsilon)=H_\gamma{}^c(\varepsilon),\quad \delta(\varepsilon)=\gamma(\varepsilon)$

Thus $(F_\alpha,A)^c\,\tilde{\cap}\,(G_\beta,B)^c\,\tilde{\subseteq}\left((F_\alpha,A)\tilde{\cup}(G_\beta,B)\right)^c$

2. Let $(F_\alpha,A)\tilde{\cap}(G_\beta,B)=(H_\gamma,C)$, where $C=A\cap B$ and $\forall\varepsilon\in C$,

$$H_\gamma(\varepsilon) = F_\alpha(\varepsilon)\cap G_\beta(\varepsilon)$$

$$=\left(\left\{x,\min\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right),\max\left(\nu_{F(\varepsilon)}(x),\nu_{G(\varepsilon)}(x)\right)\right\},\min(\alpha(\varepsilon),\beta(\varepsilon))\right)$$

Thus $\left((F_\alpha,A)\tilde{\cap}(G_\beta,B)\right)^c=(H_\gamma,C)^c=\left(H_\gamma{}^c,C\right)$, where $C=A\cap B$ and $\forall\varepsilon\in C$,

$$H^c(\varepsilon) = (F(\varepsilon)\cap G(\varepsilon))^c$$

$$=\left(\left\{x,\min\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right),\max\left(\nu_{F(\varepsilon)}(x),\nu_{G(\varepsilon)}(x)\right)\right\},\min(\alpha(\varepsilon),\beta(\varepsilon))\right)^c$$

$$=\left(\left\{x,\max\left(\nu_{F(\varepsilon)}(x),\nu_{G(\varepsilon)}(x)\right),\min\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right)\right\},1-\min(\alpha(\varepsilon),\beta(\varepsilon))\right)$$

Again, $(F_\alpha,A)^c\,\tilde{\cup}\,(G_\beta,B)^c=$ $\left(F_\alpha{}^c,A\right)\tilde{\cup}\left(G_\beta{}^c,B\right)$ $=(I_\delta,J)$, say, where $J=A\cup B$ and $\forall\varepsilon\in J$,

$$I_{\delta}(\varepsilon) = \begin{cases} F_{\alpha}{}^{c}(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G_{\beta}{}^{c}(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F_{\alpha}{}^{c}(\varepsilon) \cup G_{\beta}{}^{c}(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{x, \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right\}, 1-\alpha(\varepsilon)\right), \text{if } \varepsilon \in A\text{-}B \\ \left(\left\{x, \nu_{G(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right\}, 1-\beta(\varepsilon)\right), \text{if } \varepsilon \in B\text{-}A \\ \left\{x, \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \max\left(1-\alpha(\varepsilon), 1-\beta(\varepsilon)\right)\right\}, \\ \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \left(\left\{x, \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right\}, 1-\alpha(\varepsilon)\right), \text{if } \varepsilon \in A\text{-}B \\ \left(\left\{x, \nu_{G(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right\}, 1-\beta(\varepsilon)\right), \text{if } \varepsilon \in B\text{-}A \\ \left\{x, \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), 1-\min\left(\alpha(\varepsilon), \beta(\varepsilon)\right)\right\}, \\ \text{if } \varepsilon \in A \cap B \end{cases}$$

We see that $C \subseteq J$ and $\forall \varepsilon \in C, H_{\gamma}{}^{c}(\varepsilon) = I_{\delta}(\varepsilon)$, $\gamma(\varepsilon) \subseteq \delta(\varepsilon)$

It follows that $\left((F_{\alpha}, A) \tilde{\cap} \left(G_{\beta}, B\right)\right)^{c} \tilde{\subseteq} (F_{\alpha}, A)^{c} \tilde{\cup} \left(G_{\beta}, B\right)^{c}$

Proposition 3.4 (De Morgan Laws)

For GIFSS $\left(F_{\alpha}, A\right)$ and $\left(G_{\beta}, A\right)$ over (U, E), we have the following -

1. $\left((F_{\alpha}, A) \tilde{\cup} \left(G_{\beta}, A\right)\right)^{c} = (F_{\alpha}, A)^{c} \tilde{\cap} \left(G_{\beta}, A\right)^{c}$

2. $\left((F_{\alpha}, A) \tilde{\cap} \left(G_{\beta}, A\right)\right)^{c} = (F_{\alpha}, A)^{c} \tilde{\cup} \left(G_{\beta}, A\right)^{c}$

Proof

1. Let $\left(F_{\alpha}, A\right) \tilde{\cup} \left(G_{\beta}, A\right) = \left(H_{\gamma}, A\right)$, where $\forall \varepsilon \in A$,

$$H_{\gamma}(\varepsilon) = F_{\alpha}(\varepsilon) \cup G_{\beta}(\varepsilon)$$

$$= \left(\left\{x, \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \min\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right\}, \quad \max\left(\alpha(\varepsilon), \beta(\varepsilon)\right)\right)$$

Thus $\left((F_{\alpha}, A) \tilde{\cup} \left(G_{\beta}, A\right)\right)^{c} = \left(H_{\gamma}, A\right)^{c} = \left(H_{\gamma}{}^{c}, A\right)$, where $\forall \varepsilon \in A$,

$$H_{\gamma}{}^{c}(\varepsilon) = \left(H_{\gamma}(\varepsilon)\right)^{c}$$

$$= \left(F_{\alpha}(\varepsilon) \cup G_{\beta}(\varepsilon)\right)^{c}$$

$$= \left(\left\{x, \min\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\},\ 1 - \max(\alpha(\varepsilon), \beta(\varepsilon))\right)$$

$$= \left(\left\{x, \min\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\},\ \min(1-\alpha(\varepsilon), 1-\beta(\varepsilon))\right)$$

Again,

$(F_{\alpha}, A)^{c} \tilde{\cap} (G_{\beta}, A)^{c} = \left(F^{c}_{\alpha}, A\right) \tilde{\cap} \left(G_{\beta}^{c}, A\right) = (I_{\delta}, A)$, say, where $\forall \varepsilon \in A$,

$$I_{\delta}(\varepsilon) = F_{\alpha}^{c}(\varepsilon) \cap G_{\beta}^{c}(\varepsilon)$$

$$= \left(\left\{x, \min\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\},\ \min(1-\alpha(\varepsilon), 1-\beta(\varepsilon))\right)$$

Thus $\left((F_{\alpha}, A) \tilde{\cup} (G_{\beta}, A)\right)^{c} = (F_{\alpha}, A)^{c} \tilde{\cap} (G_{\beta}, A)^{c}$

2. Let $(F_{\alpha}, A) \tilde{\cap} (G_{\beta}, A) = (H_{\gamma}, A)$, where $\forall \varepsilon \in A$,

$$H_{\gamma}(\varepsilon) = F_{\alpha}(\varepsilon) \cap G_{\beta}(\varepsilon)$$

$$= \left(\left\{x, \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right\},\ \min(\alpha(\varepsilon), \beta(\varepsilon))\right)$$

Thus $\left((F_{\alpha}, A) \tilde{\cap} (G_{\beta}, A)\right)^{c} = (H_{\gamma}, A)^{c} = \left(H_{\gamma}^{c}, A\right)$, where $\forall \varepsilon \in A$,

$$H_{\gamma}^{c}(\varepsilon) = \left(H_{\gamma}(\varepsilon)\right)^{c}$$

$$= \left(F_{\alpha}(\varepsilon) \cap G_{\beta}(\varepsilon)\right)^{c}$$

$$= \left(\left\{x, \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\},\ 1 - \min(\alpha(\varepsilon), \beta(\varepsilon))\right)$$

$$= \left(\left\{x, \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\},\ \max(1-\alpha(\varepsilon), 1-\beta(\varepsilon))\right)$$

Again, $(F_\alpha, A)^c \tilde{\cup} (G_\beta, A)^c = (F^c{}_\alpha, A) \tilde{\cup} (G_\beta{}^c, A) = (I_\delta, A)$, say, where $\forall \varepsilon \in A$,

$$I_\delta(\varepsilon) = F_\alpha{}^c(\varepsilon) \cup G_\beta{}^c(\varepsilon)$$

$$= (\{x, \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\},$$

$$\max(1-\alpha(\varepsilon), 1-\beta(\varepsilon)))$$

Thus $((F_\alpha, A) \tilde{\cap} (G_\beta, A))^c = (F_\alpha, A)^c \tilde{\cup} (G_\beta, A)^c$

Proposition 3.5

For GIFSS (F_α, A) and (G_β, A) over (U, E), we have the following -

1. $((F_\alpha, A) \wedge (G_\beta, B))^c = (F_\alpha, A)^c \vee (G_\beta, B)^c$
2. $((F_\alpha, A) \vee (G_\beta, B))^c = (F_\alpha, A)^c \wedge (G_\beta, B)^c$

Proof

1. Let $(F_\alpha, A) \wedge (G_\beta, B) = (H_\gamma, A \times B)$, where $H_\gamma(\varepsilon_1, \varepsilon_2) = F_\alpha(\varepsilon_1) \cap G_\beta(\varepsilon_2)$,

$\forall \varepsilon_1 \in A$ and $\forall \varepsilon_2 \in B$. Thus

$$H_\gamma(\varepsilon_1, \varepsilon_2) = F_\alpha(\varepsilon_1) \cap G_\beta(\varepsilon_2)$$

$$= (\{x, \min(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)), \max(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x))\},$$

$$\min(\alpha(\varepsilon_1), \beta(\varepsilon_2)))$$

Thus $((F_\alpha, A) \wedge (G_\beta, B))^c = (H_\gamma, A \times B)^c = (H_\gamma{}^c, A \times B)$, where $\forall (\varepsilon_1, \varepsilon_2) \in A \times B$,

$$H_\gamma{}^c(\alpha, \beta) = (H_\gamma(\alpha, \beta))^c$$

$$= (\{x, \max(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)), \min(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x))\},$$

$$1 - \min(\alpha(\varepsilon_1), \beta(\varepsilon_2)))$$

Let $(F_\alpha, A)^c \vee (G_\beta, B)^c = (F_\alpha{}^c, A) \vee (G_\beta{}^c, B) = (I_\delta, A \times B)$, where

$$I_\delta(\varepsilon_1,\varepsilon_2) = F_\alpha{}^c(\varepsilon_1) \cup G_\beta{}^c(\varepsilon_2), \quad \forall(\varepsilon_1,\varepsilon_2) \in A \times B$$

$$= \left(\left\{x, \max\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right), \min\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right)\right\},\right.$$

$$\left.\max(1-\alpha(\varepsilon_1), 1-\beta(\varepsilon_2))\right)$$

$$= \left(\left\{x, \max\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right), \min\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right)\right\},\right.$$

$$\left.1-\min(\alpha(\varepsilon_1), \beta(\varepsilon_2))\right)$$

It follows that $((F_\alpha, A) \wedge (G_\beta, B))^c = (F_\alpha, A)^c \vee (G_\beta, B)^c$

2. Let $(F_\alpha, A) \vee (G_\beta, B) = (H_\gamma, A \times B)$, where $H_\gamma(\varepsilon_1,\varepsilon_2) = F_\alpha(\varepsilon_1) \cup G_\beta(\varepsilon_2)$, $\forall \varepsilon_1 \in A$ and $\forall \varepsilon_2 \in B$. Thus

$$H_\gamma(\varepsilon_1,\varepsilon_2) = F_\alpha(\varepsilon_1) \cup G_\beta(\varepsilon_2)$$

$$= \left(\left\{x, \max\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right), \min\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right)\right\},\right.$$

$$\left.\max(\alpha(\varepsilon_1), \beta(\varepsilon_2))\right)$$

Thus $((F_\alpha, A) \vee (G_\beta, B))^c = (H_\gamma, A \times B)^c = (H_\gamma{}^c, A \times B)$, where $\forall(\varepsilon_1,\varepsilon_2) \in A \times B$,

$$H_\gamma{}^c(\alpha,\beta) = (H_\gamma(\alpha,\beta))^c$$

$$= \left(\left\{x, \min\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right), \max\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right)\right\},\right.$$

$$\left.1-\max(\alpha(\varepsilon_1), \beta(\varepsilon_2))\right)$$

Let $(F_\alpha, A)^c \wedge (G_\beta, B)^c = (F_\alpha{}^c, A) \wedge (G_\beta{}^c, B) = (I_\delta, A \times B)$, where

$$I_\delta(\varepsilon_1,\varepsilon_2) = F_\alpha{}^c(\varepsilon_1) \cap G_\beta{}^c(\varepsilon_2), \quad \forall(\varepsilon_1,\varepsilon_2) \in A \times B$$

$$= \left(\left\{x, \min\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right), \max\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right)\right\},\right.$$

$$\left.\min(1-\alpha(\varepsilon_1), 1-\beta(\varepsilon_2))\right)$$

$$= \left(\left\{x, \min\left(\nu_{F(\varepsilon_1)}(x), \nu_{G(\varepsilon_2)}(x)\right), \max\left(\mu_{F(\varepsilon_1)}(x), \mu_{G(\varepsilon_2)}(x)\right)\right\},\right.$$

$$\left.1-\max(\alpha(\varepsilon_1), \beta(\varepsilon_2))\right)$$

It follows that $\left(\left(F_{\alpha}, A\right) \vee \left(G_{\beta}, B\right)\right)^{c} = \left(F_{\alpha}, A\right)^{c} \wedge \left(G_{\beta}, B\right)^{c}$

4. Conclusion

We have extended the notion of generalized intuitionistic fuzzy soft sets initiated by Dinda [2] with some modifications and some new results have been put forward in our work. It is hoped that our work will enhance this study in generalized intuitionistic fuzzy soft sets.

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